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Radiation And Chemical Reaction Effects Of Mass Transfer And Hall Current On Unsteady MHD Flow Of A Viscoelastic Fluid In A Porous Medium

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ABSTRACT: *The paper investigated the radiation and chemical reaction effects of mass transfer and Hall current on unsteady MHD flow of a viscoelastic fluid in a porous medium. The resultant equations have been solved analytically. The velocity, temperature and concentration distributions are derived, and their profiles for various physical parameters are shown through graphs. The coefficient of Skin friction, Nusselt number and Sherwood number at the plate are derived and their numerical values for various physical parameters are presented through graphs. The influence of various parameters such as the thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, viscoelasticity parameter, Hartmann number, Hall parameter, and the frequency of oscillation on the flow field are discussed qualitatively.*

Keywords:*Chemical reaction, MHD,Nusselt number, thermal Grashof number, viscoelasticity parameter.*

I. INTRODUCTION

In many transport processes existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion of heat and chemical species, the study of such processes is useful for improving a number of chemical technologies such as polymer production, enhanced oil recovery, underground energy transport, manufacturing of ceramics and food processing. Heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as drying of porous solids, thermal insulations, and cooling of nuclear reactors. At high operating temperature, radiation effects can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of reliable equipment's, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. Micro polar fluids are those consisting of randomly oriented particles suspended in a viscous medium, which can undergo a rotation that can affect the hydrodynamics of the flow, making it a distinctly non-Newtonian fluid. They constitute an important branch of non-Newtonian fluid dynamics where micro rotation effects as well as micro inertia are exhibited.The theory of micro-polar fluids originally developed by Erigen, [1]has been a popular field of research in recent years. Eringen's theory has provided a good model for studying a number of complicated fluids, such as colloidal fluids, polymeric fluids and blood. Micro polar fluid flow induced by the simultaneous action of buoyancy forces is of great interest in nature and in many industrial applications as drying processes, solidification of binary alloy as well as in astrophysics, geophysics and oceanography. When the strength of the magnetic field is strong, one cannot neglect the effect of Hall current. It is of considerable importance and interest to study how the results of the hydro dynamical problems get modified by the effect of Hall currents. Hall currents give rise to a cross flow making the flow three dimensional. Moreover, there exist flows which are caused not only by temperature differences but also by concentration differences. There are several engineering situations wherein combined heat and mass transport arise such as dehumidifiers, humidifiers, desert coolers, and chemical reactors etc. the interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. On account of their varied importance, these flows have been studied by several authors – notable amongst them are Singh and

Singh [2] they investigated MHD flow of Viscous Dissipation and Chemical Reaction over a Stretching porous plate in a porous medium numerically. Das and Jana [3] examined Heat and Mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in a porous medium. Exact Solution of MHDfrees convection flow and Mass Transfer near a moving vertical porous plate in the presence of thermal radiation was investigated by Das [4]. Sonth*et al.* [5] studied Heat and Mass transfer in a viscoelastic fluid over an accelerated surface with heat source/sink and viscous dissipation. Shateyi*et al.* [6] investigated the effects of thermal Radiation, Hall currents, Soret and Dufour on MHD flow by mixed convection over vertical surface in porous medium.

II. MATHEMATICAL FORMULATION

We consider the unsteady flow of a viscous incompressible and electrically conducting viscoelastic fluid over an infinite porous plate with oscillating temperature and mass transfer. The x- axis is assumed to be oriented vertically upwards along the plate and the y-axis is taken normal to the plane of the plate. It is assumed that the plate is electrically non – conducting and a uniform magnetic field of straight B_0 is applied normal to the plate.

The induced magnetic field is assumed constant. So that $\vec{B} = (0, B_0, 0)$.the plate is subjected to a constant suction velocity.The governing equations for the momentum, energy and concentration are as follows:

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_1 \frac{\partial^3 u}{\partial y^2 \partial t} - \sigma B_0^2 \frac{(u + mw)}{\rho(1+m^2)} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu}{k^*} u \quad (1)$$

$$\frac{\partial w}{\partial t} + v_0 \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} - k_1 \frac{\partial^3 w}{\partial y^2 \partial t} - \sigma B_0^2 \frac{(w - mu)}{\rho(1+m^2)} - \frac{u}{k^*} w \quad (2)$$

$$\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial y} = \frac{k_T}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \quad (3)$$

$$\frac{\partial C}{\partial t} + v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_r'(C - C_\infty) \quad (4)$$

The appropriate boundary conditions for the problem are

$$\begin{aligned} &u = 0, w = 0,\ T = T_\infty + (T_\omega - T_\infty)e^{\text{int}},\ C = C_\infty + (C_\omega - C_\infty)e^{\text{int}} \text{ at } y = 0 \\ &u \to 0,\ w \to 0,\ T \to 0,\ C \to 0 \qquad as\ \ y \to \infty \end{aligned} \quad (5)$$

Where u, v and w are velocity components along x, y and z-axis respectively,

g is the acceleration due to gravity, β and β* are the coefficient of volume expansion, k_1 is the kinematic viscoelasticity, ρ is the density, ν is the kinematic viscosity, k_T is the thermal conductivity, C_p is the specific heat in the fluid at constant pressure, σ is the electrical conductivity of the fluid, K^* is the permeability, T_w is the temperature of the plane and T_∞ is the temperature of the fluid far away from plane, C_w is the concentration of the plane and C_∞ is the concentration of the fluid far away from the plane, q_r is the radioactive heat flux and K_r' is the chemical reaction and D is the molecular diffusivity.

And, $v = -v_0$ the negative sign indicate that the suction is towards the plane.

By using the Rosseland approximation, the radioactive flux vector q_r can be written as:

$$q_r' = -\frac{4\sigma^*}{3k_1'}\frac{\partial T_w'^4}{\partial y'} \quad (6)$$

Where, σ^* and k_1' are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about the free stream temperature T_∞' and neglecting higher order terms, thus $T_w'^4 \cong 4T_\infty'^3 T_w' - 3T_\infty'^4$ (7) In view of equations (6) and (7), Equation (3) reduces to:

$$\frac{\partial T}{\partial t} + v_0\frac{\partial T}{\partial y} = \frac{k_T}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty'}{3k_1'\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad (8)$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$u^* = \frac{u}{v_0}, \eta = \frac{v_0 y}{\nu}, t^* = \frac{t v_0^2}{4\nu}, w^* = \frac{w\nu}{v_0^2}, N = \frac{k_1' k}{4\sigma^* T_\infty'^3}, k = \frac{k^* v_0^2}{\nu^2}, n = \frac{n'\nu}{v_0^2},$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}, K^* = \frac{k_1 v_0^2}{4\nu^2}, \Pr = \frac{\nu\rho c_p}{k}, K_r = \frac{K_r'\nu}{v_0^2}, \quad (9)$$
$$Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, Gr = \frac{\nu\beta g\left(T_w' - T_\infty'\right)}{v_0^3}, Gc = \frac{\nu\beta^* g\left(C_w' - C_\infty'\right)}{v_0^3},$$

Substituting the dimensionless variables in (9) into (1), (2), (8), (4) and (5), we get (dropping the stars)

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} - \frac{K}{4}\frac{\partial^3 u}{\partial \eta^2 \partial t} - \frac{\mathrm{M}(u + m\omega)}{\left(1 + m^2\right)} - \frac{1}{k}u + Gr\theta + Gc\phi \quad (10)$$

$$\frac{1}{4}\frac{\partial w}{\partial t} - \frac{\partial w}{\partial \eta} = \frac{\partial^2 w}{\partial \eta^2} - \frac{K}{4}\frac{\partial^3 w}{\partial \eta^2 \partial t} - \frac{M(w - mw)}{\left(1 + m^2\right)} - \frac{1}{k}u \quad (11)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial \eta} = \frac{1}{\Pr}\left(1 + \frac{4}{3N}\right)\frac{\partial^2 \theta}{\partial \eta^2} \quad (12)$$

$$\frac{1}{4}\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial \eta} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial \eta^2} - K_r\phi \quad (13)$$

The corresponding boundary conditions are

u = 0, w = 0, $\theta = e^{int}$ at $\eta = 0$ and u $\to 0, w \to 0, \theta \to 0$ $as\ \eta \to \infty$ (14)

Equations (10) and (11) can be combined into a single equation by introducing the complex velocity.

$U = u(\eta,t) + iw(\eta,t)$, where $i = \sqrt{-1}$, Thus, (15)

$$\frac{1}{4}\frac{\partial U}{\partial t} - \frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial \eta^2} - \frac{K}{4}\frac{\partial^3 U}{\partial \eta^2 \partial t} - \frac{M(1-im)U}{(1+m^2)} - \frac{U}{k} + Gr\theta + Gc\phi \quad (16)$$

With the boundary conditions

U = 0, $\theta = e^{int}$ at $\eta = 0$ and u $\to 0$, , $\theta \to 0$, $\emptyset \to 0$ $as\ \eta \to \infty$ (17)

Where $Gr, Gc, \mathrm{K}, \mathrm{Pr}, K_r, Sc, \mathrm{N}$ are the thermal Grashof number, modified Grashof number, viscoelastic parameter, Prandtl number, Chem,$\emptyset = e^{int}$ical reaction number, Schmidtnumber and radiation parameter respectively.

III. SOLUTION OF THE PROBLEM

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we assume the trial solution for the velocity, temperature and concentration as:

$U(\eta,t) = U_0(\eta)e^{int}$ ……. (18) , $\theta(\eta,t) = \theta_0(\eta)e^{int}$ … (19) $\phi(\eta,t) = \phi_0(\eta)e^{int}$ …… (20)

Substituting Equations (18), (19) and (20) in Equations (16), (14) and (15), we obtain:

$$P_1U_0'' - U_0' - P_2U_0 = -Gr\theta_0 - Gc\phi_0 \quad (21)$$

$P_3\theta_0'' + \theta_0' - \frac{in}{4}\theta_0 = 0$ ……….. (22) $\phi_0'' + Sc\phi_0' - \left(Kr + \frac{in}{4}\right)Sc\phi_0 = 0$ (23)

Here the primes denote the differentiation with respect toη.The corresponding boundary conditions can be written as $U_0 = 0$, $\theta_0 = 1$,$\emptyset_0 = 1$ at $\eta = 0$ and$U_0 \to 0$, $\theta_0 \to 0$, $\emptyset_0 \to 0$ $as\ \eta \to \infty$(24)

The analytical solutions of equations (21) – (23) with satisfying the boundary conditions (24) are given by

$$U_0(\eta) = A_1e^{-m_1\eta} + A_2e^{-m_2\eta} + A_3e^{-m_3\eta} \quad (25)$$

$\theta_0(\eta) = e^{-m_1\eta}$ (26) $\phi_0(\eta) = e^{-m_2\eta}$ (27)

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$U(\eta,t) = \left[A_1e^{-m_1\eta} + A_2e^{-m_2\eta} + A_3e^{-m_3\eta}\right]e^{\mathrm{int}} \quad (28)$$

$\theta(\eta,t) = \left(e^{-m_1\eta}\right)e^{\mathrm{int}}$ ……… (29) $\phi(\eta,\mathrm{t}) = \left(e^{-m_2\eta}\right)e^{\mathrm{int}}$ ……………… (30)

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flows.

Skin friction: Knowing the velocity field, the skin – friction at the plate can be obtained, which in non – dimensional form is given by $C_f = \left[-\frac{\partial U}{\partial \eta} \right]_{\eta=0} = \left[A_1 m_1 + A_2 m_2 + A_3 m_3 \right] e^{\text{int}}$

.**Nusselt number:** Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non –dimensional form is given, in terms of the Nusselt number, is given by $Nu = -\left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} = m_1 e^{\text{int}}$

Sherwood number: Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non –dimensional form, in terms of the Sherwood number, is given by $Sh = -\left(\frac{\partial \phi}{\partial \eta} \right)_{\eta=0} = m_2 e^{\text{int}}$

$$P_1 = 1 - \frac{Kin}{n},\ P_2 = \left[\frac{M(1-im)}{1+m^2} + \frac{1}{k} + \frac{in}{4} \right],\ \mathrm{P}_3 = \frac{1}{\mathrm{Pr}}\left(1 + \frac{4}{3N} \right)$$

Where $$m_1 = \frac{1+\sqrt{1+P_3(in)}}{2},\ m_2 = \frac{Sc+\sqrt{Sc^2+Sc(in)}}{2},\ m_3 = \frac{1+\sqrt{1+4P_1P_2}}{2P_1}$$

$$A_2 = \frac{-Gr}{P_1 m_1^2 - m_1 - P_2}, A_2 = \frac{-Gc}{P_1 m_2^2 - m_2 - P_2},\ A_3 = -\left(A_1 + A_2 \right)$$

IV. FIGURES

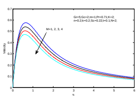

Fig-1. Velocity profilesfor different values of M .

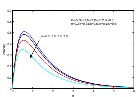

Fig-2. Velocity profiles for different values of m .

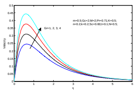

Fig-3. Velocity profiles different values for of Gr.

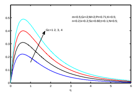

Fig-4. Velocity profiles for different values of Gc.

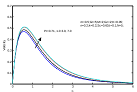

Fig-5. Velocity profiles for different values of $\Pr$.

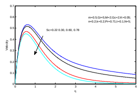

Fig-6. Velocity profiles for different values of Sc.

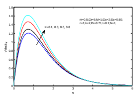

Fig-7. Velocity profiles for different values of K.

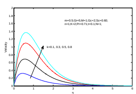

Fig-8. Velocity profiles for different values of k.

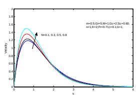

Fig-9. Velocity profilesfor different values of N.

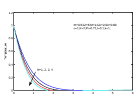

Fig-10. Temperature profiles for different values of N.

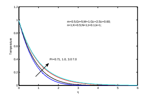

Fig-11. Temperature profiles for different values of $\Pr$.

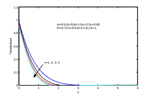

Fig-12. Temperature profilesfor different values of n.

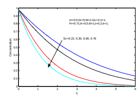

Fig13.Concentrationprofiles for different values of Sc.

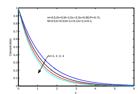

Fig14.Concentration profiles for different values of Kr.

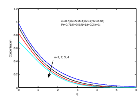

Fig15.Concentration profilesfor different values of n.

V. RESULTS AND DISCUSSION

The effect of mass transfer and Hall current on unsteady MHD flow of a viscoelastic fluid in a porous medium has been formulated and solved analytically. In order to understand the flow of the fluid, computations are performed for different parameters such as M, m, Gr, Gc, Pr, Sc, K, k, N and Kr.

Figures 1-8 represent the velocity profiles, figures 9 and 10 depict the temperature profiles and figures 11 and 12 show the concentration profiles with varying parameters respectively.

Fig-1 depicts the effect of velocity for different values of Hartmann number M. It is observed that the graph shows that velocity decreases with an increasing M. The effects of velocity profiles are for different values of *m* as shown in Fig.2. It is noticed that the velocity decreases with an increasing m. The effect of velocity for different values of Grashof number Gr is presented in Fig.3. It is seen that the velocity increases with an increases in Gr. The effect of velocity profiles for different values of modified Grashof number Gc is displayed in Fig.4. It is observed that velocity increases with the increase in Gc. The effect of velocity profiles for different values of Prandtl number Pr is presented in Fig.5. It is observed that the velocity decreases with increase in Pr. The effect of velocity for different values of Schmidt number Sc is given in Fig.6. It is noticed that the graph show that velocity decreases with the increase in Sc. Fig.7 denotes the effect of velocity for different values of viscoelastic parameter *K*. It is seen that velocity increases with the increase in *K*. The effect of velocity profiles for different values of permeability parameter *k* is shown in Fig.8. It is observed that velocity increases with the increase in *k*. In Fig. 9, the velocity profiles are presented for different values radiation parameterN. It is observed that the radiation parameter is increases, the velocity profiles also increases.

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