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Bianchi Type I Viscous Cosmological models with varying Λ in the presence of Electromagnetic Field

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ABSTRACT :*In this paper, we have investigated Bianchi Type I spacetime filled with viscous fluid in the presence of magnetic field. Here we assume an equation of state $p = \omega\rho$. We obtain the solution according to the law, the variation of the average Hubble parameter given by Berman: $H = Da^{-n}$. The geometrical and kinematical features of the model are also discussed.*

Keywords – *Bianchi Type I, Cosmological model, Electromagnetic Field, General relativity, Viscous Fluid*

I. INTRODUCTION

It is a challenging problem to determine the exact physical situation at the very early stages of the formation of our universe. The cosmological models which are spatially homogenous and anisotropic play significant roles in the description of the universe at its early stages of evolution. Bianchi I-IX spaces are very useful to constructing special homogeneous cosmological models. Spatially homogeneous cosmologies also play an important role in the attempt to understand the structure and the properties of the space of all cosmological solutions of Einstein's field equations. In the modern cosmological theories, the dynamic cosmological term $\Lambda(t)$ remains a focal point of interest as it solves the cosmological constant problem in a natural way. There is significant observational evidence towards identifying Einstein's cosmological constant Λ or a component of material content of the universe that varies slowly with time and space and so acts like Λ. Recent cosmological observations by the High-Z Supernova Team and the Supernova Cosmological Project [1–7] suggest the existence of a positive cosmological constant Λ with magnitude $\Lambda \left(\frac{Gh}{c^3}\right) \approx 10^{-123}$.

Viscosity is important for number of reasons. Heller and Klimek [8] have investigated viscous fluid cosmological model without initial singularity. They have shown that the introduction of bulk viscosity effectively remove the initial singularity. Roy and Singh [9] have investigated LRS Bianchi Type-V cosmological model with viscosity, Santos et al. [10] investigated isotropic homogeneous cosmological model with bulk viscosity assuming viscous coefficient as power function of mass density. Bali and Singh [11] have investigated Bianchi Type –V viscous fluid string dust cosmological model assuming the condition that the bulk coefficient (ξ) is inversely proportional to the expansion (θ) in the model and shown the existence of the model.

Magnetic field has important role at the cosmological scale and is present in galactic and intergalactic spaces. The importance of the magnetic field for various astrophysical phenomena has been studied by Banerjee et al. [12] , Chakraborty[13], Tikekar and Patel [14], Patel and Maharaj [15] and Singh and Singh[16] . Further Melvin [17] has pointed out that during the evolution of the universe, the matter was highly ionized state and is smoothly coupled with the field, subsequently forming neutral matter as a result of expansion of the universe Recently, Kandalkar et. al [18] investigated Bianchi type-V string dust cosmological models with bulk viscous magnetic field. Also Samdurkar and Sen [19] observed the effect of bulk viscosity on Bianchi Type V cosmological model filled with viscous fluid with varying cosmological term Λ. Suresh Kumar and C. P. Singh [20] used the variation of the average Hubble parameter $H = Da^{-n}$ to present the exact solutions for Bianchi I space-time with anisotropic dark energy and perfect fluid.

In this paper, we have studied Bianchi Type I magnetized bulk viscous cosmological model to get determinate solution, coefficient of bulk viscosity (ξ) is inversely proportional to the expansion (θ) is considered. Here, we have used the variation of the average Hubble parameter given by Berman [21] and investigated the cases $n \neq 0$ and n = 0. The behavior of the models in the presence of magnetic field and bulk viscosity are observed. The physical and geometrical aspects of the models are also discussed.

II. METRIC AND FIELD EQUATIONS

We consider the Bianchi type I space time in the form $ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2$ (1)

Einstein's equation we consider here is $R_i^j - \frac{1}{2}Rg_i^j = -\left(8\pi T_i^j - \Lambda g_i^j\right)$ (2)

The energy momentum tensor with magnetic field along the x-direction is given by

$$T_i^j = (\rho + p')u_i u^j - p'\delta_i^j + \eta g^{j\beta}\left(u_{i;\beta} + u_{\beta;i} - u_i u^j u_{\beta;\alpha} - u_\beta u^\alpha u_{i;\alpha}\right) + E_i^j \tag{3}$$

where $p' = p - \left(\xi - \frac{2}{3}\eta\right)u^i_{;i}$ (4)

Here ρ is the energy density, p is pressure and η and ξ are coefficients of shear and bulk viscosity resp.

u_i is the flow vector satisfying the relation $g_{ij}u^iu^j = 1$ (5)

E_i^j is the energy momentum for magnetic field given by $E_i^j = \frac{1}{4\pi}\left(F_{i\alpha}F^{j\beta}g^{\alpha\beta}g_{ij} - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\right)$ (6)

where E_{ij} is the electromagnetic field tensor which satisfies the Maxwell's equations

$$F[ij,\alpha] = 0,\ \left(F^{ij}\sqrt{-g}\right)_{;j} = 0 \tag{7}$$

In co-moving co-ordinates, the incident magnetic field is taken along x-axis, with the help of Maxwell equation (7), the only non-vanishing component of F_{ij} is F_{23} = constant = K (8)

We assume that the fluid obeys an equation of state of the form $p = \omega\rho \quad (0 \le \omega \le 1)$ (9)

The Einstein field equations (2) for the line element (1) has been set up as

$$\frac{\dot A\dot B}{AB} + \frac{\dot A\dot C}{AC} + \frac{\dot B\dot C}{BC} + \Lambda = 8\pi\rho - \frac{K}{A^2B^2} \quad \text{---- (10)}$$

$$\frac{\ddot B}{B} + \frac{\ddot C}{C} + \frac{\dot B\dot C}{BC} + \Lambda = 8\pi\left(-p' + 2\eta\frac{\dot A}{A}\right) - \frac{K}{A^2B^2} \quad \text{-- (11)}$$

$$\frac{\ddot A}{A} + \frac{\ddot C}{C} + \frac{\dot A\dot C}{AC} + \Lambda = 8\pi\left(-p' + 2\eta\frac{\dot B}{B}\right) - \frac{K}{A^2B^2} \quad (12)$$

$$\frac{\ddot A}{A} + \frac{\ddot B}{B} + \frac{\dot A\dot B}{AB} + \Lambda = 8\pi\left(-p' + 2\eta\frac{\dot C}{C}\right) - \frac{K}{A^2B^2} \quad (13)$$

We define the average scale factor a and generalized Hubble parameter H, expansion factor θ, shear σ, deceleration parameter q and anisotropy parameter $\bar{A}$ for Bianchi I universe as,

$$V = a^3 = ABC \quad \text{------------------------ (14)}$$

$$H = \frac{\dot a}{a} = \frac{1}{3}\left(\frac{\dot A}{A} + \frac{\dot B}{B} + \frac{\dot C}{C}\right) \quad \text{-------- (15)}$$

$$\theta = \frac{\dot A}{A} + \frac{\dot B}{B} + \frac{\dot C}{C} \quad \text{------------------------- (16)}$$

$$2\sigma^2 = \left(H_1^2 + H_2^2 + H_3^2\right) - \frac{\theta^2}{3} \quad \text{------ (17)}$$

$$q = \frac{-a\ddot a}{\dot a^2} \quad \text{------------------------------- (18)}$$

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad \text{-------------------(19)}$$

We assume the coefficient of shear viscosity is proportional to the scale of expansion $\eta \alpha \theta$ i. e $\eta = l\theta$ (20) where l is proportionality constant.

Solving (11), (12) and (13), we get $A = D_1V^{\frac{1}{3}}\exp\left[X_1\int\frac{dt}{V^{16\pi l+1}}\right]$ (21)

$$B = D_2V^{\frac{1}{3}}\exp\left[X_2\int\frac{dt}{V^{16\pi l+1}}\right] \quad \text{----- (22)}$$

$$C = D_3V^{\frac{1}{3}}\exp\left[X_3\int\frac{dt}{V^{16\pi l+1}}\right] \quad \text{--- (23)}$$

where $D_1=\dfrac{1}{d_1^{\frac{-2}{3}}d_2^{\frac{-1}{3}}}, D_2=\dfrac{1}{d_1^{\frac{1}{3}}d_2^{\frac{-1}{3}}}, D_3=\dfrac{1}{d_1^{\frac{1}{3}}d_2^{\frac{2}{3}}}, X_1=\dfrac{2x_1+x_2}{3}, X_2=\dfrac{-x_1+x_2}{3}, X_3=\dfrac{-x_1-2x_2}{3}$

such that $X_1+X_2+X_3=0,\ D_1D_2D_3=1$

Also to get deterministic solution bulk viscosity is assumed to be a simple power function of the energy density

$$\xi=\xi_0\rho^m \quad \text{(where } \xi_0 \text{ and m are constants)} \tag{24}$$

The variation of the average Hubble parameter is given by $H=Da^{-n}$ (25)

Case I: Cosmology for $n\neq 0$

Solving (25) we get $a(t)=(nDt+c_1)^{\frac{1}{n}}$ (26)

Using equation (21), (22) and (23), we get $A=D_1(nDt+c_1)^{\frac{1}{n}}\exp\left[\dfrac{X_1}{D(n-48\pi l-3)}(nDt+c_1)^{\frac{n-3(16\pi l+1)}{n}}\right]$

(27)

$$B=D_2(nDt+c_1)^{\frac{1}{n}}\exp\left[\frac{X_2}{D(n-48\pi l-3)}(nDt+c_1)^{\frac{n-3(16\pi l+1)}{n}}\right] \tag{28}$$

$$C=D_3(nDt+c_1)^{\frac{1}{n}}\ xp\left[\frac{X_3}{D(n-48\pi l-3)}(nDt+c_1)^{\frac{n-3(16\pi l+1)}{n}}\right] \tag{29}$$

From equation (10) we get

$$8\pi\rho=\frac{(X_1X_2+X_2X_3+X_1X_3)}{(nDt+c_1)^{\frac{6(16\pi l+1)}{n}}}+\frac{3D^2}{(nDt+c_1)^2}+\frac{K}{D_1^2D_2^2(nDt+c_1)^{\frac{4}{n}}\exp\left[\frac{2(X_1+X_2)}{D(n-48\pi l-3)}(nDt+c_1)^{\frac{n-3(16\pi l+1)}{n}}\right]}+\Lambda \tag{30}$$

From equation (11) and (24), we get

$$8\pi p=\frac{24\pi\xi_0\rho^m D}{(nDt+c_1)}-\frac{(X_2^2+X_3^2+X_2X_3)}{(nDt+c_1)^{\frac{6(16\pi l+1)}{n}}}+\frac{(2n-3)D^2}{(nDt+c_1)^2}-\frac{K}{D_1^2D_2^2(nDt+c_1)^{\frac{4}{n}}\exp\left[\frac{2(X_1+X_2)}{D(n-48\pi l-3)}(nDt+c_1)^{\frac{n-3(16\pi l+1)}{n}}\right]}-\Lambda \tag{31}$$

Model 1: Solution for $\xi=\xi_0, m=0$

$$\rho=\frac{1}{8\pi(\omega+1)}\left[\frac{24\pi\xi_0 D}{(nDt+c_1)}+\frac{(X_1X_2+X_1X_3-X_2^2-X_3^2)}{(nDt+c_1)^{\frac{6(16\pi l+1)}{n}}}+\frac{2nD^2}{(nDt+c_1)^2}\right] \tag{32}$$

$$\Lambda=\frac{1}{(\omega+1)}\left\{\begin{array}{l}\dfrac{24\pi\xi_0 D}{(nDt+c_1)}-\dfrac{((X_2^2+X_3^2+X_2X_3)+\omega(X_1X_2+X_2X_3+X_1X_3))}{(nDt+c_1)^{\frac{6(16\pi l+1)}{n}}}+\dfrac{(2n-3\omega-3)D^2}{(nDt+c_1)^2}\\ \qquad -\dfrac{K(\omega+1)}{D_1^2D_2^2(nDt+c_1)^{\frac{4}{n}}\exp\left[\frac{2(X_1+X_2)}{D(n-48\pi l-3)}(nDt+c_1)^{\frac{n-3(16\pi l+1)}{n}}\right]}\end{array}\right\} \tag{33}$$

Model 2: Solution for $\xi=\xi_0\rho$, m = 1

$$\rho=\frac{1}{8\pi\left[(\omega+1)-\frac{3\xi_0 D}{(nDt+c_1)}\right]}\left\{\frac{(X_1X_2+X_1X_3-X_2^2-X_3^2)}{(nDt+c_1)^{\frac{6(16\pi l+1)}{n}}}+\frac{2nD^2}{(nDt+c_1)^2}\right\} \tag{34}$$

$$\Lambda=\frac{1}{\left[(\omega+1)-\frac{3\xi_0 D}{(nDt+c_1)}\right]}\left\{\frac{(X_1X_2+X_1X_3-X_2^2-X_3^2)}{(nDt+c_1)^{\frac{6(16\pi l+1)}{n}}}+\frac{2nD^2}{(nDt+c_1)^2}\right\} \tag{35}$$

$$-\frac{(X_1X_2+X_2X_3+X_1X_3)}{(nDt+c_1)^{\frac{6(16\pi l+1)}{n}}}-\frac{3D^2}{(nDt+c_1)^2}-\frac{K}{D_1^2D_2^2(nDt+c_1)^{\frac{4}{n}}\exp\left[\frac{2(X_1+X_2)}{D(n-48\pi l-3)}(nDt+c_1)^{\frac{n-3(16\pi l+1)}{n}}\right]}$$

Some physical and geometrical aspects of the model

The spatial volume V, Hubble parameter H, expansion factor θ, shear σ, deceleration parameter q and anisotropy parameter $\overline{A}$ are given by

$$V = D_1 D_2 D_3 (nDt + c_1)^{\frac{3}{n}} \qquad (36)$$

$$\theta = \frac{3D}{(nDt + c_1)} \quad \text{----------} \quad (37)$$

$$\sigma^2 = \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{2(nDt + c_1)^{\frac{6(16\pi l+1)}{n}}} \quad \text{--------}(38)$$

$$q = n-1 \quad \text{--------------------------} \quad (39)$$

$$\overline{A} = \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{3D^2 (nDt + c_1)^{\frac{6(16\pi l+1)-2n}{n}}} \qquad (40)$$

Case II: Cosmology for n = 0

Solving equation (25) we get, $a = c_0 e^{Dt}$ (41)

From (21), (22) and (23), $A = c_0 D_1 \exp\left[Dt - \frac{X_1}{Lc_1} e^{-c_1 t}\right]$ (42)

$$B = c_0 D_2 \exp\left[Dt - \frac{X_2}{Lc_1} e^{-c_1 t}\right] \quad (43) \qquad C = c_0 D_3 \exp\left[Dt - \frac{X_3}{Lc_1} e^{-c_1 t}\right] \quad (44)$$

where $L = c_0 e^{3(16\pi l+1)}$ and $c_1 = 3D(16\pi l + 1)$

From equation (10) we get,

$$\rho = \frac{1}{8\pi}\left\{3D^2 + \frac{(X_1 X_2 + X_1 X_3 + X_2 X_3)}{L^2} e^{-2c_1 t} + \frac{K}{c_0{}^4 D_1{}^2 D_2{}^2 \exp\left[4Dt + \frac{2X_3}{Lc_1} e^{-c_1 t}\right]} + \Lambda\right\} \qquad (45)$$

From equation (11), we get

$$p = \frac{1}{8\pi}\left\{24\pi\xi_0 \rho^m D - 3D^2 - \frac{\left(X_2{}^2 + X_3{}^2 + X_2 X_3\right)}{L^2} e^{-2c_1 t} - \frac{K}{c_0{}^4 D_1{}^2 D_2{}^2 \exp\left[4Dt + \frac{2X_3}{Lc_1} e^{-c_1 t}\right]} - \Lambda\right\} \qquad (46)$$

Model 1: Solution for $\xi = \xi_0, m = 0$,

$$\rho = \frac{1}{8\pi(\omega+1)}\left\{24\pi\xi_0 D + \frac{\left(X_1 X_2 + X_1 X_3 - X_2{}^2 - X_3{}^2\right)}{L^2} e^{-2c_1 t}\right\} \qquad (47)$$

$$\Lambda = \frac{1}{(\omega+1)}\left\{\begin{array}{l} 24\pi\xi_0 D - 3(\omega+1)D^2 \\ - \frac{\left[\left(X_2{}^2 + X_3{}^2 + X_2 X_3\right) + \omega(X_1 X_2 + X_1 X_3 + X_2 X_3)\right] e^{-2c_1 t}}{L^2} - \frac{K(\omega+1)}{c_0{}^4 D_1{}^2 D_2{}^2 \exp\left[4Dt + \frac{2X_3}{Lc_1} e^{-c_1 t}\right]} \end{array}\right\} \qquad (48)$$

Model 2: Solution for $\xi = \xi_0 \rho$, m = 1

$$\rho = \frac{\left(X_1 X_2 + X_1 X_3 - X_2{}^2 - X_3{}^2\right)}{8\pi[(\omega+1) - 3\xi_0 D]L^2} e^{-2c_1 t} \qquad (49)$$

$$\Lambda = \frac{\left\{\left(X_1 X_2 + X_1 X_3 - X_2{}^2 - X_3{}^2\right) - [(\omega+1) - 3\xi_0 D](X_1 X_2 + X_1 X_3 + X_2 X_3)\right\}}{[(\omega+1) - 3\xi_0 D]L^2} e^{-2c_1 t} - \frac{K}{c_0{}^4 D_1{}^2 D_2{}^2 \exp\left[4Dt + \frac{2X_3}{Lc_1} e^{-c_1 t}\right]} - 3D^2 \qquad (50)$$

Some physical and geometrical aspects of the model

The spatial volume V, Hubble parameter H, expansion factor θ, shear σ, deceleration parameter q and anisotropy parameter $\overline{A}$ are given by

$$V = c_0{}^3 D_1 D_2 D_3 e^{3Dt} \quad \text{------------ (51)}$$

$$H = D \quad \text{-------- (52)}$$

$$\theta = 3D \quad \text{------------ (53)}$$

$$\sigma^2 = \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{2L^2 e^{2c_1 t}} \quad \text{-------- (54)}$$

$$q = -1 \quad \text{------------ (55)}$$

$$\overline{A} = \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{3D^2 L^2} e^{-2c_1 t} \quad \text{----- (56)}$$

III. CONCLUSION

In this paper, we discuss Bianchi Type I models with viscous fluid and varying cosmological term Λ in the presence of electromagnetic field by assuming equation of state $p = \omega\rho$. This shows that value of deceleration parameter is constant in both the cases. It is interesting to note that the works of Perlmutter et al. [22-23], Riess et al. [6, 24], Tonry et al. [25]Knop et al. [26] and John [27] on the basis of recent observations of SNe Ia, reveal the approximate value of deceleration parameter in the range -1< q < 0, which can be obtained from the relation q = n -1 by restricting n in the range 0 < n < 1. Thus, the relevance of relation $H = Da^{-n}$ is justified. Here we investigated two cases for n $(n \neq 0, n = 0)$.

The solutions for the scale factors have a combination of power law term and exponential term in the product form. For $n \neq 0$, we observe that at $t = \frac{-c_1}{nD}$, the spatial volume vanishes while all the parameters diverge. Therefore the model has big bang singularity at $t = \frac{-c_1}{nD}$, which can be shifted to t = 0 by choosing $c_1 = 0$. The directional scale factors and volume of the universe increase exponentially with cosmic time whereas the mean Hubble parameter and expansion scalar are constant throughout the evolution. Therefore, uniform exponential expansion takes place. Also, we observe that at $t \to \infty$, the scale factors and volume of the universe becomes infinitely large whereas shear scalar, Anisotropy parameter and Hubble parameters becomes constant. By taking suitable values of constant, we get $\Lambda > 0, \rho > 0$.

For n = 0, we observe that the model has no initial singularity. The directional scale factors and volume of the universe increase exponentially with the cosmic time whereas the mean Hubble parameter and expansion scalars are constant throughout the evolution. Therefore, uniform exponential expansion takes place. As $t \to \infty$, the scale factors and volume of the universe becomes infinitely large whereas mean Hubble parameter and expansion scalars become constant. The pressure and energy density and cosmological constant also become constant and for $t \to \infty$ the shear scalar tends to zero. For n = 0, we get q = -1. This value of deceleration parameter leads $\frac{dH}{dt} = 0$ which states that the greatest value of Hubble parameter and the fastest rate of expansion of the universe. Therefore, the universe exhibits the fastest rate of expansion among all the values of n.

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