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Some Characteristics on Join of Intuitionistic Fuzzy Graphs

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ABSTRACT: *In this paper, we derived some results like the join of two complete Intuitionistic Fuzzy G raphs (IFG) is complete and which is isomorphic to the join of their complements. The nature of edge set in the complement of a complete IFG is analyzed. We study about the join of two IFGs when they are regular, irregular or complete and discuss some theorems. Also we discuss some more properties of the join of two intuitionistic fuzzy graphs using the regularity and irregularity. The minimum and maximum degrees of an IFG and its complement are examined.*

Keywords: *Intuitionistic Fuzzy Graph (IFG), degree, total degree, Complete IFG, regular IFG, irregular IFG, neighbourly irregular IFG, highly irregular IFG, Complement, Join of two IFG.*

I. INTRODUCTION

Intuitionistic fuzzy sets are generalization of fuzzy sets. Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). M. G. Karunambigai and R. Parvathi [4] introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. In [6], these concepts had been applied to find the shortest path in networks using dynamic programming problem approach. A. Nagoor Gani and S.R. Latha [12] introduced Irregular fuzzy graphs and discussed some of its properties. In this paper, we define the concept of complement of an Intuitionistic Fuzzy Graph (IFG) and Join of two IFGs and dicussed its properties. Also some properties of Irregular Intuitionistic fuzzy graphs and neighbourly irregular intuitionistic fuzzy graphs are studied and some results on totally Irregular intuitionistic fuzzy graphs are established.

II. PRELIMINARIES

Definition 2.1: An intuitionistic fuzzy graph (IFG) is of the form $G = \langle V, E\rangle$ where (i) $V = \{v_1, v_2, \ldots\ldots v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non – membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ …………. (1) for every $v_i \in V$, (i = 1, 2, …n),
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that

$$\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)], \ \ldots\ldots\ldots\ldots\ (2)$$
$$\gamma_2(v_i, v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)] \ \ldots\ldots\ldots\ldots\ (3)$$
and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ …………..(4) for every $(v_i, v_j) \in E$, (i, j = 1, 2, …..n)

Note 1: If one of the inequalities (1) or (2) or (3) or (4) is not satisfied, then the graph G is not an intuitionistic fuzzy graph.

Note 2: The triple $\langle v_i, \mu_{1i}, \gamma_{1i}\rangle$ represent the degree of membership and non – membership of vertex v_i. Also the triple $\langle e_{ij}, \mu_{2ij}, \gamma_{2ij}\rangle$ represent the degree of membership and non – membership of edge $e_{ij} = (v_i, v_j)$ on V.

Here $\mu_{1i} = \mu_1(v_i)$, $\gamma_{1i} = \gamma_1(v_i)$
and $\mu_{2ij} = \mu_2(v_i, v_j)$, $\gamma_{2ij} = \gamma_2(v_i, v_j)$

Note 3: If $\mu_{2ij} = \gamma_{2ij} = 0$, for some i and j, then there is no edge between the vertices v_i and v_j.

Definition 2.2: Let $G = \langle V, E\rangle$ be an Intuitionistic Fuzzy Graph. The degree of a vertex u is defined by $d(u) = (d\mu(u), d\gamma(u))$ where $d\mu(u) = \sum_{v\ne u} \mu_2(u, v)$ and $d\gamma(u) = \sum_{v\ne u} \gamma_2(u, v)$.

Definition 2.3: The minimum degree of an Intuitionistic Fuzzy Graph $G = \langle V, E\rangle$ is $\delta(G) = (\delta\mu(G), \delta\gamma(G))$ where $\delta\mu(G) = min\{d\mu(v)/v \in V\}$ and $\delta\gamma(G) = min\{d\gamma(v)/v \in V\}$.

Definition 2.4: The maximum degree of an Intuitionistic Fuzzy Graph $G = \langle V, E\rangle$ is $\Delta(G) = \left(\Delta\mu(G), \Delta\gamma(G)\right)$ where $\Delta\mu(G) = max\{d\mu(v)/v \in V\}$ and $\Delta\gamma(G) = max\{d\gamma(v)/v \in V\}$.

Definition 2.5: An edge $e = (u, v)$ of an Intuitionistic Fuzzy Graph $G = \langle V, E\rangle$ is called an effective edge if $\mu_2(u, v) = min\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u, v) = max\{\gamma_1(u), \gamma_1(v)\}$.

Definition 2.6: An Intuitionistic Fuzzy Graph is complete if $\mu_{2ij} = min\{\mu_{1i}, \mu_{1j}\}$ and $\gamma_{2ij} = max\{\gamma_{1i}, \gamma_{1j}\}$ for all $v_i, v_j \in V$.

Definition 2.7: The total degree of a vertex 'u' in an Intuitionistic Fuzzy Graph $G = \langle V, E\rangle$ is defined as $td(u) = \left(td_\mu u, td_\gamma u\right)$, where $td_\mu u = \sum_{u\neq v} \mu_2(u, v) + \mu_1(u)$ and $td_\gamma u = \sum_{u\neq v} \gamma_2(u, v) + \gamma_1(u)$.

Definition 2.8: The complement of an Intuitionistic Fuzzy Graph $G = \langle V, E\rangle$ is denoted by $\overline{G} = \langle \overline{V}, \overline{E}\rangle$ and is defined as (i) $\overline{\mu}_1(u) = \mu_1(u)$ and $\overline{\gamma}_1(u) = \gamma_1(u)$ for every $u \in V$.

(ii) $\overline{\mu}_2(u, v) = min\{\mu_1(u), \mu_1(v)\} - \mu_2(u, v)$ and

$\overline{\gamma}_2(u, v) = max\{\gamma_1(u), \gamma_1(v)\} - \gamma_2(u, v)$ for all $(u, v) \in E$

Example 2.1: Let $G = \langle V, E\rangle$ be an Intuitionistic Fuzzy Graph. The membership and non membership values of G are as follows:

$\left(\mu_1(u), \gamma_1(u)\right) = (0.3, 0.6), \left(\mu_1(v), \gamma_1(v)\right) = (0.7, 0.2), \left(\mu_1(w), \gamma_1(w)\right) = (0.9, 0.0), \left(\mu_1(x), \gamma_1(x)\right) = (0.0,1.0)$, $\left(\mu_2(u, v), \gamma_2(u, v)\right) = (0.3, 0.6)$, $\left(\mu_2(v, w), \gamma_2(v, w)\right) = (0.6, 0.1), \left(\mu_2(w, x), \gamma_2(w, x)\right) = (0.0, 0.9)$, $\left(\mu_2(x, u), \gamma_2(x, u)\right) = (0.0, 0.8)$

$$\therefore d(u) = (0.3, 1.4), d(v) = (0.9, 0.7), d(w) = (0.6, 1.0), d(x) = (0.0, 1.7)$$

$$\delta(G) = (0.0, 0.7) \quad \text{and} \quad \Delta(G) = (0.9, 1.7).$$

Here, edge (u, v) is an effective edge.

Also total degrees of vertices are

$$td(u) = (0.6, 2.0), td(v) = (1.6, 0.9), td(w) = (1.5, 1.0), td(x) = (0.0, 2.7)$$

The complement of Intuitionistic Fuzzy Graph G is the graph $\overline{G}$ with membership and non membership values are as follows:

$\left(\mu_1(u), \gamma_1(u)\right) = (0.3, 0.6), \left(\mu_1(v), \gamma_1(v)\right) = (0.7, 0.2), \left(\mu_1(w), \gamma_1(w)\right) = (0.9, 0.0)$, $\left(\mu_1(x), \gamma_1(x)\right) = (0.0,1.0), \left(\mu_2(u, w), \gamma_2(u, w)\right) = (0.3, 0.6)$, $\left(\mu_2(v, w), \gamma_2(v, w)\right) = (0.1, 0.1), \left(\mu_2(w, x), \gamma_2(w, x)\right) = (0.0, 0.1)$, $\left(\mu_2(x, u), \gamma_2(x, u)\right) = (0.0, 0.2)$, $\left(\mu_2(v, x), \gamma_2(v, x)\right) = (0.0, 1.0)$.

Definition 2.9: An Intuitionistic Fuzzy Graph $G = \langle V, E\rangle$ is said to be regular, if every vertex adjacent to vertices with same degree.

Example 2.2: Let $G = \langle V, E\rangle$ be an Intuitionistic Fuzzy Graph. The membership and non membership values of G are as follows:

$\left(\mu_1(u), \gamma_1(u)\right) = (0.5, 0.4), \left(\mu_1(v), \gamma_1(v)\right) = (0.5, 0.5), \left(\mu_1(w), \gamma_1(w)\right) = (0.4, 0.5)$, $\left(\mu_1(x), \gamma_1(x)\right) = (0.4,0.5)$, $\left(\mu_2(u, v), \gamma_2(u, v)\right) = (0.4, 0.4)$, $\left(\mu_2(v, w), \gamma_2(v, w)\right) = (0.1, 0.3), \left(\mu_2(w, x), \gamma_2(w, x)\right) = (0.4, 0.4)$, $\left(\mu_2(x, u), \gamma_2(x, u)\right) = (0.1, 0.3)$.

$$\therefore d(u) = d(v) = d(w) = d(x) = (0.5, 0.7)$$

So G is a regular Intuitionistic Fuzzy Graph.

Definition 2.10: An Intuitionistic Fuzzy Graph $G = \langle V, E \rangle$ is said to be irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

Example 2.3: In example 2.1, u adjacent to the vertices v and x which are having distinct degrees. So the Intuitionistic Fuzzy Graph $G = \langle V, E \rangle$ in example 2.1 is irregular.

Definition 2.11: Let $G = \langle V, E \rangle$ be a connected Intuitionistic Fuzzy Graph. G is said to be a neighbourly irregular Intuitionistic Fuzzy Graph if every two adjacent vertices of G have distinct degrees.

Example 2.4:

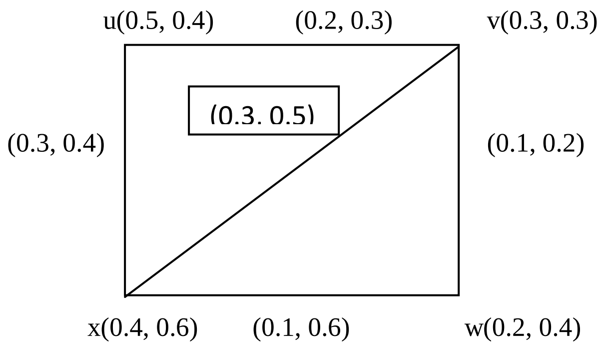

Figure 2.1

$\therefore d(u) = (0.5, 0.7), d(v) = (0.6, 1.0), d(w) = (0.2, 0.8), d(x) = (0.7, 1.5)$

Here u, v and x are adjacent vertices with distinct degrees. Also v, w and x are adjacent vertices with distinct degrees. So G is a neighbourly irregular Intuitionistic Fuzzy Graph.

Definition 2.12: Let $G = \langle V, E \rangle$ be a connected Intuitionistic Fuzzy Graph. Then G is said to be a highly irregular Intuitionistic Fuzzy Graph if every vertex of G is adjacent to vertices with distinct degrees.

Example 2.5:

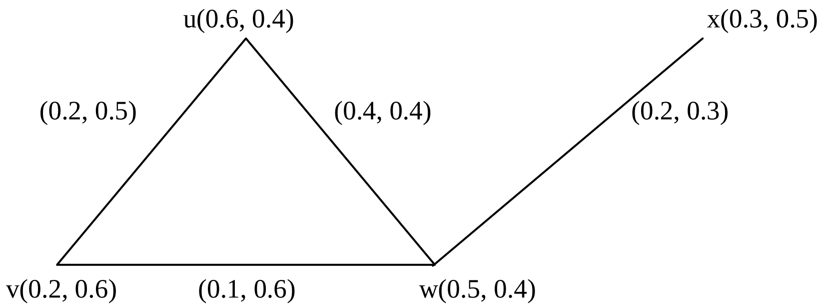

Figure 2.2

$\therefore d(u) = (0.6, 0.9), d(v) = (0.3, 1.1), d(w) = (0.7, 1.3), d(x) = (0.2, 0.3)$

Since every vertex of IFG is adjacent to vertices with distinct degrees, G is highly irregular Intuitionistic Fuzzy Graph.

Definition 2.13: An Intuitionistic Fuzzy Graph $G = \langle V, E \rangle$ is said to be a complete IFG if $\mu_{2ij} = min\{\mu_{1i}, \mu_{1j}\}$ and $\gamma_{2ij} = max\{\gamma_{1i}, \gamma_{1j}\}$, for every $v_i, v_j \in V$.

Example 2.6:

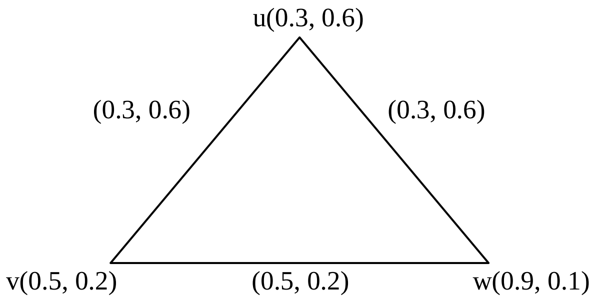

Figure 2.3 Complete Intuitionistic Fuzzy Graph G

Definition 2.14: The join of two Intuitionistic Fuzzy Graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ is an Intuitionistic Fuzzy Graph $= G_1 + G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \cup E' \rangle$, defined by

$$\begin{aligned}
(\mu_1 + {\mu_1}')(v) &= (\mu_1 \cup {\mu_1}')(v), \text{ if } v \in V_1 \cup V_2 \\
(\gamma_1 + {\gamma_1}')(v) &= (\gamma_1 \cup {\gamma_1}')(v), \text{ if } v \in V_1 \cup V_2 \\
(\mu_2 + {\mu_2}')(v_i v_j) &= (\mu_2 \cup {\mu_2}')(v_i v_j), \text{ if } v_i v_j \in E_1 \cup E_2 \\
&= min\{\mu_1(v_i), {\mu_1}'(v_j)\}, \text{ if } v_i v_j \in E'
\end{aligned}$$

and

$$\begin{aligned}
(\gamma_2 + {\gamma_2}')(v_i v_j) &= (\gamma_2 \cup {\gamma_2}')(v_i v_j), \text{ if } v_i v_j \in E_1 \cup E_2 \\
&= max\{\gamma_1(v_i), {\gamma_1}'(v_j)\}, \text{ if } v_i v_j \in E'
\end{aligned}$$

Where (μ_1, γ_1) and $({\mu_1}', {\gamma_1}')$ refer the vertex membership and non – membership of G_1 and G_2 respectively; (μ_2, γ_2) and $({\mu_2}', {\gamma_2}')$ refer the edge membership and non – membership of G_1 and G_2 respectively; E' refer the complement of $E_1 \cup E_2$.

Example 2.7: Let $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$ be two Intuitionistic Fuzzy Graphs with $V_1 = \{u_1, u_2, u_3\}$ and $V_2 = \{v_1, v_2\}$ such that $V_1 \cap V_2 = \emptyset$. The membership and non membership values of G_1 are as follows:

$\big(\mu_1(u_1), \gamma_1(u_1)\big) = (0.3, 0.6), \big(\mu_1(u_2), \gamma_1(u_2)\big) = (0.5, 0.2), \big(\mu_1(u_3), \gamma_1(u_3)\big) = (0.9, 0.1),$
$\big(\mu_2(u_1, u_2), \gamma_2(u_1, u_2)\big) = (0.1, 0.6),$ $\big(\mu_2(u_2, u_3), \gamma_2(u_2, u_3)\big) = (0.5, 0.0), \big(\mu_2(u_3, u_1), \gamma_2(u_3, u_1)\big) =$
$(0.2, 0.6).$

The membership and non membership values of G_2 are as follows:

$\big(\mu_1(v_1), \gamma_1(v_1)\big) = (0.6, 0.4), \big(\mu_1(v_2), \gamma_1(v_2)\big) = (0.2, 0.8), \big(\mu_2(v_1, v_2), \gamma_2(v_1, v_2)\big) = (0.1, 0.7)$

The membership and non membership values of $G_1 + G_2$ are as follows:

$\big(\mu_1(u_1), \gamma_1(u_1)\big) = (0.3, 0.6),$ $\big(\mu_1(u_2), \gamma_1(u_2)\big) = (0.5, 0.2),$ $\big(\mu_1(u_3), \gamma_1(u_3)\big) = (0.9, 0.1),$
$\big(\mu_1(v_1), \gamma_1(v_1)\big) = (0.6, 0.4),$ $\big(\mu_1(v_2), \gamma_1(v_2)\big) = (0.2, 0.8),$ $\big(\mu_2(u_1, v_1), \gamma_2(u_1, v_1)\big) = (0.3, 0.6),$
$\big(\mu_2(u_1, v_2), \gamma_2(u_1, v_2)\big) = (0.2, 0.8),$ $\big(\mu_2(u_1, u_2), \gamma_2(u_1, u_2)\big) = (0.1, 0.6),$ $\big(\mu_2(u_1, u_3), \gamma_2(u_1, u_3)\big) =$
$(0.2, 0.8), \big(\mu_2(u_2, u_3), \gamma_2(u_2, u_3)\big) = (0.5, 0.0),$ $\big(\mu_2(u_2, v_1), \gamma_2(u_2, v_1)\big) = (0.5, 0.4),$
$\big(\mu_2(u_2, v_2), \gamma_2(u_2, v_2)\big) = (0.2, 0.8),$ $\big(\mu_2(u_3, v_1), \gamma_2(u_3, v_1)\big) = (0.6, 0.4), \big(\mu_2(u_3, v_2), \gamma_2(u_3, v_2)\big) =$
$(0.2, 0.8),$ $\big(\mu_2(v_1, v_2), \gamma_2(v_1, v_2)\big) = (0.1, 0.7)$

III. CHARECTERISTICS ON JOIN OF TWO IFG

Theorem 3.1:

If $G_1 = \langle V_1, E_1\rangle$ *and* $G_2 = \langle V_2, E_2\rangle$ are two complete IFGs, then $G_1 + G_2$ is also a complete IFG.

Proof:

Let $G_1 = \langle V_1, E_1\rangle$ *and* $G_2 = \langle V_2, E_2\rangle$ be two complete IFGs.

So,

$\mu_{2ij} = min\{\mu_{1i}, \mu_{1j}\}$ and $\gamma_{2ij} = max\{\gamma_{1i}, \gamma_{1j}\}$, for every $v_i, v_j \in V_1$.

and

$\mu_2{'}_{ij} = min\left\{\mu_1{'}_i, \mu_1{'}_j\right\}$ and $\gamma_2{'}_{ij} = max\left\{\gamma_1{'}_i, \gamma_1{'}_j\right\}$, for every $v_i, v_j \in V_2$

But, $G_1 + G_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \cup E'\rangle$ is defined by,

$$(\mu_1 + \mu_1{'})(v) = (\mu_1 \cup \mu_1{'})(v), \text{ if } v \in V_1 \cup V_2$$
$$(\gamma_1 + \gamma_1{'})(v) = (\gamma_1 \cup \gamma_1{'})(v), \text{ if } v \in V_1 \cup V_2$$
$$(\mu_2 + \mu_2{'})(v_i v_j) = (\mu_2 \cup \mu_2{'})(v_i v_j), \text{ if } v_i v_j \in E_1 \cup E_2$$
$$= min\{\mu_1(v_i), \mu_1{'}(v_j)\}, \text{ if } v_i v_j \in E'$$

and

$$(\gamma_2 + \gamma_2{'})(v_i v_j) = (\gamma_2 \cup \gamma_2{'})(v_i v_j), \text{ if } v_i v_j \in E_1 \cup E_2$$
$$= max\{\gamma_1(v_i), \gamma_1{'}(v_j)\}, \text{ if } v_i v_j \in E'$$

Thus in $G_1 + G_2$, all the vertices of G_1 and G_2 exists without any change in the vertex membership and non – membership values and all the edges of G_1 and G_2 exists without changing its edge membership and non – membership values. The remaining edges of $G_1 + G_2$ satisfies the definition of complete IFGs because they are in E'.

Thus, $G_1 + G_2$ is a complete IFG.

Theorem 3.2:

The complement of a complete IFG is an IFG with no edges.

OR

If G is a complete IFG then in $\overline{G}$ the edge set is empty.

Proof:

Let $G = \langle V, E\rangle$ be a complete IFG.

So,

$\mu_{2ij} = min\{\mu_{1i}, \mu_{1j}\}$ and $\gamma_{2ij} = max\{\gamma_{1i}, \gamma_{1j}\}$, for every $v_i, v_j \in V$.

Hence in $\overline{G}$,

$$\overline{\mu}_{2ij} = min\{\mu_{1i}, \mu_{1j}\} - \mu_{2ij} \quad \text{for all i, j = 1, 2, …..n}$$
$$= min\{\mu_{1i}, \mu_{1j}\} - min\{\mu_{1i}, \mu_{1j}\} \quad \text{for all i, j = 1, 2, …..n}$$
$$= 0 \quad \text{for all i, j = 1, 2, …..n}$$

and

$$\overline{\gamma}_{2ij} = max\{\gamma_{1i}, \gamma_{1j})\} - \gamma_{2ij} \quad \text{for all i, j = 1, 2, …..n}$$

$$= max\{\gamma_{1i}, \gamma_{1j})\} - max\{\gamma_{1i}, \gamma_{1j})\} \quad \text{for all i, j = 1, 2, …..n}$$

$$= 0 \quad \text{for all i, j = 1, 2, …..,n}$$

Thus $\left(\overline{\mu}_{2ij},\ \overline{\gamma}_{2ij}\right) = (0, 0)$

Hence the edge set of $\overline{G}$ is empty if G is a complete IFG.

Theorem 3.3:

If $G_1 = \langle V_1, E_1\rangle$ *and* $G_2 = \langle V_2, E_2\rangle$ are two complete IFGs, then $G_1 + G_2 = \overline{G_1} + \overline{G_2}$

Proof:

Let $G_1 = \langle V_1, E_1\rangle$ *and* $G_2 = \langle V_2, E_2\rangle$ be two complete IFGs.
∴ By Theorem 3.1, $G_1 + G_2$ is also a complete IFG.
Also by Theorem 3.2, $\overline{G_1}$ and $\overline{G_2}$ are two IFGs with $\overline{E_1} = \emptyset$ and $\overline{E_2} = \emptyset$.
But $\overline{V_1} = V_1$ and $\overline{V_2} = V_2$ without any change in membership and non membership values.
In $\overline{G_1} + \overline{G_2}$, the edge membership and non membership values are defined by

$$(\mu_2 + \mu_2{}')(v_i v_j) \quad = min\{\mu_1(v_i), \mu_1{}'(v_j)\}$$

and

$$(\gamma_2 + \gamma_2{}')(v_i v_j) = max\{\gamma_1(v_i), \gamma_1{}'(v_j)\}$$

So,
$\overline{G_1} + \overline{G_2}$ is a complete IFG with

$$V = V_1 \cup V_2 \text{ and } (\mu_1 + \mu_1{}')(v) = \begin{cases} \mu_1(v), \text{if } v \in V_1 - V_2 \\ \mu_1{}'(v), \text{if } v \in V_2 - V_1 \end{cases}$$

&

$$(\gamma_1 + \gamma_1{}')(v) = \begin{cases} \gamma_1(v), \text{if } v \in V_1 - V_2 \\ \gamma_1{}'(v), \text{if } v \in V_2 - V_1 \end{cases}$$

Hence $\overline{G_1} + \overline{G_2} = G_1 + G_2$

Proposition 3.1:

The join of two regular Intuitionistic Fuzzy Graphs need not be regular.

Example 3.1:

Let $G_1 = \langle V_1, E_1\rangle$ and $G_2 = \langle V_2, E_2\rangle$ be two regular Intuitionistic Fuzzy Graphs with $V_1 = \{u_1, u_2, u_3, u_4\}$ and $V_2 = \{v_1, v_2\}$ such that $V_1 \cap V_2 = \emptyset$. The membership and non membership values of G_1 are as follows:

$(\mu_1(u_1), \gamma_1(u_1)) = (0.4, 0.5), (\mu_1(u_2), \gamma_1(u_2)) = (0.5, 0.5), (\mu_1(u_3), \gamma_1(u_3)) = (0.5, 0.3), (\mu_1(u_4), \gamma_1(u_4)) = (0.5, 0.4), (\mu_2(u_1, u_2), \gamma_2(u_1, u_2)) = (0.3, 0.3)$, $(\mu_2(u_2, u_3), \gamma_2(u_2, u_3)) = (0.4, 0.2)$, $(\mu_2(u_3, u_4), \gamma_2(u_3, u_4)) = (0.3, 0.3)$, $(\mu_2(u_4, u_1), \gamma_2(u_4, u_1)) = (0.4, 0.2)$.

Here,

$$d(u_1) = d(u_2) = d(u_3) = d(u_4) = (0.7, 0.5)$$

The membership and non membership values of G_2 are as follows:

$(\mu_1(v_1), \gamma_1(v_1)) = (0.6, 0.4), (\mu_1(v_2), \gamma_1(v_2)) = (0.2, 0.8), (\mu_2(v_1, v_2), \gamma_2(v_1, v_2)) = (0.1, 0.7)$

Here,

$$d(v_1) = d(v_2) = (0.1, 0.7)$$

The membership and non membership values of $G_1 + G_2$ are as follows:

$(\mu_1(u_1), \gamma_1(u_1)) = (0.4, 0.5), (\mu_1(u_2), \gamma_1(u_2)) = (0.5, 0.5), (\mu_1(u_3), \gamma_1(u_3)) = (0.5, 0.3), (\mu_1(u_4), \gamma_1(u_4)) = (0.5, 0.4), (\mu_1(v_1), \gamma_1(v_1)) = (0.6, 0.4)$, $(\mu_1(v_2), \gamma_1(v_2)) = (0.2, 0.8)$, $(\mu_2(u_1, v_1), \gamma_2(u_1, v_1)) = (0.4, 0.5)$, $(\mu_2(u_1, v_2), \gamma_2(u_1, v_2)) = (0.2, 0.8)$, $(\mu_2(u_1, u_2), \gamma_2(u_1, u_2)) = (0.3, 0.3)$, $(\mu_2(u_1, u_3), \gamma_2(u_1, u_3)) = (0.4, 0.5)$, $(\mu_2(u_1, u_4), \gamma_2(u_1, u_4)) = (0.4, 0.2)$, $(\mu_2(u_2, u_3), \gamma_2(u_2, u_3)) = (0.4, 0.2)$, $(\mu_2(u_2, v_1), \gamma_2(u_2, v_1)) = (0.5, 0.5)$, $(\mu_2(u_2, v_2), \gamma_2(u_2, v_2)) = (0.2, 0.8)$, $(\mu_2(u_2, u_4), \gamma_2(u_2, u_4)) = (0.5, 0.5)$, $(\mu_2(u_3, v_1), \gamma_2(u_3, v_1)) = (0.5, 0.4)$, $(\mu_2(u_3, v_2), \gamma_2(u_3, v_2)) = (0.2, 0.8)$,

$\big(\mu_2(u_3,u_4),\gamma_2(u_3,u_4)\big)=(0.3,0.3)$, $\big(\mu_2(u_4,v_1),\gamma_2(u_4,v_1)\big)=(0.5,0.4)$, $\big(\mu_2(u_4,v_2),\gamma_2(u_4,v_2)\big)=(0.2,0.8)$, $\big(\mu_2(v_1,v_2),\gamma_2(v_1,v_2)\big)=(0.1,0.7)$.

In G_1+G_2,

$$d(u_1)=(1.7,2.3), d(u_2)=(1.9,2.3), d(u_3)=(1.8,2.2), d(u_4)=(2.2,1.8),$$
$$d(v_1)=(2.0,2.5), d(v_2)=(0.9,3.9)$$

Thus, G_1+G_2 is an irregular Intuitionistic Fuzzy Graph.

Theorem 3.4:

Let $G=\langle V,E\rangle$ be a highly irregular and neighbourly irregular Intuitionistic Fuzzy Graph if and only if the degrees of all vertices of G are distinct.

Proof:

Let $G=\langle V,E\rangle$ be an Intuitionistic Fuzzy Graph with $V=\{v_1,v_2,\ldots..v_n\}$. Assume that G is highly irregular and neighbourly irregular Intuitionistic Fuzzy Graph. Let the adjacent vertices of v_1 be $v_2,v_3,\ldots..v_n$ with degrees $(h_2,k_2),(h_3,k_3),\ldots\ldots,(h_n,k_n)$ respectively.
Since G is a highly irregular Intuitionistic Fuzzy Graph,

$h_2\neq h_3\neq\cdots\neq h_n$ and $k_2\neq k_3\neq\cdots\neq k_n$.

Thus, $d(v_1)$ is neither of $(h_2,k_2),(h_3,k_3),\ldots\ldots,(h_n,k_n)$.
So, the degrees of all vertices of G are distinct.

Conversely, assume that the degrees of all vertices of G are distinct.
Thus, every two adjacent vertices have distinct degrees and to every vertex, the adjacent vertices have distinct degrees.
Hence, G is a highly irregular and neighbourly irregular Intuitionistic Fuzzy Graph.

Theorem 3.5:

The join of two highly irregular and neighbourly irregular Intuitionistic Fuzzy Graphs having no common vertices is irregular.

Proof:

Let $G_1=\langle V_1,E_1\rangle$ and $G_2=\langle V_2,E_2\rangle$ be two highly and neighbourly irregular Intuitionistic Fuzzy Graphs such that $V_1\cap V_2=\emptyset$.

Since G_1 is highly irregular Intuitionistic Fuzzy Graph and neighbourly irregular Intuitionistic Fuzzy Graph, by Theorem 3.4, the degrees of all vertices of G_1 are distinct.

Similarly, the degrees of all vertices of G_2 are distinct.

Since there is no common vertex for G_1 and G_2, all the edges of them will exist in $G=G_1+G_2$ without changing the membership and non membership values.

For the remaining vertices if G,

$$(\mu_2+\mu_2{}')(v_iv_j)=min\{\mu_1(v_i),\mu_1{}'(v_j)\}, \text{ if } v_iv_j\in E'$$

and

$$(\gamma_2+\gamma_2{}')(v_iv_j)=max\{\gamma_1(v_i),\gamma_1{}'(v_j)\}, \text{ if } v_iv_j\in E'$$

Hence there exists at least one vertex in G which is adjacent to vertices with distinct degrees.

So, G is irregular.

Proposition 3.2:

The complement of a regular Intuitionistic Fuzzy Graph need not be regular.

Example 3.2:

Let $G = \langle V, E \rangle$ be a regular Intuitionistic Fuzzy Graph. The membership and non membership values of G are as follows:

$\big(\mu_1(u), \gamma_1(u)\big) = (0.4, 0.5), \big(\mu_1(v), \gamma_1(v)\big) = (0.5, 0.5), \big(\mu_1(w), \gamma_1(w)\big) = (0.5, 0.3), \big(\mu_1(x), \gamma_1(x)\big) = (0.5, 0.4), \big(\mu_2(u, v), \gamma_2(u, v)\big) = (0.3, 0.3), \big(\mu_2(v, w), \gamma_2(v, w)\big) = (0.4, 0.2), \big(\mu_2(w, x), \gamma_2(w, x)\big) = (0.3, 0.3), \big(\mu_2(x, u), \gamma_2(x, u)\big) = (0.4, 0.2)$.
Here,

$$d(u) = d(v) = d(w) = d(x) = (0.7, 0.5)$$

Thus, G is a regular Intuitionistic Fuzzy Graph.

The complement of G is the graph $\overline{G}$ with membership and non membership values are as follows:

$\big(\mu_1(u), \gamma_1(u)\big) = (0.4, 0.5), \big(\mu_1(v), \gamma_1(v)\big) = (0.5, 0.5), \big(\mu_1(w), \gamma_1(w)\big) = (0.5, 0.3), \big(\mu_1(x), \gamma_1(x)\big) = (0.5, 0.4), \big(\mu_2(u, v), \gamma_2(u, v)\big) = (0.1, 0.2), \big(\mu_2(u, w), \gamma_2(u, w)\big) = (0.4, 0.5), \big(\mu_2(v, w), \gamma_2(v, w)\big) = (0.1, 0.3), \big(\mu_2(w, x), \gamma_2(w, x)\big) = (0.2, 0.1), \big(\mu_2(x, u), \gamma_2(x, u)\big) = (0.0, 0.3), \big(\mu_2(v, x), \gamma_2(v, x)\big) = (0.5, 0.5)$.
Here,

$$d(u) = (0.5, 1.0), d(v) = (0.7, 1.0), d(w) = (0.7, 0.9), d(x) = (0.7, 0.9)$$

Thus, $\overline{G}$ is not a regular Intuitionistic Fuzzy Graph.

Proposition 3.3:

The degree of a vertex u in an Intuitionistic Fuzzy Graph G and in its complement $\overline{G}$ need not be same.

Example 3.3:

Let $G = \langle V, E \rangle$ be an Intuitionistic Fuzzy Graph. The membership and non membership values of G are as follows:

$\big(\mu_1(u), \gamma_1(u)\big) = (0.3, 0.6), \big(\mu_1(v), \gamma_1(v)\big) = (0.7, 0.2), \big(\mu_1(w), \gamma_1(w)\big) = (0.9, 0.0), \big(\mu_1(x), \gamma_1(x)\big) = (0.0, 1.0), \big(\mu_2(u, v), \gamma_2(u, v)\big) = (0.3, 0.6), \big(\mu_2(v, w), \gamma_2(v, w)\big) = (0.6, 0.1), \big(\mu_2(w, x), \gamma_2(w, x)\big) = (0.0, 0.9), \big(\mu_2(x, u), \gamma_2(x, u)\big) = (0.0, 0.8)$.

The complement of IFG, G is the graph $\overline{G}$ with membership and non membership values are as follows:

$\big(\mu_1(u), \gamma_1(u)\big) = (0.3, 0.6), \big(\mu_1(v), \gamma_1(v)\big) = (0.7, 0.2), \big(\mu_1(w), \gamma_1(w)\big) = (0.9, 0.0), \big(\mu_1(x), \gamma_1(x)\big) = (0.0, 1.0), \big(\mu_2(u, w), \gamma_2(u, w)\big) = (0.3, 0.6), \big(\mu_2(v, w), \gamma_2(v, w)\big) = (0.1, 0.1), \big(\mu_2(w, x), \gamma_2(w, x)\big) = (0.0, 0.1), \big(\mu_2(x, u), \gamma_2(x, u)\big) = (0.0, 0.2), \big(\mu_2(v, x), \gamma_2(v, x)\big) = (0.0, 1.0)$.

	In G	In $\overline{G}$
$d(u)$	$(0.3, 1.4)$	$(0.3, 0.8)$
$d(v)$	$(0.9, 0.7)$	$(0.1, 1.1)$
$d(w)$	$(0.6, 1.0)$	$(0.4, 0.2)$
$d(x)$	$(0.0, 1.7)$	$(0.0, 1.3)$

Proposition 3.4:

The minimum degree of an Intuitionistic Fuzzy Graph G and that of its complement $\overline{G}$ need not be same. Also, the maximum degree of an Intuitionistic Fuzzy Graph G and that of its complement $\overline{G}$ need not be same.

Example 3.4:

In example 3.3,

$$\delta(G) = (0.0, 0.7) \text{ and } \delta(\overline{G}) = (0.0, 0.2)$$
$$\Delta(G) = (0.9, 1.7) \text{ and } \Delta(\overline{G}) = (0.9, 1.7).$$

IV. CONCLUSION

Here we derived some theorems on complete Intuitionistic Fuzzy Graphs and their join. Also we analyzed and discussed some more characteristics of the operation join of two IFG based on regularity and irregularity of Intuitionistic Fuzzy Graphs. The complement of IFG and some properties are also studied. The theory of IFG has more applications in Communication Networks, Information Technology, Pattern Clustering, Image Retrieval and so on. So we are planning to extend our investigation to more properties on operations like composition and union of two IFGs.

References

[1] K. T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Applications. Physica – Verlag, New York, 1999.
[2] J. George Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall of India, New Delhi, 2001.
[3] H. J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer – Nijhoff, Boston, 1985.
[4] M. G. Karunambigai and R.Parvathi, Intuitionistic Fuzzy Graphs, Journal of Computational Intelligence: Theory and Applications, 20, 2006, pp. 139 – 150.
[5] M. S. Sunitha and A. Vijayakumar, Complement of a Fuzzy Graph, Indian Journal of Pure and Applied Mathematics, 33(9), 2002, pp. 1451 – 1464.
[6] R.Parvathi, M. G. Karunambigai and K. T. Atanassov, Operations on Intuitionistic Fuzzy Graphs, FUZZ – IEEE, 2009, pp. 1396 – 1401.
[7] A. Nagoor Gani, R. Jahir Hussain and S. Yahya Mohamed, Irregular Intuitionistic Fuzzy Graph, IOSR Journal MathematicsVol. 9, Issue 6, Jan. 2014, pp. 47 – 51.
[8] R. Jahir Hussain and S. Yahya Mohamed, More on Irregular Intuitionistic Fuzzy Graphs and its Complements, Discovery Publication, Vol. 21, July 2014, pp. 101 – 107.
[9] F. Harary, Graph Theor, Addition Wesley, Third Printing, October 1972.
[10] K. R. Bhutani, Automorphism of Fuzzy Graphs, Pattern Recognition Letter 9: 1989, 159 – 162.
[11] R. Jahir Hussain and S. Yahya Mohamed, Properties on Irregular Intuitionistic Fuzzy Graphs, Applied Mathematical Sciences (International Journal), Hikari Ltd., Bulgeria, Vol. 8, No. 8, 2014, pp. 379 – 389.
[12] A. Nagoor Gani and Latha S. R, On Irregular Fuzzy Graphs, Applied Mathematical Sciences, Vol. 6, 2012, no. 11, 517 – 523.