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Graceful V^*2F_n-tree

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Abstract: *The concept of graceful labeling was introduced by Solomon Wolf Golomb (May 30, 1932). In this paper we discuss graph and its graceful labeling. Finally we show that V^*2F_n is graceful for an n.*
Keywords: *Graceful tree, graceful labeling, root, stem, branch, leaf, flower.*

I. Introduction

The last two decades have witnessed an upsurge of interest and activity in graph theory, particularly among applied mathematicians and engineers. The past 30 years has been a period of intense activity in graph theory both pure and applied. A great deal of research has been done and is being done in this area.

Graph theory is one of the most flourishing branches of mathematics with wide application to combinational problems. Graphs are usually represented by diagrams using a point for each vertex and a line for each edge. A graph G is an ordered triple of $(V(G), E(G), \psi)$. If e is an edge and $\psi(e) = (u, v)$, then we say that e is an edge joining u and v and the vertices u and v are called the ends of e. Now label each edge with the absolute difference of the endpoints of the concerned edge. The labeling is graceful, if the edges are labeled $1, 2, \ldots, n$ inclusive.

A graceful labeling on a graph with p vertices and q edges is a one to one map taking the vertices into the integers $0, 1, \ldots. q$ with the property that each edge uv is assigned by the label $|f(u)-f(v)|$.

A graceful graph is a graph that has atleast one graceful labeling.

1.1 Definition

A *walk* is a list $v_0,e_1,v_1,e_2,\ldots,e_k,v_k$ of vertices and edges such that, for $1 \le i \le k$, the edge e_i has end points v_{i-1} and v_i. A *trail* is a walk with no repeated edge. A u, v-walk or u, v-trail has first vertex u and last vertex v; that is u. v are its endpoints. A *u, v-path* is a path whose vertices of degree 1(its *endpoints are u and v*); the other vertices are internal vertices and of degree 2. The *length* of a walk, trail, path, or cycle is its number of edges. A walk or trail is closed if its endpoints are the same.

1.2 Example

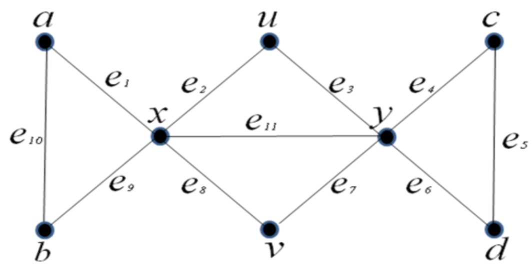

Fig.1: Path graph

In the above graph $a,e_1,x,e_2,u,e_3,y,e_4,c,e_5,d,e_6,y,e_7,v,e_8,x,e_9,b,e_{10},a$ specifies a closed walk of length 10. Omitting the first two steps yields a closed trail and it has five cycle (a,b,x),(c,y,d),(u,x,y),(x,y,v),(u,x,v,y)

1.3 Definition

A graph G is *connected* if there is a path between every pair of vertices.

1.4 Definition

A graph with no cycle is *acyclic*. Collection of trees is called forest. A *tree* is a connected acyclic graph. A *leaf* in a tree (or *pendent vertex*) is a vertex of degree 1.

1.5 Definition

If in a tree there is one vertex which is distinguished from all other vertices then the vertex is called *root* and the tree is called a *rooted tree* and also the root is denoted by *r*.

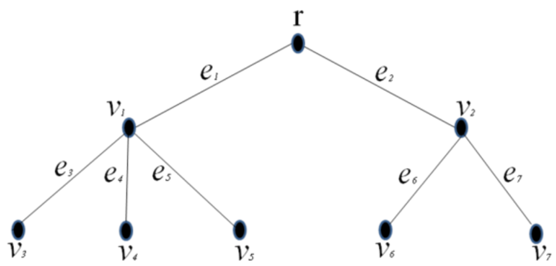

Fig.2: Rooted tree

1.6 Definition

A *binary tree* is a rooted tree where each vertex has atmost two children.

1.7 Definition

A *star* is a special kind of tree consisting of one vertex adjacent to all the other vertices and a star is denoted by $K_{1,\, n-1}$.

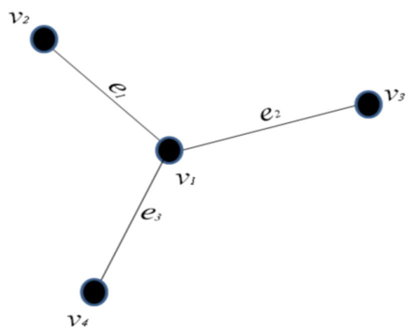

Fig.3: $K_{1,3}$ – Star graph

1.8 Definition

A *matching* in a graph is a subset of edges in which no two edges are adjacent. The vertices incident to the edges of a matching *M* are *saturated* by *M;* the others are *unsaturated* (we say *M*-saturated and *M*-unsaturated vertices)

1.9 Definition

A *perfect matching* in a graph is a matching that saturates every vertex of a graph *G*.

1.10 Definition

Given a matching *M*, an *M-alternating path* is a path that alternates between edges in *M* and edges not in *M*. An *M-alternating path* whose endpoints are unsaturated by *M* is an *M*-augmenting.

1.11 Definition

Let *M* be a matching in a graph *G*, and let *u* be an *M*-unsaturated vertex. A *flower* is the union of two *M*-alternating paths from *u* that reach a vertex *x* on steps of opposite parity. The *stem* of the flower is the maximal common initial path (of non negative even length).The *blossom* of the flower is the odd cycle obtained by deleting the stem.

1.12 Theorem (Berge)

A matching *M* in *G* has maximum size if and only if *G* has no *M*-augmenting Path.

1.13 Example

In the graph below, the matching M indicated in bold, a search for shortest M-augmenting paths from u reaches x via the unsaturated edge ax. If we do not also consider a longer path reaching x via a saturated edge, then we miss the augmenting path $u,e_1,v,e_2,a,e_3,b,e_4,c,e_5,d,e_6,e,e_7,f,e_8,g,e_9,h,e_{10},i,e_{11},j,e_{12},x,e_{14},y$.

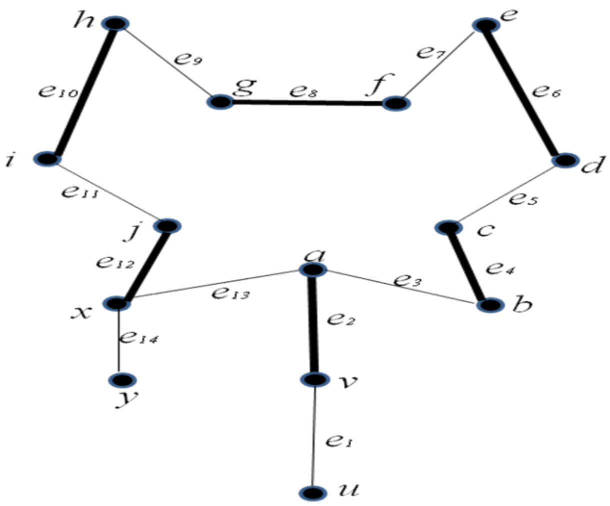

Fig.4: Flower graph

If an exploration of M-alternating paths from u reaches a vertex x by an unsaturated edge in one path and by a saturated edge in another path, then x belongs to an odd cycle. Alternating paths from u can diverge only when the next edge is unsaturated; when the next edge is saturated there is only one choice for it.

P_1 and P_2 are different paths from u reaches x and its union makes a flower.

$P_1 = \{ u,e_1,v,e_2,a,e_3,b,e_4,c,e_5,d,e_6,e,e_7,f,e_8,g,e_9,h,e_{10},i,e_{11},j,e_{12},x \}$
$P_2 = \{ u.e_1,v,e_2,a,e_{13},x\}$

$P = P_1 \cup P_2$
$= \text{flower}$

1.14 Definition

A *graceful labeling* of a graph G with m edges is a function $f : v(G) \rightarrow \{0, \ldots. m\}$ such that, distinct vertices receive distinct numbers and $\{|f(x) - f(y)| : xy \in E(G)\} = \{1, \ldots m\}$ and a graph is graceful, if it has a *graceful labeling*.

1.15 Definition

A *rooted tree* (*Bi-tree*) is a V-tree if it has only two branches which are leaves.

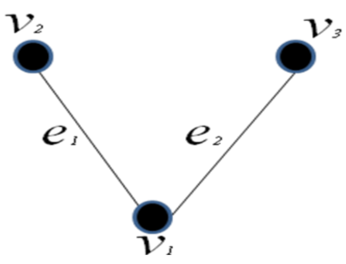

Fig.5 : V-tree

1.16 Definition

The V^*2F_n is a V-tree, it combined with n-stars in each of the two branches.

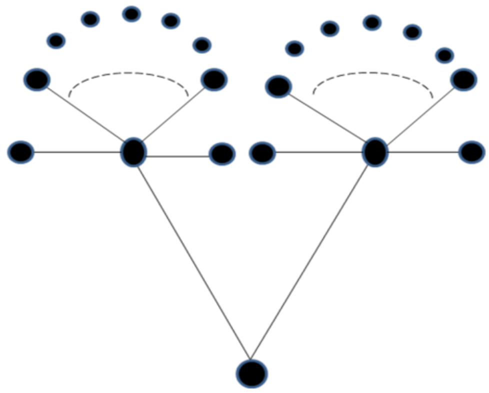

Fig. 6(a) : V^*2F_n

1.17 Theorem

The tree V^*2F_n is graceful.

Proof:

Let the tree V^*2F_n be obtained by merging n copies of stars F_n with V- tree, which is given as bellow.

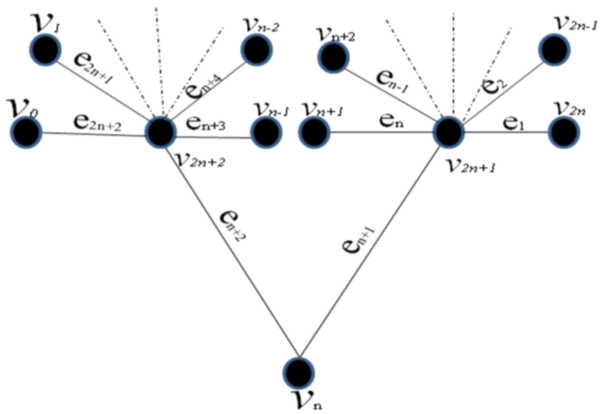

Fig. 6(b) : Graceful labeling V^*2F_n

Let $p = 2n + 3$ be the number of vertices and $q = 2n + 2$ be the number of edges in the V^*2F_n - tree. Now, define a labeling f on the vertex set of V^*2F_n by $f(v_i) = i$ such that $i = 0$ to $2n + 2$. **→ (1)**

The edge set labeling e is defined by $e(uv) = |f(u) - f(v)|$, for any edge uv in the tree V^*2F_n. Thus the given tree with the vertex labeling f and edge labeling e becomes a graceful tree. Hence V^*2F_n is a graceful tree.

1.18 Example

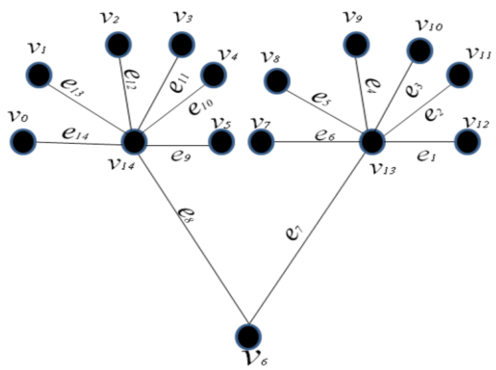

Fig.6(c) : Graceful labeling V^*2F_6

Let $p = 15$, $q = 14$ be the number of vertices and edges respectively and $n = 6$ be the number of stars merges in each branch of the V- *tree*. Define a vertex labeling 'f' on the V^*2F_6 by $f(v_i) = i$, $i = 0$ to 14 and edge labeling by $e(uv) = |f(u) - f(v)|$ for any edge uv in the tree, V^*2F_6.

1.19 Theorem:

V^*2F_n is graceful if n is even.

Proof:

Let the tree V^*2F_n be obtained by merging n copies of stars F_n with V - tree, which is given as bellow

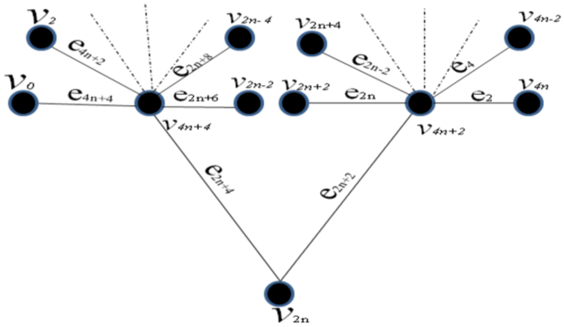

Fig.7(a) : Even graceful labeling V^*2F_n

Let $p = 4n+5$ be the number of vertices labeling by even integers and $q = 4n+4$ be the number of edges labeling by even integers in the V^*2F_n - tree. Now, define a labeling f on the vertex set of V^*2F_n by $f(v_i) = 2i$ such that $i = 0$ to $4n+4$ $\rightarrow$ **(2)**

The edge set labeling e is defined by $e(uv) = |f(u) - f(v)|$, for any edge uv in the tree V^*2F_n.Thus the given tree with the vertex labeling f and edge labeling e becomes a graceful tree if n is even. Hence V^*2F_n is a graceful tree.

1.20 Example

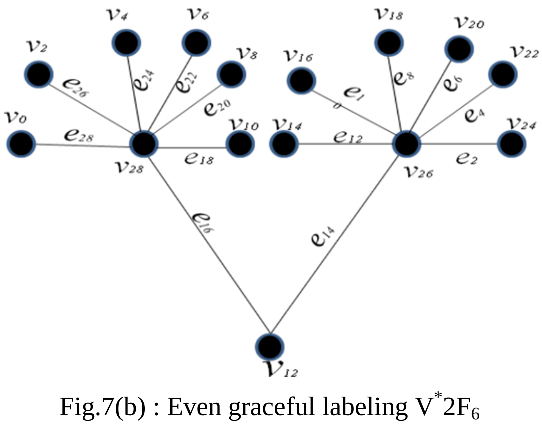

Fig.7(b) : Even graceful labeling V^*2F_6

Let $p = 15$, $q = 14$ and p, q are having only even number of vertices and edges respectively and $n = 6$ be the number of stars merges in each branch of the V- *tree*. Define a vertex labeling 'f' on the V^*2F_6 by $f(v_i) = 2i$, $i = 0$ to 28 and edge labeling by $e(uv) = |f(u) - f(v)|$ for any edge uv in the tree, V^*2F_6.

1.21 Theorem:

V^*2F_n is graceful if n is odd.

Proof:

Let the tree V^*2F_n be obtained by merging n copies of stars F_n with V - tree, which is shown as below.

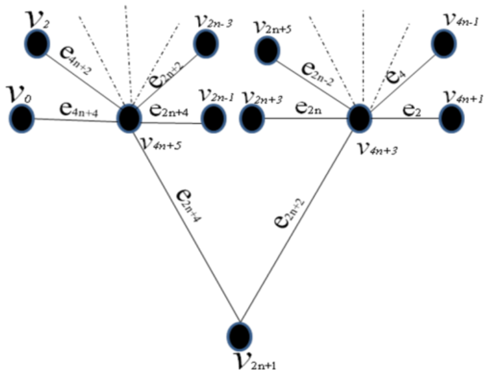

Fig.8(a) : Odd graceful labeling V^*2F_n

Let p be the number of vertices with odd integers of labeling and q be the number of edges with even number of labeling in V^*2F_n - tree. According the definition of graceful labeling f on the vertex set of V^*2F_n by $f(v_i) = 2i+1$ such that $i = 0$ to $4n+4$ → **(3)**

The edge set labeling e is defined by $e(uv) = |f(u) - f(v)|$, for any edge uv in the tree V^*2F_n.Thus the given tree with the vertex labeling f and edge labeling e becomes a graceful tree if n is odd. Hence V^*2F_n is a graceful tree.

1.22 Example

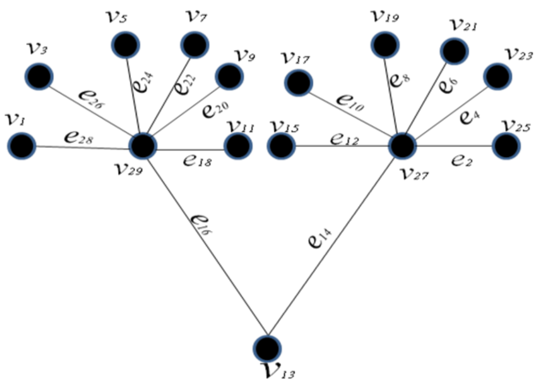

Fig.8(b) : Odd graceful labeling V^*2F_6

Let p and q be the number of vertices and edges. The vertices are labeling by odd integers and the edges are labeling by even numbers respectively. Because the difference between two odd integers is even and if $n = 6$ be the number of stars in each branch of the V-tree. Define a vertex labeling 'f' on the V^*2F_6 by $f(v_i) = 2i + 1$, $i = 0$ to 28 and edge labeling by $e(uv) = |f(u) - f(v)|$ for any edge uv in the tree, V^*2F_6 is graceful tree.

II. Conclusion

In this paper we have proved that V^*2F_n is graceful when n is odd or even. So we conclude that V^*2F_n is graceful for an n.

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