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Some distances and sequences in a weighted graph

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ABSTRACT: *In a weighted graph, the arcs are mainly classified into α, β and δ. In this article, some new distances and sequences in weighted graphs are introduced. These concepts are based on the above classification. With respect to the distances, the concepts of centre and self centered graphs are introduced and their properties are discussed. It is proved that, only partial blocks with even number of vertices can be self centered. Using the sequences, a characterization for partial blocks and precisely weighted graphs (PWG) are obtained.*
Keywords: *α distance, β distance, partial blocks, partial trees, PWG.*

I. Introduction and preliminaries

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving a combinatorial problem in different areas such as geometry, algebra, number theory, topology, optimization and computer science. Most important thing which is to be noted is that, any problem which can be solved by any graph technique can only be modeled as a weighted graph problem. Distance and centre concepts play an important role in applications related with graphs and weighted graphs. Several authors including Bondy and Fan [1, 2, 3], Broersma, Zhang and Li [14], Sunil Mathew and M. S. Sunitha [8, 9, 10, 11, 12, 13] introduced many connectivity concepts in weighted graphs following the works of Dirac [4] and Grotschel [5].

In this article we introduce three new distance concepts in weighted graphs. These concepts are derived by using the notion of connectivity in weighted graphs. In a weighted graph model, for example, in an information network or an electric circuit, the reduction of flow between pairs of nodes is more relevant and may frequently occur than the total disruption of the entire network [7, 10, 11]. Finding the centre of a graph is useful in facility location problems where the goal is to minimize the distance to the facility. For example, placing a hospital at a central point reduces the longest distance the ambulance has to travel. This concept is our motivation. As weighted graphs are generalized structures of graphs, the concepts introduced in this article also generalize the classic ideas in graph theory.

A weighted graph $G:(V,E,W)$ is a graph in which every arc e is assigned a nonnegative number $w(e)$, called the *weight* of e [1]. The *distance* between two nodes u and v in G is defined and denoted by $d(u,v)=\min\{\sum_{e\in P}w(e)/P$ is a $u-v$ path in $G\}$ [1, 6]. The *eccentricity* of a node u in G is defined and denoted by $e(u)=\max\{d(u,v)/v$ is any other node of $G\}$ [6]. The minimum and maximum of the eccentricities of nodes are respectively called *radius*, $r(G)$ and *diameter*, $d(G)$ of the graph G [6]. A node u is called *central* if $e(u)=r(G)$ and *diametral* if $e(u)=d(G)$ [6]. The subgraph induced by the set of all central nodes is called the *centre* of G, $c(G)$ [6]. G is called *self centered* if it is isomorphic with it's centre [6].

In a weighted graph $G:(V,E,W)$ the *strength of a path* $P=v_0e_1v_1e_2v_2...e_nv_n$ is defined and denoted by $S(P)=\min\{w(e_1),w(e_2),w(e_3),...w(e_n)\}$ [12]. The *strength of connectedness* of a pair of nodes u and v in G is defined and denoted by $CONN_G(u,v)=\max\{S(P)/P$ is a $u-v$ path in $G\}$ [11]. A $u-v$ path P is called a *strongest path* if $S(P)=CONN_G(u,v)$ [10]. A node w is called a *partial cut node (p - cut node)* of G if there exists a pair of nodes u, v in G such that $u\neq v\neq w$ and

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$CONN_{G-w}(u,v)CONN_G(u,v)$ [11]. A graph without p - cut nodes is called a *partial block(p - block)* [11]. It is also proved in [11] that a node w in a weighted graph G is a p -cut node if and only if w is an internal node of every maximum spanning tree. A connected weighted graph $G:(V,E,W)$ is called a *partial tree* if G has a spanning subgraph $F:(V,E',W')$ which is a tree, where for all arcs $e=(u,v)$ of G which are not in F, we have $CONN_G(u,v) > w(e)$ [11]. An arc $e=(u,v)$ is called α *-strong* if $CONN_{G-e}(u,v) < w(e)$ and β *-strong* if $CONN_{G-e}(u,v) = w(e)$ and a δ arc if $CONN_{G-e}(u,v) > w(e)$. An arc is called *strong* if it is either α - strong or β - strong [11]. A precisely weighted graph (PWG) is a complete weighted graph $G:(V,E,W_1,W_2)$, where the weight functions $W_1:V \to \Re^+$ and $W_2:E \to \Re^+$ such that for every edge $e=(u,v)$ of G, we have $W_2(e) = W_1(u) \wedge W_1(v)$, where $\wedge$ denotes the minimum[9].

II. α, β and strong distances

In this section, we give the definitions of the distances along with an example.

Definition 2.1. *Let $G:(V,E,W)$ be a weighted graph. Let u and v be any two nodes of G. Then the $\alpha - distance$ between u and v is defined and denoted by $d_\alpha(u,v) = \min \sum_{e\in P} w(e)$ where P is any α - strong path between u and v, 0 if $u=v$, ∞ if there exists no α - strong $u-v$ path.*

Clearly d_α satisfies all the axioms of a metric.

1. $d_\alpha(u,v) \geq 0$ for all u and v.
2. $d_\alpha(u,v) = 0$ if and only if $u=v$.
3. $d_\alpha(u,v) = d_\alpha(v,u)$ for all u and v.
4. $d_\alpha(u,v) \leq d_\alpha(u,w) + d_\alpha(w,v)$ for all u, v and w in G.

Hence $(V(G), d_\alpha)$ is a metric space.

Definition 2.2. *Let $G:(V,E,W)$ be a weighted graph. Let u and v be any two nodes of G. Then the $\beta - distance$ between u and v is defined and denoted by $d_\beta(u,v) = \min \sum_{e\in P} w(e)$ where P is any β - strong path between u and v, 0 if $u=v$, ∞ if there exists no β - strong $u-v$ path.*

Clearly d_β satisfies all the axioms of a metric.

1. $d_\beta(u,v) \geq 0$ for all u and v.
2. $d_\beta(u,v) = 0$ if and only if $u=v$.
3. $d_\beta(u,v) = d_\beta(v,u)$ for all u and v.
4. $d_\beta(u,v) \leq d_\beta(u,w) + d_\beta(w,v)$ for all u, v and w in G.

Hence $(V(G), d_\beta)$ is a metric space.

Definition 2.3. *Let $G:(V,E,W)$ be a weighted graph. Let u and v be any two nodes of G. Then the $strong - distance$ between u and v is defined and denoted by $d_s(u,v) = \min \sum_{e\in P} w(e)$ where P is any strong path between u and v, 0 if $u=v$, ∞ if there exists no strong $u-v$ path.*

Clearly d_s satisfies all the axioms of a metric.

1. $d_s(u,v) \geq 0$ for all u and v.

2. $d_s(u,v) = 0$ if and only if $u = v$.
3. $d_s(u,v) = d_s(v,u)$ for all u and v.
4. $d_s(u,v) \leq d_s(u,w) + d_s(w,v)$ for all u, v and w in G.

Hence $(V(G), d_s)$ is a metric space.

In the following example (Figure. 1:), we find that these three distances are generally different.

Example 2.1.

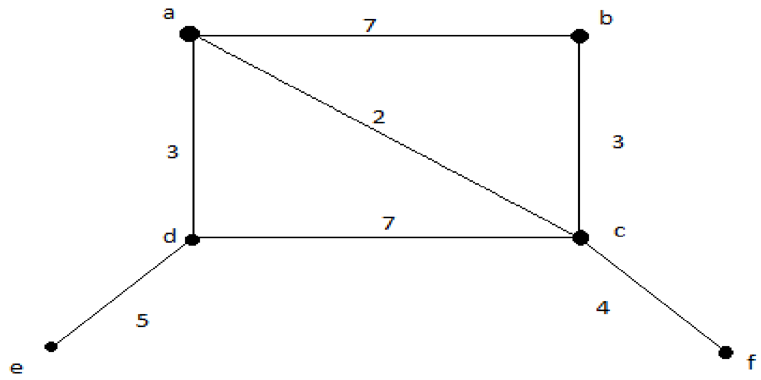

Figure. 1: A Weighted graph

Definition 2.4. *The α - distance matrix of a connected weighted graph $G:(V,E,W)$, $|V| = n$ is a square matrix of order n defined and denoted by $D_\alpha = (d_{ij})$, where $d_{ij} = \alpha$ - distance between the vertices v_i and v_j.*

Note that D_α is a symmetric matrix.

In the same manner D_β, the β - distance matrix and D_S, the strong distance matrix can be defined.
The three distance matrices of the above weighted graph (Figure. 1) are given below.

$$D_\alpha = \begin{pmatrix} 0 & 7 & \infty & \infty & \infty & \infty \\ 7 & 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 7 & 15 & 4 \\ \infty & \infty & 7 & 0 & 5 & 11 \\ \infty & \infty & 15 & 5 & 0 & 16 \\ \infty & \infty & 4 & 11 & 16 & 0 \end{pmatrix}, \quad D_\beta = \begin{pmatrix} 0 & \infty & \infty & 3 & \infty & \infty \\ \infty & 0 & 3 & \infty & \infty & \infty \\ \infty & 3 & 0 & \infty & \infty & \infty \\ 3 & \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty & 0 \end{pmatrix},$$

$$D_s = \begin{pmatrix} 0 & 7 & 10 & 3 & 8 & 14 \\ 7 & 0 & 3 & 10 & 15 & 7 \\ 10 & 3 & 0 & 7 & 12 & 4 \\ 3 & 10 & 7 & 0 & 5 & 11 \\ 8 & 15 & 12 & 5 & 0 & 16 \\ 14 & 7 & 4 & 11 & 16 & 0 \end{pmatrix}.$$

Remark 2.1. In any connected weighted graph $G:(V,E,W)$ there exists a strong path between any two nodes [10]. Hence all the elements in the strong distance matrix are finite. That is $0 \le d_s(u,v) < +\infty$.

III. Strong centre and self centered graphs

In this section, we introduce the concept of centre with respect to the distances which are defined in the above section. Also we present a characterization for a strong self centered weighted graph, which is valid for both α and β - self centered weighted graphs.

Definition 3.1. *The* $\alpha-eccentricity$ *of a node* u *in* G *is defined and denoted by* $e_\alpha(u) = \max\{d_\alpha(u,v)/v \in V, 0 \le d_\alpha(u,v) < \infty\}$.

Similarly the $\beta-eccentricity$ and $strong\ \ eccentricity$ are defined below.

$e_\beta(u) = \max\{d_\beta(u,v)/v \in V, 0 \le d_\beta(u,v) < \infty\}$.

$e_s(u) = \max\{d_s(u,v)/v \in V, 0 \le d_s(u,v) < \infty\}$.

Definition 3.2. *A node* v *is called the* $\alpha-eccentric\ \ node$ *of* u *if* $e_\alpha(u) = d_\alpha(u,v)$.

Set of all $\alpha-eccentric$ nodes of u is denoted by u_α^*.

Similarly we can define $\beta-eccentric\ \ node$ and $strong\ \ eccentric\ \ node$.

Definition 3.3. *Among the* $\alpha-eccentricity$ *of all the nodes of a graph, the minimum is called the* $\alpha-radius$ *of* G. It is denoted by $r_\alpha(G)$.

That is $r_\alpha(G) = \min\{e_\alpha(u)/u \in V\}$.

Also the $\beta-radius$ of G, denoted and defined by $r_\beta(G) = \min\{e_\beta(u)/u \in V\}$ and the $strong\ \ radius$ of G is denoted and defined by $r_s(G) = \min\{e_s(u)/u \in V\}$.

As the radius is the minimum eccentricity, the maximum eccentricity is called the diameter of the graph.

Definition 3.4. *Among the* $\alpha-eccentricity$ *of all the nodes of a graph, the maximum is called the* $\alpha-diameter$ *of* G. it is denoted by $d_\alpha(G)$.

That is $d_\alpha(G) = \max\{e_\alpha(u)/u \in V\}$.

Also the $\beta-diameter$ of G ,denoted and defined by $d_\beta(G) = \max\{e_\beta(u)/u \in V\}$ and the $strong\ \ diameter$ of G is denoted and defined by $d_s(G) = \max\{e_s(u)/u \in V\}$.

Definition 3.5. *A node* u *is called* $\alpha-central$ *if* $e_\alpha(u) = r_\alpha(G)$ *, and called* $\beta-central$ *if* $e_\beta(u) = r_\beta(G)$ *and* $strong\ \ central$ *if* $e_s(u) = r_s(G)$.

Definition 3.6. *The subgraph induced by the set of all* $\alpha - central$ *nodes is called the* $\alpha - cetre$ *of* G. It is denoted by $< C_\alpha(G) >$. Analogously the $\beta - centre$ of G is the subgraph induced by the set of all $\beta - central$ nodes of G, denoted by $< C_\beta(G) >$. Also the $strong \quad centre$ of G is denoted by $< C_s(G) >$ and is defined as the subgraph induced by the set of all strong central nodes.

Example 3.1.

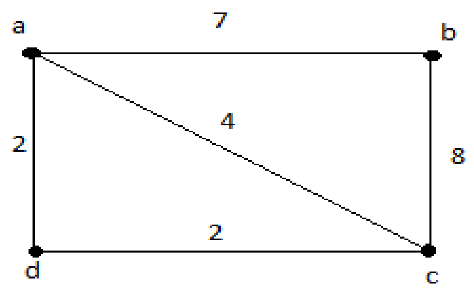

Figure. 2: A weighted graph

$e_\alpha(a) = 15$, $e_\alpha(b) = 8$, $e_\alpha(c) = 15$, $e_\alpha(d) = \infty$, $e_\beta(a) = 4$, $e_\beta(b) = \infty$, $e_\beta(c) = 4$, $e_\beta(d) = 2$, $e_s(a) = 7$, $e_s(b) = 9$, $e_s(c) = 8$, $e_s(d) = 9$, $r_\alpha(G) = 8$, $r_\beta(G) = 2$, $r_s(G) = 7$.

b is the α - central node, d is the β - central node and a is the strong central node.

The following proposition is trivial and valid for both α and β distances.

Preposition 3.1. *In any connected weighted graph* G *with at least one strong arc incident on every vertex* u*, then* $e_s(u) \geq e(u)$ *for every node* u *in* G*. Where* $e(u)$ *is the eccentricity of* u *in the underlying graph of* G*.*

As in the classical concept of distance we have the following inequalities. We omit their proofs as they are obvious.

Theorem 3.1. *Let* $G(V, E, W)$ *be a connected weighted graph, then*

$$r_\alpha(G) \leq d_\alpha(G) \leq 2r_\alpha(G).$$

$$r_\beta(G) \leq d_\beta(G) \leq 2r_\beta(G).$$

$$r_s(G) \leq d_s(G) \leq 2r_s(G).$$

Definition 3.7. *A weighted graph* G *is called* $\alpha - self \quad centered$ *if* G *is isomorphic with its* $\alpha - centre$ *and* $\beta - self \quad centered$ *if* G *is isomorphic with its* $\beta - centre$*. Also* G *is called* $strong \quad self \quad centered$ *if* G *is isomorphic with its* $strong \quad centre$*.*

The following theorem is true for both β and strong self centered graphs.

Theorem 3.2. *Let* $G : (V, E, W)$ *be a connected weighted graph such that there exists exactly one strong arc incident on every node and that all the strong arcs are of equal weight, then* G *is strong self centered.*

Proof. Given that all the nodes of G are incident with exactly one strong arc, and all the strong arcs are of equal weight. That means if $e = (u, v)$ is a strong, then there will be no other strong arcs incident on u and v. Hence $e_s(u) = w(e) = e_s(v)$. By this same argument we get this same equality for any other strong arc. Thus

$e_s(u) = w(e)$ for every node u in G. This proves that G is α - self centered.

The next theorem is a characterization for these self centered graphs.

Theorem 3.3. A connected weighted graph $G:(V,E,W)$ is strong self centered if for any two nodes u and v such that u is a strong eccentric node of v, then v should be one of the strong eccentric nodes of u.

Proof. First assume that G is strong self centered. Also assume that u is a strong eccentric node of v. This means $e_s(v) = d_s(v,u)$. Since G is strong self centered, all nodes will be having the same strong eccentricity. Therefore $e_s(v) = e_s(u)$. From the above two equations we get, $e_s(u) = d_s(v,u) = d_s(u,v)$. Thus $e_s(u) = d_s(u,v)$. That is v is a strong eccentric node of u.
Next assume that u is a strong eccentric node of v. Then v is a strong eccentric node of u. Thus $e_s(u) = d_s(u,v)$, and $e_s(v) = d_s(v,u)$. But $d_s(u,v) = d_s(v,u)$. Therefore $e_s(v) = e_s(u)$, where u and v are two arbitrary nodes of G. Thus all nodes of G have the same strong eccentricity, and hence G is strong self centered.
Similarly, we can prove this result for α and β self centered graphs.

IV. Self Centered Partial Blocks

In this section, we present some necessary conditions for the α - centre of partial blocks. Also we included some characterizations for α - self centered partial blocks.
In the following two theorems, we characterize partial cut nodes of a weighted graph.

Theorem 4.1. *If a node is common to more than α - strong arcs, then it is a partial cut node.*

Proof. In [11] it is proved that, if z is a common node of at least two partial bridges, then it is a partial cut node. Also in [10], we can see, an arc e in a weighted graph G is partial bridge if and only if it is α - strong. Hence the proof is completed.

Example 4.1.

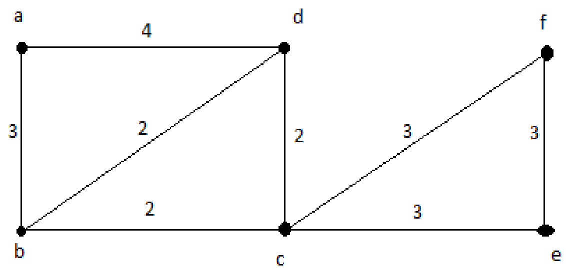

Figure .3: A weighted graph having both cut nodes and p- cut nodes

In the above example (Figure.3), the node a is common to two α - strong arcs, and by theorem 4.1, it is a partial cut node. ($CONN_{G-a}(b,d) = 2 < CONN_G(b,d) = 3$). But the node c is a cut node and hence a partial cut node even though it is not incident with any α - strong arc.
The condition in the above theorem is not sufficient. If we restrict the underlying graph G^* of G to be a block (A graph having no cut nodes), then this condition will be sufficient.

Theorem 4.2. *If $G:(V,E,W)$ is a connected weighted graph such that G^* is a block, then every partial cut node is the common node of at least two α - strong arcs.*

Proof. Suppose that $G:(V,E,W)$ is a connected weighted graph such that G^* is a block. Let u be a partial cut node of G. We have to prove that u is a common node of at least two α - strong arcs. If possible, suppose the contrary.

Case 1. When u is incident with exactly one α - strong arc.
Let $e=(u,v)$ be the α - strong arc which is incident on u. Since G has no cut nodes, u cannot be a cut node, and hence it lies on a cycle, say, C. Let w be the other node in C which is incident on u. By the assumption, (u,w) cannot be α - strong. Therefore the weight of the arc (u,w) can be at the most the minimum of all the weights of arcs in C other than that of the arc (u,v). Hence the deletion of the node u from G, will not reduce the strength of connectedness between any other nodes. It is a contradiction to the fact that u is a partial cut node. Hence our assumption is wrong.

Case 11. When u is incident with no α - strong arc.
Since G has no cut nodes, u cannot be a cut node, and hence u lies on a cycle, say, C. Since u is incident with no α - strong arcs, the two arcs which are incident with u will have the minimum weight among the arcs in C. Therefore u cannot be a partial cut node of G as it does not reduce the strength of connectedness between any pair of nodes of G. This is a contradiction to our assumption that u is a partial cut node of G. So our assumption is wrong.
Hence in all the cases, we have proved that the partial cut node u is incident with at least two α - strong arcs.
In the following theorem, we can see only α - strong arcs are present in the α - centre of partial blocks.

Theorem 4.3. The α - centre of a partial block G contains all α - strong arcs with minimum weight.

Proof. Suppose that $G:(V,E,W)$ is a partial block. Therefore G has no partial cut nodes. We know that, if a node u in a connected weighted graph is common to more than one α - strong arcs, then it is a partial cut node [12]. As G is free from partial cut nodes, at most one α - strong arc can be incident on every node of G. Thus the α - eccentricity, e_α of a node u is the weight of the α - strong arc incident on u. So the α - radius of G, that is $r_\alpha(G)$ is the weight of the smallest α - strong arc. Hence the α - centre of G, $< C_\alpha(G) >$ contains all α - strong arcs of G with minimum weight. This completes the proof of the theorem.

In the next theorem, we present a characterization for α - self centered partial blocks.

Theorem 4.4. Let $G:(V,E,W)$ be a partial block with exactly k number of α - strong arcs. Then G is α - self centered if and only if the following 2 conditions are satisfied.

1. $k = (|V|)/2$
2. All the α - strong arcs have equal weight.

Proof. Let $G:(V,E,W)$ be a partial block with exactly k number of α - strong arcs.
First suppose that, G is α - self centered. We have to prove conditions 1 and 2. Since G is a partial block, it has no partial cut nodes. Also if a node in a weighted graph is common to more than α - strong arc, then it is a partial cut node [12]. So as G is independent of partial cut nodes, no node in G can be an end node of more than one α - strong arc.Since G is α - self centered, the α eccentricity of each node, $e_\alpha(u) = w$ for every u in G. Thus there exists exactly one α - strong arc incident on every node of G. Moreover the above equality holds only when all the α - strong arcs are of equal weight. Therefore conditions 1 and 2 are true.

Conversely suppose that conditions 1 and 2 hold. We have to prove that G is α - self centered. That means we have to prove that $e_\alpha(u) = e_\alpha(v)$ for every u,v in G. Since G is a partial block, each node of G is adjacent with at most one α - strong arc. Also by condition 1, $(|V|)/2 = k$. These two conditions are simultaneously satisfied only when exactly one α - strong arc is incident on each node. By condition 2, all the α - strong arcs are of equal weight. Thus $e_\alpha(u) =$ weight of the α - strong arc incident on u = weight of the α - strong arc incident on $v = e_\alpha(v)$. This is true for all nodes in G. This proves that G is α - self centered.

The following result is a corollary of the above theorem. It is also used as an easy check for α - self centered partial blocks.

Corollary 4.1. If a partial block $G:(V,E,W)$ is α - self centered, then $|V|$ is even.
That means there does not exist a self centered partial block with odd order.

Proof. Let $G(V,E,W)$ be a self centered partial block. Let there be k number of α - strong arcs. Then by the above theorem, $k = (|V|)/2$, which implies that $|V| = 2k$, an even integer. This proves the corollary.

V. Sequences in a weighted graph

In this section, we introduce three types of sequences in a weighted graph and make a characterization of PWGs using the α - sequence.

Definition 5.1. *Let $G:(V,E,W)$ be a connected weighted graph with $|V| = p$. Then a finite sequence $\alpha_s(G) = (n_1, n_2, n_3, ..., n_p)$ is called the α - sequence of G if $n_i =$ number of α strong arcs incident on v_i and = 0, if no α strong arcs are incident on v_i.*
If there is no confusion regarding G, we use the notation α_s instead of $\alpha_s(G)$.
Definition 5.2. *Let $G:(V,E,W)$ be a connected weighted graph with $|V| = p$. Then a finite sequence $\beta_s(G) = (n_1, n_2, n_3, ..., n_p)$ is called the β - sequence of G if $n_i =$ number of β strong arcs incident on v_i and = 0, if no β strong arcs are incident on v_i.*

If there is no confusion regarding G, we use the notation β_s instead of $\beta_s(G)$.

Definition 5.3. *Let $G:(V,E,W)$ be a connected weighted graph with $|V| = p$. Then a finite sequence $S_s = (n_1, n_2, n_3, ..., n_p)$ is called the strong sequence of G if $n_i =$ number of α or β strong arcs incident on v_i and = 0, if no α and β strong arcs are incident on v_i.*
If there is no confusion regarding G, we use the notation S_s instead of $S_s(G)$.

In the following example (Figure.4), we find these sequences.

Example 5.1.

$$\alpha_s = (1,2,0,1)$$
$$\beta_s = (0,1,2,1)$$
$$s_s = (1,3,2,2)$$

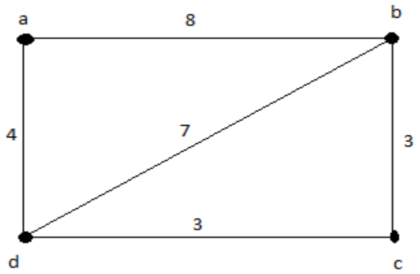

Figure 4. A weighted graph with all types of arcs

As a PWG can have at most one α - strong arc and has no δ arcs[9], we have the following prepositions.

Preposition 5.1. *If* $G:(V,E,W_1,W_2)$ *is a PWG, then either* $\alpha_s(G)=(0,0,0,...,0)$ *or* $\alpha_s(G)=(1,1,0,...,0)$.

Preposition 5.2. *If* $G:(V,E,W_1,W_2)$ *is a PWG, then* $\sum_{n_i\in\beta_s(G)} n_i$ *is either* nC_2 *or* (nC_2-1)

Preposition 5.3. *If* $G:(V,E,W_1,W_2)$ *is a PWG, then for every node* u *in* G *, we have* $e_\beta(u)+e_\alpha(u)=e_s(u)$

Also, $\sum_{n_i\in\alpha_s(G)} n_i+\sum_{n_i\in\beta_s(G)} n_i=\sum_{n_i\in\beta_s(G)} n_i=nC_2$.

VI. Conclusion

In this article, three new distances and three new sequences in weighted graphs are introduced. As reduction in strength between two nodes is more important than total disconnection of the graph, the authors made use of the connectivity concepts in defining the distances and sequences. A special focus on self centered graphs can be seen as they are applied widely. In sections 4 and 5, discussions about the partial block structure and Precisely weighted graph structure have made as they have got many practical applications.

References

[1] J. A. Bondy, H. J. Broersma, J. van den Heuvel and H. J. Veldman, Heavy cycles in weighted graphs, Discuss. Math. Graph Theory, 22, pp. 7-15 (2002).

[2] J. A. Bondy, G. Fan, Cycles in weighted graphs, Combinatorica 11, pp. 191-205 (1991).

[3] J. A. Bondy, G. Fan, Optimal paths and cycles in weighted graphs, Ann. Discrete Mathematics 41, pp. 53-69 (1989).

[4] G. A. Dirac, Some theorems on abstract graphs, Proc. London Math. Soc. (3) 2, pp. 69 - 81 (1952).

[5] M. Grotschel, Gaphs with cycles containing given paths, Ann. Discrete Mathematics 1, pp. 233-245 (1977).

[6] Frank Harary, Graph Theory, Addison Wesley, New York (1969).

[7] John N. Mordeson and Premchand S. Nair, Fuzzy Graphs and Fuzy Hypergraphs, Physica-verlag Heidelberg (2000).

[8] Sunil Mathew, M. S. Sunitha, Bonds in graphs and fuzzy graphs, Advances in Fuzzy Sets and Systems 6(2), pp. 107-119 (2010).

[9] Sunil Mathew, M. S. Sunitha, On totally weighted interconnection networks, Journal of interconnection networks 14,(2010).

[10] Sunil Mathew, M. S. Sunitha, Some Connectivity concepts in weighted graphs, Advances and Applications in Discrete Mathematics 6(1), pp. 45-54 (2010).

[11] Sunil Mathew, M. S. Sunitha, Partial trees in weighted graphs I, Proyecciones Journal of Mathematics30, 163-174 (2011).

[12] Sunil Mathew, M. S. Sunitha, Cycle connectivity in Weighted graphs, Proyecciones Journal of Mathematics 30, 1, pp. 1-17 (2009).

[13] Sunil Mathew, M. S. Sunitha, Partial trees in weighted graphs -I, Proyecciones Journal of Mathematics 30, 2, pp. 163-174 (2011).

[14] S. Zang, X. Li, H. Broersma, Heavy paths and cycles in weighted graphs, Discrete Math. 223, pp. 327-336 (2000).