

Invention of the Plane geometrical formulae - Part III

Mr. Satish M. Kaple

Asst..teacher,Mahatma Phule High School Kherda Jalgaon (Jamod)-443402,Dist – Buldana
Maharashtra (India)

Abstract: In this paper , I have invented the first new type of a scalene quadrilateral and also developed the two new formulae for finding the area of that's new type of quadrilateral. My finding are based on phthagoras theorem.

I. Introduction

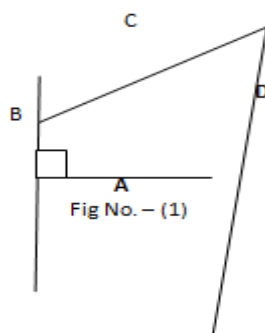
The types of quadrilaterals named on the basis of their angles and side are square, rectangle, rhombus, parallelogram, trapezium and kite respectively. These various types of quadrilaterals are also exist in educational curriculum.

I Have invented the first new type of a scalene quadrilateral and also developed the two new formulae for finding the area of it's scalene quadrilateral with the help of Pythagoras theorem.I Used Pythagoras theorem with geometrical figures and algebraic equations for the invention of the new type of a quadrilateral.

I Proved it by using geometrical formulae & figures, 10 examples & verifications (Proofs).
Here, myself is giving you the summary of the plane geometrical formulae-Part-III

First new type of a quadrilateral is as follows:-

1) **Scalene quadrilateral :-**



If the four sides of a quadrilateral are of different lengths. The lengths of all four sides of that quadrilateral are a, b, c and d respectively. The sides a,b, c and d are positive integers. The lengths of the first three consecutive sides(a, b, c) is a set of Pythagorean Triple. In Pythagorean Triple, a is the length of smallest side.

In this new type of quadrilateral , a is the length of smallest side also and the length fourth side (d) is twice the length of smallest side(d) . The smallest side and the fourth side are also adjacent sides.

Similarly , a and b are the lengths of both sides forming the right angle. It means that one angle of that's quadrilateral is a right angle means measure 90° . It's diagonals are not congruent.

A quadrilateral with all four sides of different lengths and one angle a right angle, then this quadrilateral is named as a scalene quadrilateral.

Hence, this scalene quadrilateral is the first new type of a quadrilateral.

Construction :-

Construct \square PQRS such that

$l(QR) = 3\text{cm}$, $l(PQ) = 4\text{ cm}$

$l(PS) = 5\text{ cm}$, $l(RS) = 6\text{ cm}$

and $m \angle PQR = 90^{\circ}$

Steps of construction

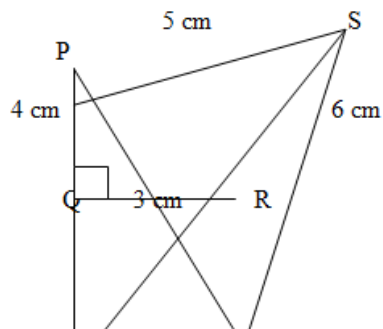


Fig No.- (2)

- 1) Seg QR is drawn measuring 3 cm .
 - 2) At Point Q of seg QR , a ray is drawn making an angle of 90^0 with it, as shown.
 - 3) On this ray, point P lies at a distance of 4 cm from point Q . Therefore, with point Q as the centre and radius of 4 cm , an arc is drawn to intersect the ray .The point of intersection is labeled as P.
 - 4) Draw an arc on one side of seg PQ with center P and radius 5 cm.
 - 5) Draw another arc with center R and radius 6 cm to cut the first arc.
- The Point of intersection is labeled as S.
- 6) Thus, the position of all four vertices of \square PQRS are fixed.
 - 7) Seg PS and seg RS are drawn and the required \square PQRS is obtained.
 - 8) Draw seg PR and seg QS.

Explanation :-

In \square PQRS ,
 $L(QR) = 3\text{ cm}$, $l(PQ) = 4\text{ cm}$
 $l(PS) = 5\text{ cm}$, $l(RS) = 6\text{ cm}$
 and $m \angle PQR = 90^0$

In above required \square PQRS ,

The four sides of a quadrilateral are of different lengths. The lengths of all four sides of \square PQRS are 3 cm, 4 cm, 5 cm and 6 cm respectively. The lengths of the first three consecutive sides (3 cm, 4 cm, 5 cm) is a set of Pythagorean Triple . In Pythagorean Triple , 3 cm is the length of smallest side.

In \square PQRS , 3 cm is the length of smallest side QR also and the length of fourth side RS (6cm) is double the length of smallest side QR (3cm).

Similarly , side PQ and side QR are both sides forming the right angle. It means that, one angle $\angle PQR$ of \square PQRS is a right angle means measure 90^0 .Seg PR and seg QS are the diagonals of required \square PQRS.

\square PQRS with all four sides of different lengths and one angle a right angle ,then this quadrilateral is a scalene quadrilateral.

Hence, \square PQRS is the first new type of a scalene quadrilateral.

Properties of a scalene quadrilateral

- 1) Four sides of a scalene quadrilateral are of different lengths.
- 2) Out of four angles , one angle is a right angle means measure of 90^0 .
- 3) The diagonals of a scalene quadrilateral are not congruent.
- 4) The diagonals of a scalene quadrilateral intersect each other. But they do not have perpendicular bisector with each other.
- 5) The diagonals of a scalene quadrilateral do not bisect each other.

Now using this above construction of \square PQRS ,

We shall obtain the new formulae for finding the area of a scalene quadrilateral (first new type).

Method :

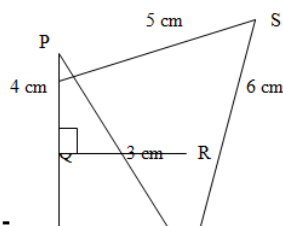
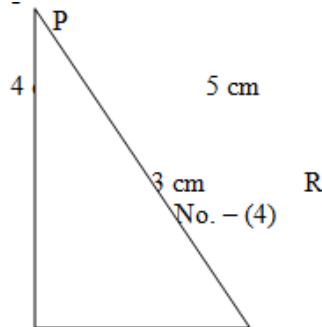


Fig No. - (3)

In \square PQRS, the diagonal PR is drawn. Due to the diagonal \square PR, PQRS is divided into two triangles, namely, \triangle PQR and \triangle PRS.

From \square PQRS,

1) Taking First \triangle PQR.



In \triangle PQR,

Seg PQ and seg QR are both sides forming the right angle. Seg PR is its hypotenuse.

$L(QR) = 3 \text{ cm}$, $l(PQ) = 4 \text{ cm}$

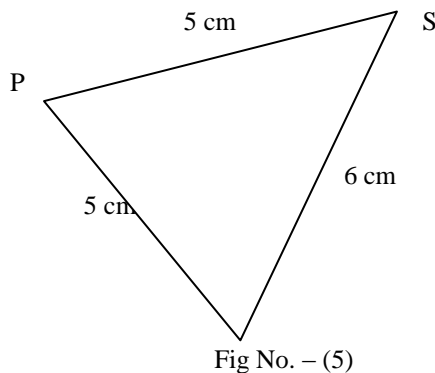
And $m \angle PQR = 90^\circ$

(3 cm, 4 cm, 5 cm) is a Pythagorean Triple.

Therefore, the length of hypotenuse is 5 cm.

$L(PR) = 5 \text{ cm}$.

II. Let us now taking \triangle PRS.



In \triangle PRS,

The length of seg PS and the length of seg PR are equal.

$L(PS) = l(PR) = 5 \text{ cm}$.

Hence, \triangle PRS is an isosceles triangle.

III. Let us now taking an isosceles triangle \triangle PRS.

Similarly,

In an isosceles triangle \triangle PRS, perpendicular PM is drawn on side RS from the vertex P.

By the length of perpendicular PM drawn on the side RS from the vertex P, two congruent right angled triangles are formed. These two congruent right angled triangles are namely \triangle PMR and \triangle PMS.

Due to the perpendicular PM drawn on side RS, side RS is divided into two equal segments. Therefore these two equal segments are seg MR and seg MS.

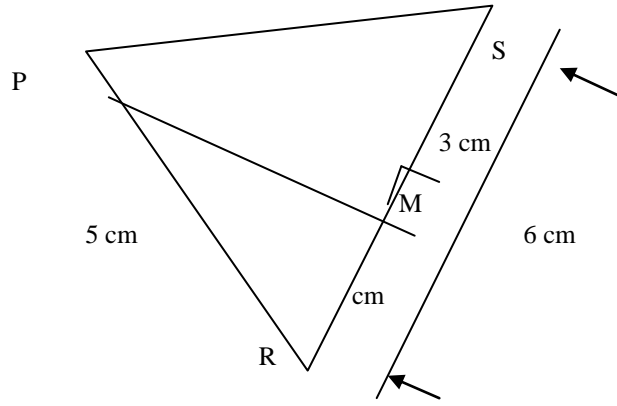


Fig No. - (6)

$L(MR) = l(MS) = \frac{1}{2} \times l(RS)$
 $l(MR) = l(MS) = \frac{1}{2} \times 6$
 $l(MR) = l(MS) = 3 \text{ cm.}$
 But the length of side QR of $\triangle PQR \cong \triangle S$ 3 cm
 $l(QR) = 3 \text{ cm}$
 $l(MR) = l(MS) = l(QR) = 3 \text{ cm}$

IV. Now taking a right angled triangle $\triangle PMR$.

In $\triangle PMR$
 Seg PM and seg MR are both sides forming the right angle. Seg PR is it's hypotenuse.
 $l(MR) = 3 \text{ cm}$, Hypotenuse, $l(PR) = 5 \text{ cm}$
 and $m \angle PMR = 90^\circ$

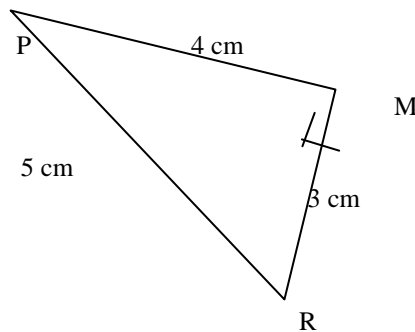


Fig No. - (7)

BY Using Pythagoras Theorem,

$PR^2 = PM^2 + MR^2$
 $PM^2 + MR^2 = PR^2$
 $PM^2 + 3^2 = 5^2$
 $PM^2 + 9 = 25$
 $PM^2 = 16$
 $PM^2 = 4^2$

Removing the square on both side
 $PM = 4 \text{ cm}$

In an isosceles $\triangle PRS$, \triangle

Seg PM is perpendicular to side RS. Therefore, PM is the height.

The length of perpendicular, PM and the length of side PQ of $\triangle PQR \cong \triangle S$ equal.

$l(PM) = 4 \text{ cm}$

From above information, formed two right angled triangles, namely, $\triangle PMR$ and $\triangle PMS$ are congruent to another right angled triangle, $\triangle PQR$. It means that, three congruent right angled triangles, namely,

$\triangle PMR, \triangle PMS$ and $\triangle PQR$ are also formed in $\triangle PQR$. \square
 $m \angle PQR = \angle PMR = \angle PMS = 90^\circ$

Generalization of a scalene quadrilateral (PQRS) □

Method :-

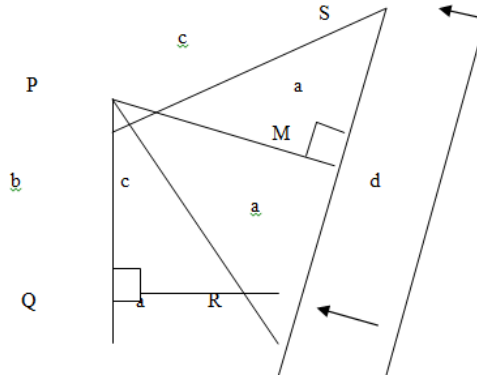


Fig No.- (8)

In □ PQRS , the lengths of all four sides are 3 cm , 4 cm , 5 cm, and 6 cm respectively. Suppose that , the lengths of all four sides are a, b, c and d respectively. The length of seg PS and length of seg PR are equal. Similarly, the length of seg PQ and the length seg PM are equal. The length of seg MR and the length MS are also equal.

Here taking,

$$l(QR) = 3 \text{ cm} = a$$

$$l(PQ) = 4 \text{ cm} = b$$

$$l(PS) = 5 \text{ cm} = c$$

$$l(RS) = 6 \text{ cm} = d$$

But,

$$l(PS) = l(PR) = 5 \text{ cm}$$

$$l(PR) = 5 \text{ cm} = c$$

similarly,

$$l(QR) = l(MR) = l(MS) = 3 \text{ cm}$$

$$l(MR) = l(MS) = 3 \text{ cm} = a$$

and,

$$l(PQ) = l(PM) = 4 \text{ cm}$$

$$l(PM) = 4 \text{ cm} = b$$

From PQR □

Step (1)Taking first a right angled triangle PQR △

In PQR △

$$l(QR) = a, l(PQ) = b, l(PR) = c$$

$$m \angle PQR = 90^\circ$$

Side PQ and side QR are both sides forming the right angle. Seg PR is it hypotenuse.

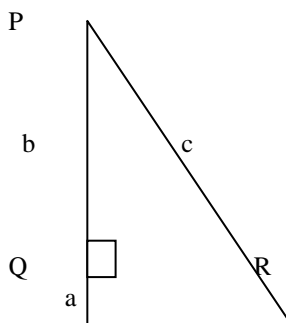


Fig No. - (9)

Area of PQR △ (PQR △)

$$= \frac{1}{2} \times \text{Product of the lengths of sides forming the right angle.}$$

$$\begin{aligned}
 &= \frac{1}{2} \times \text{One side forming the right angle} \times \text{Other side forming the right angle} \\
 &= \frac{1}{2} \times l(QR) \times l(PQ) \\
 &= \frac{1}{2} \times a \times b \\
 &= ab/2 \\
 \text{Area of } \triangle PQR &= \frac{ab}{2} \dots \dots \dots (1)
 \end{aligned}$$

Step (2) Let us now taking an isosceles triangle $\triangle PRS$

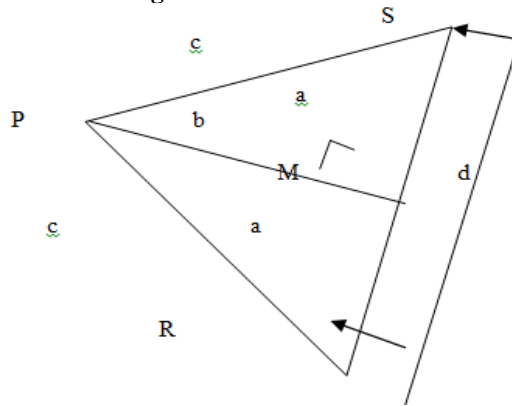


Fig No. – (10)

In $\triangle PRS$,
 The length of seg PS and the length of seg PR are equal.
 Similarly,
 The length of seg MR and the length of seg MS are equal. $\triangle PMR$ and $\triangle PMS$ are right angled triangles.
 These two right angled triangles are congruent also. Seg PM is the height and side RS is the base.
 Height , $l(PM) = b$
 Base , $l(RS) = d$
 Area of $\triangle PRS = \frac{1}{2} \times \text{Base} \times \text{height}$
 $= \frac{1}{2} \times \text{Base} \times \text{height}$
 $= \frac{1}{2} \times l(RS) \times l(PM)$
 $= \frac{1}{2} \times d \times b$
 $= db/2$
 Area of $\triangle PRS = \frac{db}{2} \dots \dots \dots (2)$

From equation no. (1) and (2)
 Area of $\square PQRS = \text{Area}(\triangle PQR) + \text{Area}(\triangle PRS)$
 $= ab/2 + db/2$
 $= b/2 (a + d)$
 Area of $\square PQRS = b/2 (a + d)$

New Formula no . (1)

Area of a scalene quadrilateral = $b/2 (a + d)$
 From an isosceles triangle $\triangle PRS$,

Step (3) (A) now taking a right angled triangle $\triangle PMR$
 In $\triangle PMR$
 Seg PM and seg MR are both sides forming the right angle. Seg PR is it's hypotenuse.
 $l(MR) = a$, $l(PM) = b$ $l(PR) = c$ and $m \angle PMR = 90^\circ$

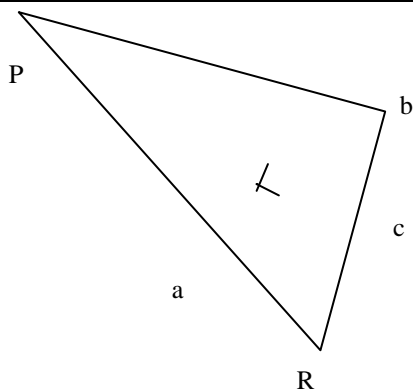


Fig. No - (11)

Area of $\triangle PMR$ ($\triangle PMR$)
 = $\frac{1}{2}$ x Product of the lengths of the sides forming the right angle.
 = $\frac{1}{2}$ x One side forming the right angle x other side forming the right angle.
 = $\frac{1}{2}$ x l(MR) x l(PM)
 = $\frac{1}{2}$ x a x b
 = $ab/2$
 Area of $\triangle PMR = ab/2$ -----(3)

(B) Taking a right angled triangle $\triangle PMS$ also

Similarly, in $\triangle PMS$, \triangle
 Seg PM and seg MS are both sides forming the right angle. Seg PS is it's hypotenuse.
 l(MS) = a, l(PM) = b, l(PS) = c
 and $\angle PMS = 90^\circ$

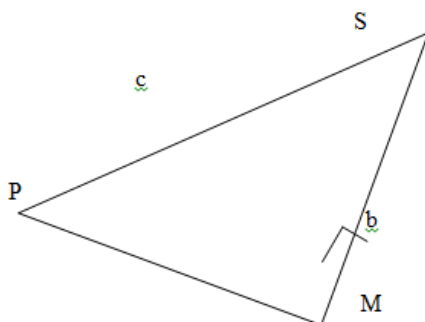


Fig No. - (12)

Fig No. - (12)

Area of $\triangle PMS = A(\triangle PMS)$
 = $\frac{1}{2}$ x product of \triangle gths of the \triangle des forming the right angle.
 = $\frac{1}{2}$ x one side forming the right angle x other side forming the right angle.
 = $\frac{1}{2}$ x l(MS) x l(PM)
 = $\frac{1}{2}$ x a x b
 = $ab/2$

Area of $\triangle PMS = ab/2$ -----(4)

From equation (3) and (4)

Area of $\triangle PRS = \triangle(\triangle PMR) + A(\triangle PMS)$
 = $ab/2 + ab/2$
 = $ab + ab/2$
 = $2ab/2$
 = ab

Area of $\triangle PRS = ab$ -----(5)

From equations no. (1) and (5)

Area of $\square PQRS = A(\triangle PQR) + A(\triangle PRS)$
 = $ab/2 + ab$

$$\begin{aligned}
 &= ab/2 + 2ab/2 \\
 &= ab+2ab/2 \\
 &= 3ab/2 \\
 &= 3/2 ab \\
 \text{Area of } \square PQRS &= 3/2 ab
 \end{aligned}$$

New Formula no. (2)

Area of scalene quadrilateral = 3/2 ab

Now consider the following example :-

Ex. (1) Find the area of $\square ABCD$.

In $\square ABCD$, $l(BC) = 6$ cm.

$l(AB) = 8$ cm $l(AD) = 10$ cm,

$l(CD) = 12$ cm and $m \sphericalangle ABC = 90^\circ$

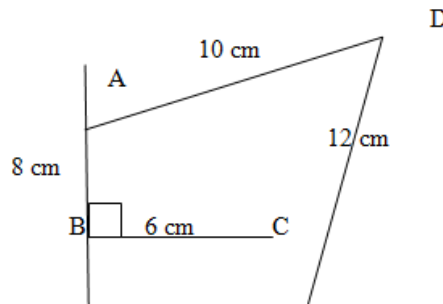


Fig No. - (13)

$\square ABCD$ is scalene quadrilateral.

In $\square ABCD$,

Out of the four sides, the length of first three consecutive sides (6cm, 8cm, 10cm) is a Pythagorean Triple.

And, the length of fourth side CD (12cm) is double the length of the smallest side BC (6cm)

Let, $a = 6$ cm, $b = 8$ cm, $c = 10$ cm and $d = 12$ cm

By using the new formula no (1)

$$\text{Area of } \square ABCD = (\square ABCD)$$

$$= b/2(a + d)$$

$$= 8/2(6 + 12)$$

The simplest form of $8/2$ is 4

$$= 4 \times 18 = 72 \text{ sq.cm.}$$

$$\text{Area of } \square ABCD = 72 \text{ sq.cm}$$

By using the new formula no. (2)

$$\text{Area of } \square ABCD = A(\square ABCD)$$

$$= 3/2 ab$$

$$= 3/2 \times 6 \times 8$$

$$= 3 \times 6 \times 8/2$$

$$= 3 \times 48/2$$

$$= 3 \times 48/2$$

The simplest form of $48/2$ is 24

$$= 3 \times 24 = 72 \text{sq.cm}$$

$$\text{Area of } \square ABCD = 72 \text{ sq.cm}$$

Verification :-

To find the area of $\square ABCD$,

Diagonal AC is drawn, so that this quadrilateral is divided into convenient two triangles, namely $\triangle ABC$ and $\triangle ACD$.

1) Let us first find the area of $\triangle ABC$

Area of a right angled triangle ABC

$$= A(\triangle ABC)$$

$$= 1/2 \times \text{Product of the lengths of the sides forming the right angle.}$$

$$= 1/2 \times ab$$

$$\begin{aligned}
 &= \frac{1}{2} \times 6 \times 8 \\
 &= 6 \times 8/2 \\
 &= 48/2 \\
 &= 24 \text{ sq.cm} \\
 \text{Area of } \triangle ABC &= 24 \text{ sq.cm}
 \end{aligned}$$

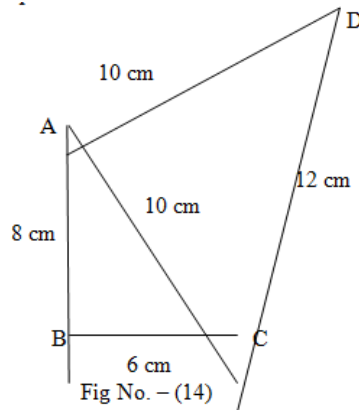


Fig No. - (14)

2) Let us now find the area of $\triangle ACD$ in a right angled triangle ABC ,

By using Pythagoras theorem

$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 \\
 &= 8^2 + 6^2 \\
 &= 64 + 36
 \end{aligned}$$

$$AC^2 = 100$$

$$AC = 10^2$$

Removing the square on both sides,

$$AC = 10 \text{ cm}$$

In $\triangle ACD$, $l(AC) = l(AD) = 10 \text{ cm}$ and $l(CD) = 12 \text{ cm}$

$\triangle ACD$ is an isosceles triangle.

By using the formula of Heron's

$$\text{Perimeter of } \triangle ACD = 10+12+10 = 32 \text{ cm.}$$

$$\text{Semiperimeter of } \triangle ACD, s = 32/2 = 16 \text{ cm}$$

$$\begin{aligned}
 \text{Area of } \triangle ACD &= \sqrt{16(16-10)(16-12)(16-10)} \\
 &= \sqrt{16 \times 6 \times 4 \times 6} \\
 &= \sqrt{(16 \times 4) \times (6 \times 6)} \\
 &= \sqrt{64 \times 36} \\
 &= \sqrt{64} \times \sqrt{36}
 \end{aligned}$$

The square root of 64 is 8 and the square root of 36 is 6

$$= 8 \times 6 = 48 \text{ sq.cm.}$$

$$\text{Area of } \triangle ACD = 48 \text{ sq.cm.}$$

$$\text{Area of } \triangle ACD = 48 \text{ sq.cm.}$$

$$\text{Area of } \square ABCD = A(\triangle ABC) + A(\triangle ACD)$$

$$= 24 + 48$$

$$= 72 \text{ sq.cm.}$$

$$\text{Area of } \square ABCD = 72 \text{ sq.cm.}$$

V. Explanation :-

This above example are solved using the new formulae No. (1) and (2) for finding the area of new quadrilateral.

From above solved example and it's verification, we note that – the value of solved example and the value of it's verification are equal.

Hence, the new formulae No (1) and (2) are proved.

VI. Conclusion :-

These above two new formulae are invented for finding the area of a scalene quadrilateral.

New formulae no. (1) and (2) are as follows :-

Area of PQRS = $\frac{b}{2} (a + d)$

New formula no (1)

Area of a scalene quadrilateral = $\frac{b}{2} (a + d)$

OR

Area of PQRS = $\frac{3}{2} ab$

New formula no. (2)

Area of a scalene quadrilateral = $\frac{3}{2} ab$

From above two new formulae, we can find out the area of a scalene quadrilateral of first new type. Out of two new formulae, any one formula can use to find the area of such quadrilateral.

These new formulae are useful in educational curriculum , building and bridge construction and department of land records.

References:-

- [1]. Geometry concept & Pythagoras theorem.