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Operational techniques upon certain single integral transform

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***Abstract**: The aim of this paper is to evaluate certain results on integral transform by using operational technique, Next the result obtained here are quite general in nature due to presence of operational technique which are basic in nature. A large number of new results and special cases have seen obtained by operational technique using proper choice of parameters.*
***Keywords**: Laplace transform, Generalized Hypergeometric functions, integral Laplace transform, Inverse Laplace transform, Lagurre Polynomials, inverse Laplace transform integral*

I. Introduction

We define two base points in favors of some main problems on integral transform such that sequentially continuous and Laplace transformation, see ref. [1]. A function $f(t)$ is called sequentially continuous or piecewise continues in any interval $[a,b]$, If it is continuous and has finite left and right hand limits in every sub-intervals of $[a,b]$.

A function $f(x)$ is said to be of exponential order as $t \to \infty$

(1.1) $$if \lim_{n\to\infty} (e)^{-\alpha t} f(t) = finit\ quantity$$

We say that $f(t)$ is of exponential order. Let us function $g(t)$ is piecewise continuous on the closed interval $0 \le t \le T\ for\ every\ T \ge 0\ also$

(1.2) $$let\ f(t) = o(e)^{\alpha t}\ as\ t \to \infty\ for\ some\ \alpha$$

Also see ref. [1], the Laplace transformation of a function $f(t)$ is define for all real numbers $t \ge 0$ is $F(s)$ defined by

(1.3) $$F(s) = \mathcal{L}\{f(t):s\} = \int_0^\infty e^{-st} f(t)dt, \qquad Re(s) > \alpha$$

Where the parameter S is complex number, $S = \sigma + \boldsymbol{i}\omega$ with real numbers ω & σ

We will obtain special function in the classical Laplace transformation by appealing to Euler's transformation (see ref. [1], cf. 1.1(1))

(1.4) $$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt, \qquad Re(z) > 0$$

(1.5) $$therfore \int_0^\infty t^{\lambda-1} e^{-st} dt = \frac{\Gamma(\lambda)}{S^\lambda}, \qquad \min\{Re(\lambda), Re(s)\} > 0$$

On the other hand, assuming $f(t)$ to be continuous for each $t \ge 0$ and to satisfy (1.2), computation of the inverse Laplace transform

(1.6) $$f(t) = \mathcal{L}^{-1}\{F(t):s\} = \frac{1}{2\pi i}\int_{\sigma-iT}^{\sigma+iT} e^{st} F(s)ds$$

We can also write above

(1.7) $$f(t) = \mathcal{L}^{-1}\{F(t):s\} = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{st} F(s)ds, \qquad \sigma > \alpha$$

Is usually based upon Hankel's contour integral in the equivalent form see [Watson (1927), P. 245] and also Luke ([2], (1959). Vol.1, P.17, Eq. (5))

(1.8) $$\frac{1}{\Gamma(z)} = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} e^s s^{-z} ds, \qquad \sigma > 0,\ Re(z) > 0$$

II. Main results

Our aim in this research paper is to solve some results on integral transform using operational technique and proceeding on further. Thus it has seen some main results due to application of the integral formula (1.5) and (1.8) to the case of the generalized hypergeometric ${}_pF_q$ function readily yield the operational relation see Eede′lyi et al.(ref. [3], 1954, Vol.1, P. 219, Eq. (17)) are present as

(2.1) $$\mathcal{L}\left\{ t^{\lambda-1}\, {}_pF_q \begin{bmatrix} \alpha_1,\dots,\alpha_p; & zt \\ \beta_1,\dots,\beta_q; & \end{bmatrix} : s \right\} = \frac{\Gamma(\lambda)}{S^\lambda}\, {}_{p+1}F_q \begin{bmatrix} \lambda,\alpha_1,\dots,\alpha_p; & \frac{z}{s} \\ \beta_1,\dots,\beta_q; & \end{bmatrix}$$

Where $Re(\lambda) > 0, p \leq q, Re(s) > 0\ if\ p < q;\ Re(s) > Re(z)\ if\ p = q$ and see (ref. [1], P. 297, Eq. (1))

(2.2) $$\mathcal{L}^{-1}\left\{ S^{-\lambda}\ {}_pF_q \begin{bmatrix} \alpha_1, \dots, \alpha_{p;} & z \\ \beta_1, \dots, \beta_q; & \frac{}{s} \end{bmatrix} : t \right\} = \frac{t^{\lambda-1}}{\Gamma(\lambda)}\ {}_pF_{q+1}\begin{bmatrix} \alpha_1, \dots, \alpha_{p;} & zt \\ \lambda, \beta_1, \dots, \beta_q; & \end{bmatrix}$$

Where $Re(\lambda) > 0,\ p \leq q + 1$

In term of Lagurre polynomials $L_n^{(\alpha)}(x)$ see (ref. [1], 1.3 (34)) yields the operational relationship Erde'lyi et al. (1954), Vol.1, P. 175, Eq. (3))

(2.3) $$\mathcal{L}^{-1}\left\{\frac{(S-x-\zeta)^n}{(S-\zeta)^{\alpha+n+1}} : t\right\} = \frac{n!\ t^\alpha}{\Gamma(\alpha+n+1)} e^{t\zeta} L_n^{(\alpha)}(xt)$$

Where $Re(\alpha) > -1,\ Re(S-\zeta) > 0, n = 0,1,2, \dots$

While (2.1) $with\ (p = q = 1,\ \alpha_1 = -n,\ \beta_1 = \alpha + 1)$ reduce to Erde'lyi et al. [(1953), vol. II, P. 19, eq. (33)]

(2.4) $$\mathcal{L}\left\{ t^{\lambda-1} L_n^{(\alpha)}(xt) : s\right\} = \binom{\alpha+n}{n}\frac{\Gamma(\lambda)}{S^\lambda}\ {}_2F_1\begin{bmatrix} -n,\ \lambda; & \frac{x}{s} \\ \alpha+1; & \end{bmatrix}$$

Where $Re(\alpha) > 0, Re(s) > 0$

Setting $\lambda = \alpha + \beta + n + 1$ and replacing $x\ by\ \frac{1}{2}(1-x)\ thus\ (2.4)\ can\ be\ rewritten\ as$

(2.5) $$\mathcal{L}\left\{ t^{\alpha+\beta+n} L_n^{(\alpha)}\left(\frac{1}{2}(1-x)t\right) : s\right\} = \frac{\Gamma(\alpha+\beta+n+1)}{S^{\alpha+\beta+n+1}}\ P_n^{(\alpha,\beta)}\left(\frac{x+s-1}{s}\right)$$

Where $Re(\alpha+\beta) > -1,\ Re(S) > 0$

III. Proof of Main results

Proof of (2.1) lets us assume first clearly

(3.1) $$S = \mathcal{L}\left\{ t^{\lambda-1}\ {}_pF_q \begin{bmatrix} \alpha_1, \dots, \alpha_{p;} & zt \\ \beta_1, \dots, \beta_q; & \end{bmatrix} : s\right\}$$

Thus in view of (1.3) and using generalized hypergeometric function see (ref. [1], PP. 42, 1.4(1)) is defined by

(3.2) $${}_pF_q\begin{bmatrix} \alpha_1, \dots, \alpha_{p;} & z \\ \beta_1, \dots, \beta_q; & \end{bmatrix} = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \dots (\alpha_p)_n}{(\beta_1)_n \dots (\beta_q)_n} \cdot \frac{z^n}{n!},\ p, q\epsilon z^+$$

Where

(3.3) $\alpha_{i,} \beta_j \in C, \beta_1 \neq 0, -1, -2, \dots\ and\ i = 1,2,3, \dots, p\ ; j = 1,2,3, \dots, q$

In above results (3.1) can be rewritten as

(3.4) $$= \int_0^\infty t^{\lambda-1} e^{-st} \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \dots (\alpha_p)_n}{(\beta_1)_n \dots (\beta_q)_n} \cdot \frac{(zt)^n}{n!} dt,$$

Again in view of (1.5) and using one more formula see ref. [1] which is defined by

(3.5) $\Gamma(\lambda + n) = (\lambda)_n \Gamma(\lambda) in\ above\ result\ (3.4) can\ br\ represent\ as$

(3.6) $$= \frac{\Gamma(\lambda)}{S^\lambda} \sum_{n=0}^{\infty} \frac{(\lambda)_n (\alpha_1)_n \dots (\alpha_p)_n}{(\beta_1)_n \dots (\beta_q)_n} \left(\frac{z}{s}\right)^n, Re(\lambda) > 0, Re(s) > 0$$

Lastly from generalized hypergeometric function which is defined in (3.2) use in above results (3.6). Hence which gives the complete proof of (2.1)

Similarly proof of each results (2.2) to (2.5), we first consider L.H.S. in each results like as (2.1) then involving (3.2), (1.3), (3.5), (1.5), [(3.2) with $p = q = 2, t = 1$] and change the order of integration & summation in each results except the results of (2.2) and (2.3)[here we use only (1.7), (2.3), (1.8) $and\ setting\ ST = Z \Rightarrow Tds = dZ$]], therefore lastly to use one more formula for the proof of each results (2.2) to (2.5) see ref. [1,4] which is defined by

(3.7) $$(1-z)^{-\alpha} = \sum_{n=0}^{\infty} \frac{(\alpha)_n z^n}{n!}$$

(3.8) $$P_n^{(\alpha,\beta)}(x) = \frac{(\alpha+1)_n}{n!}\ {}_2F_1\begin{bmatrix} -n,\ \alpha+\beta+n+1; & \frac{1-x}{s} \\ \alpha+1; & \end{bmatrix}$$

(3.9) $$L_n^{(\alpha)}(x) = \frac{(1+\alpha)_n}{n!}\ {}_2F_1\begin{bmatrix} -n,\ ; & x \\ \alpha+1; & \end{bmatrix}$$

IV. Special cases

In order to obtain special cases from the main results with operational relationship (2.1) is worthy of see (ref. [3], vol. 1, P. 217, eq. (1))

(4.1) $$\mathcal{L}\left\{ t^{C-1}\ {}_1F_1\begin{bmatrix} a; & zt \\ c; & \end{bmatrix} : s\right\} = \Gamma(c)\, S^{-c}\left(1 - \frac{z}{s}\right)^{-a}$$

Where $\min\{Re(z), Re(s), Re(c)\} > 0$

While upon a simple change of variables in above results becomes as

(4.2) $$\mathcal{L}\left\{ t^{C-1} e^{\zeta t} {}_1F_1 \begin{bmatrix} a; & \\ c; & xt \end{bmatrix} : s \right\} = \Gamma(c)\,(S-\zeta)^{-c}\left(1 - \frac{x}{s-\zeta}\right)^{-a}$$

Where $\min\{Re(x), Re(S-\zeta), Re(c)\} > 0$

(4.3) $$\mathcal{L}^{-1}\left\{ (S-\zeta)^{-c}\left(1 - \frac{x}{s-\zeta}\right)^{-a} : t \right\} = \frac{t}{\Gamma(c)}^{C-1} e^{\zeta t} {}_1F_1 \begin{bmatrix} a; & \\ c; & xt \end{bmatrix}$$

Where $\min\{Re(x), Re(S-\zeta), Re(c)\} > 0$

Proof of special cases (4.1) to (4.3) is much akin to that of main results (2.1) to (2.5), which we have already presented in a reasonably detailed manner, but lastly we use another one more formula for the proof of special case (4.4) see ref. [1] defined by

(4.4) $$\sum_{n=0}^{\infty} \frac{z^n}{n!} = e^z$$

Accoundingly to obtain more special cases from main results in this paper

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