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COMPETITION IN COMMUNICATION NETWORK: A GAME WITH PENALTY

Sapana P. Dubey

(Department of applied mathematics,PIET, Nagpur,India)

ABSTRACT : *We are proposing a new network transmission model wherein we have a number of users who wish to send their throughput demand in the form of packets through one or more links with minimum cost or more efficiently than the other users. This situation introduces the role of non-cooperative games in the communication network. We introduce three aspects which distinguish this model from other transmission network models studied elsewhere. These aspects are regarding introduction of time (discrete) and penalty to the users, and cost of transmission to be an increasing function of time.*

Keywords: *Convex, Nash equilibrium, Non-cooperative game*

I. INTRODUCTION

Routing in a communication network is a game with imperfect information where the complete knowledge of strategies of other users/players is impossible. In this game decisions are made simultaneously by all the users and cost functions are known to every player. There are few works using game theoretical concept in a communication network. For example, in [1], A. Orda, R. Rom and N. Shimkin provide Nash equilibrium for the system of two node multiple link using non-cooperative game. They have proved uniqueness of Nash Equilibrium Point (NEP) under reasonable convexity conditions. In this paper each user can measure the load on the network links and change their behavior based on the state of network.

I. Sahin, M.A. Simaan [2] have derived an optimal flow and routing control policy for two node parallel link communication networks with multiple competing users. In this paper network consists of several parallel links and each user uses different preference constant for different links. The review paper by F.N. Pavlidou and G. Koltsidas [3] presents different routing models that use game theoretical methodologies for conventional and wireless networks as well. Time dependent behavior has an impact on the performance of telecommunication models and queuing theory is also used for communication perspective by Messey [4]. Bottleneck routing games and Nash equilibrium is discussed in [5] for splittable and unsplittable flow. In [6] authors considered that the cost function for the link is polynomial and they have established the uniqueness of Nash equilibrium.

This work presented here deals with routing of data packets in a communication network. The players/users come to the "game" with the knowledge of the number of packets they wish to send through one or more network link over the m chances/shots. As usual each link at a given chance/shot has a finite capacity to carry the packets. In all other models, the users are dissuaded from sending number of packets exceeding capacity of a link by making the cost infinity for such a situation and in such a situation there is a transmission failure in the sense no one's packets are sent through the link. In our model, the cost of transmission remains finite even if the sum of packets wished to be sent by the users through a given link in a given slot/shot exceeds the specified capacity of the link. However, we consider a number of scenarios to deal with such a situation. In this simple model, the game becomes competitive as the users would want to send their packets in earlier and few shots possible. The game would be very competitive and interesting if the total number of packets wished to be sent by all the players together over all the shots exceeds the total capacity of all link summed for all the shots.

The paper comprises seven sections. In section 2, we present a mathematical modeling for communication network. In section 3, Routing Scheme and the cost function is discussed showing explicitly the relation of cost function with time and number of packets, being transmitted. In sections 4, Extended cost function is introduced for packets more than capacity of the link. In section 5, we present a number of scenarios to deal with the situation of violating the capacity of link. In section 6, we prove some theorems to show existence of Nash equilibrium point in this game and in section 7 we provide some concluding remarks.

II. MATHEMATICAL MODELING

In the present communication network model, we consider p number of users sharing k parallel links connecting a common source node to a common destination node. We assume that the network/links are available to the users over a discrete range of time known as time slots. We assume that there are m time slots in a single cycle. Let $L = \{1, 2, 3, \ldots . k\}$ be the set of parallel links, $N = \{1, 2, 3, \ldots \ldots p\}$ be the set of users and S={1,2,3,…..m} be the time slots. We assume that users are rational and selfish for this competitive game. Each user $n \in N$ has throughput demand $D^{(n)}$ which he/she wants to ship from source to destination. A user sends his/her throughput demand in the form of data packets through the communication links and is able to decide at any time how the data packets will be transmitted and what fraction of throughput demand should be sent at that time through each link.

We assume that each link is available for all the users after a unique interval of time. Capacity λ_l of each link is fixed. Let $P_l^n(t)$ denotes the number of packets transmitted through link l at time t by user n .

$P_l^n(t)$ satisfies the following conditions :

P₁ : $P_l^n(t) \geq 0$ (Non – negative constraint)

P₂ : $\sum_n P_l^n(t) \leq \lambda_l$ (Capacity constraint for each link l)

P₃ : $D^{(n)} = \sum_t \sum_n P_l^n(t)$ (Demand constraint for each user n)

Turning our attention to a link $l \in L$, let $P_l(t)$ be the total transmitted packets on that link at time t and s_l be the speed of data packets for link l, which remains fixed in time. We visualize this problem as a non-cooperative game in which each user wants to minimize his/her cost. Let $C^{(n)}$, denote the cost borne by n^{th} user.The following general assumptions on the cost function $C^{(n)}$ of each user $n \in N$ will be imposed throughout the paper.

A₁ : $\mathbf{C^{(n)} = \sum_t \sum_l C(P_l^n, t)}$

It is a sum of cost of routed packets over each link $l \in L$ by user $n \in N$.

A₂ : Cost function is non- linear and non- negative function **.**

A₃ : Cost function is strictly increasing with no. of packets i and time t.We shall consider the cost of transmitting packets (i in number) at time t to be of the type

$$C_{p_i} = f(t) \, . \, \emptyset(i, i^-)$$

Additional assumptions concerning the time function f(t) are

T₁ : Each user get m number of instances called discrete time slots

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T_2 : Flow of packets is continuous which implies that there is no congestion in the system.

T_3 : Users can transmit more than one packets on the link at the same time slot. They must obey the capacity constraint (P_2) and non-negative constraint (P_1).

III. ROUTING SCHEME AND COST FUNCTION

We describe routing scheme in a single link communication network for users. Users can route one or more packets on this link at time slot t. (fig. 3.1)

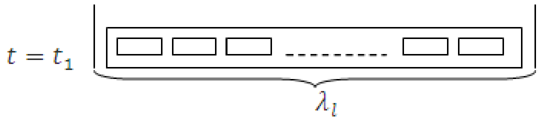

fig. 3.1: Available space at $t = t_1$

The expected cost C for the user n at t^{th} time slot depends on the number of packets i to be routed by user n and number of packets i^- by other users.

$$C_t^{(n)} = f(t).\emptyset(i, i^-) = \frac{\sqrt{t}\,(e^i - 1)}{\lambda_l + 1 - (i + i^-)} \qquad \text{when } i + i^- \le \lambda_l \qquad \text{Where } t \in S.$$

Therefore total cost for user n to send i packets through link l is

$$C^{(n)} = \sum_t \sum_i f(t).\,\phi(i, i^-)$$

The time function $f(t)$ is the square root of time slot (or instance).

IV. EXTENDED COST FUNCTION

The cost function is presented as link availability and time based formulation. The capacity constraint (P_2) for each link l is provided that the total number of packets by all users on the link l should be less than or equal to link capacity. In Wardop's first principle (1952) states that every user seeks minimum travel cost under their individual prospection. The following facts may arise:

1. The game is considered as game with imperfect information and non-cooperative, therefore the complete knowledge of strategies of other players is impossible. In this game decisions are made simultaneously by all the users and hence without interaction it is very difficult to maintain the capacity constraint i.e. $i + i^- \le \lambda_l$.
2. When $i + i^- > \lambda_l$ i.e. number of packets shipped on the link exceeds link capacity then effect on the cost function is not defined.
3. If number of packets exceeds the link capacity then there may be a possibility that some packets will be transmitted or none of them will be transmitted.
4. Since decisions are made simultaneously by all the users, it is difficult to decide responsible user for violating capacity constraint as well as to decide who has to pay more?
5. Will game be over or not for that responsible user? If game is over for the user then what about the packets of that user?

To consider all above points there is need to extend the cost function for user n to transmit i packets at the time slot t. Now the extended cost function is

$$C_t^{(n)} = f(t).\emptyset(i,i^-) = \begin{cases} \frac{\sqrt{t}\,(e^i - 1)}{\lambda_l + 1 - (i + i^-)} & \text{when } i + i^- \le \lambda_l \\ \sqrt{t}\,(e^i - 1) & \text{when } i + i^- > \lambda_l \\ 0 & \text{when } i = 0 \end{cases} \quad - - - - - (1)$$

Also we consider that game will not be over until all the packets are transmitted or fixed number of time slot is over. And hence the efficiency of user is measured by the term performance which depends on total cost, number of packets and time slots. i.e.

$$\text{Performance} = \frac{\text{Total Cost}}{(\text{No.of transmitted packets}) * (\text{No.of Time Slots})} = \frac{C^{(n)}}{(\sum_t i) * m} = \frac{\sum_t \sum_i f(t).\phi(i,i^-)}{(\sum_t P_l^n(t)) * m}$$

$$\text{Efficient user} = \min(\text{Performance}(\text{user1}, \text{user2}))$$

V. PENALTIES

To avoid the violation of capacity constraint we introduce the term penalty for the users. Now we will discuss different types of penalties.

5.1 Type A: Transmission failure for all user (when $i + i^- > \lambda_l$)

1. No packet will be transmitted at that time slot.
2. Cost for user n can be expressed as $C_t^{(n)} = \sqrt{t}.(e^i - 1)$

Penalty constant x will give the number of packets which should be routed in the next time slot by the user with maximum packet in the transmission failure situation. i.e.

$$x = \max(i^n, i^m) - 1$$

EXAMPLE: Let the capacity of link l is 3 (i.e. $\lambda_l = 3$) and user u_1 and u_2 wants to transmit 4 packets in 4 time slots. Transmission may be fail at any time slot. In the Transmission failure situation penalty $x = \max(i^n, i^m) - 1$ will be activated for the next time slot. The user is more efficient if he/she will transmit maximum packets (approximately 4 packets) in less number of time slots. The efficiency of user is measured by the term performance in the following games.

$$\text{Where Performance} = \frac{\text{Total Cost}}{(\text{No.of transmitted packets}) * (\text{No.of Time Slots})} = \frac{\sum_t \sum_i f(t).\phi(i,i^-)}{(\sum_t P_l^n(t)) * m}$$

$$\text{Efficient user} = \min(\text{Performance}(\text{user1}, \text{user2}))$$

GAME -1

	User 1		User 2		Situation
Time Slot	U1(No. of packets)	Cost(U1)	U2 (No. of packets)	Cost(U2)	
1	3	19.08554	2	6.389056	No Transmission
2	2	9.03549	2	9.03549	No Transmission
3	1	1.488076	1	1.488076	Transmitted
4	1	3.436564	2	12.77811	Transmitted
Total	2	33.04567	3	29.69073	

performance		4.130708			2.474228	

Efficient user = User 2

5.2 Type B: Transmission failure only for user with maximum packets (when $i + i^{-} > \lambda_l$)

In type B, we consider that data packets will not be transmitted for the user with maximum number of packets, but other user's data packets will be transmitted.

1. No packet will be transmitted for user with maximum packets at that time slot.

   $P_l^n(t) = \max(P_l^{n1}(t), P_l^{n2}(t) \dots \dots)$ or $i = \max(i_1, i_2, i_3, \dots\dots)$
2. Cost for user n can be expressed as

   $C_t^{(n)} = \sqrt{t}.(e^i - 1)$ For any other user

   $C_t^{(n_1)} = \frac{\sqrt{t}\,(e^{i_1} - 1)}{\lambda_l + 1 - (i_1 + i^-)}$ where i^- no. of packets except i which is not being transmitted.

Therefore total cost for user n to send i packets through link l is $C^{(n)} = \sum_t \sum_i f(t).\phi(i, i^-)$

Example : Consider the same example in which condition is applied according to type B

Game 1(Penalty Type –B)

	User 1			User 2		
Time Slot	U1	Cost(U1)	Situation	U2	Cost(U2)	Situation
1	3	19.08553692	No Trans	2	3.194528049	Transmitted
2	2	9.035489786	Transmitted	1	2.430017466	Transmitted
3	2	11.06616978	Transmitted	1	2.976151429	Transmitted
4	0	0		0	0	
Total	4	39.18719648		4	8.600696944	
performance		2.44919978			0.537543559	

Efficient user = $\min(\text{Performance}(\text{user1}, \text{user2}))$ = user 2

5.3 Type C: Adjustment of packets by retransmission on the free link (when $i + i^{-} > \lambda_l$)

We can consider a smart system which can retransmit data packets in another free link if number of packets exceeds the link capacity. This smart system maintains a record of status (st) of all links available in the network.

If a link *l* has no free space then its status is set as st =1 otherwise st = 0.

When $i + i^{-} > \lambda_l$ system search another link with status st = 0, and find $i = \max(i_1, i_2, i_3, \dots\dots)$. It divides i into two parts i' and i'' such that $i' + i^{-} = \lambda_l$ and i'' will be retransmitted in the link with st = 0.

In this case cost for n[th] user with i data packets

$$C_t^{(n)} = \frac{\sqrt{t}(e^{i'}-1)}{\lambda_{l_1}+1-(i'+i^-)} + 2.\frac{\sqrt{t}(e^{i''}-1)}{\lambda_{l_2}+1-(i''+i^-)}$$

VI. EXISTENCE OF NASH EQUILIBRIUM

The following theorems establish the existence of Nash Equilibrium Point for the communication network.

Theorem 1*:* In a communication network cost function $C_t^{(n)}: I \times I \to R^+$ for each time slot, defined as (1) is convex.

Proof : To prove that cost function $C_t^{(n)}$ is convex we will use following theorem (by [8])

"A function $\boldsymbol{f}(\boldsymbol{X})$ is convex if the Hessian matrix $H(\boldsymbol{X}) = \left[\frac{\partial^2 \boldsymbol{f}(\boldsymbol{X})}{\partial \boldsymbol{x_i} \partial \boldsymbol{x_j}}\right]$ is positive semidefinite. If $\boldsymbol{H}(\boldsymbol{X})$ is positive definite, the function $\boldsymbol{f}(\boldsymbol{X})$ will be strictly convex."

By equation (1) extended cost function can be expressed as

$$C_t^{(n)} = f(t).\emptyset(i,i^-) = \begin{cases} \dfrac{\sqrt{t}\,(e^i-1)}{\lambda_l+1-(i+i^-)} & when\; i+i^- \le \lambda_l \\ \sqrt{t}\,(e^i-1) & when\; i+i^- > \lambda_l \\ 0 & when\; i=0 \end{cases}$$

For a fixed time slot, $f(t)$ is constant, therefore $C_t^{(n)}$ (for simplicity consider $C_t^{(n)} = C$) will be a function of two variables $i\; and\; i^-$, and Hessian Matrix for C is

$$H(C) = \begin{bmatrix} \frac{\partial^2 C}{\partial i^2} & \frac{\partial^2 C}{\partial i \partial i^-} \\ \frac{\partial^2 C}{\partial i \partial i^-} & \frac{\partial^2 C}{\partial i^{-2}} \end{bmatrix}$$

Case I: When $i + i^- \le \lambda_l$

$$C = \frac{\sqrt{t}(e^i-1)}{\lambda_l+1-(i+i^-)}$$

Let $\sqrt{t} = K'\; and\; \lambda_l + 1 - (i+i^-) = M$

$$\frac{\partial^2 C}{\partial i^2} == K'\left[e^i\left[\frac{1}{M}+\frac{2}{M^2}+\frac{2}{M^3}\right]-\frac{2}{M^3}\right], \quad \frac{\partial^2 C}{\partial i^{-2}} = \frac{2K'(e^i-1)}{M^3}, \quad \frac{\partial^2 C}{\partial i \partial i^-} = K'\left[e^i\left[\frac{1}{M^2}+\frac{2}{M^3}\right]-\frac{2}{M^3}\right]$$

Now Determinant of Hessian Matrix $|H(C)| = \frac{K'^2 e^i}{M^4}[e^i - 2] > 0 \quad (\because\; e^1 = 2.71828)$

Case II: When $i + i^- > \lambda_l$

$$C = \sqrt{t}\,(e^i - 1) = K'(e^i - 1)$$

$$\frac{\partial^2 C}{\partial i^2} = K' e^i \text{ and } \frac{\partial^2 C}{\partial i^{-2}} = \frac{\partial^2 C}{\partial i \partial i^-} = 0$$

Therefore $|H(C)| = 0$ which is non-negative.

Case III: When $i = 0$ $\quad C = 0$

$$\frac{\partial^2 C}{\partial i^2} = \frac{\partial^2 C}{\partial i^{-2}} = \frac{\partial^2 C}{\partial i \partial i^-} = 0$$

Therefore $|H(C)| = 0$ which is non-negative.

Since $H(C)$ is positive definite in all cases therefore the function $C_t^{(n)}$ will be strictly **convex** for each time slot.

Theorem 2: Cost function $C_t^{(n)}$ is continuous in each time slot.

Proof : We will use $\varepsilon - \delta$ definition to prove that $C_t^{(n)}: I \times I \to R^+$ is continuous in each time slot.

Let (i_o, i_o^-) is any point in the domain of $C_t^{(n)} = C(let)$ such that

$$|i - i_o| < \delta \text{ and } |i^- - i_o^-| < \delta' \text{ where } \delta > 0, \delta' > 0$$

Now

$$|C(i, i^-) - C(i_o, i_o^-)| = \left| \frac{K'(e^i - 1)}{\lambda_l + 1 - (i + i^-)} - \frac{K'(e^{i_o} - 1)}{\lambda_l + 1 - (i_o + i_o^-)} \right|$$

$$\leq \left| K'(e^i - 1) - K'(e^{i_o} - 1) \right| = K' \left| e^i - e^{i_o} \right|$$

$$\leq K'^{e^{\lambda_l}|i - i_o|} < K'^{e^{\lambda_l}} \delta < \varepsilon \text{ since } K' \text{ is positive integer.}$$

Where $\varepsilon = K'^{e^{\lambda_l}} \delta > 0$. Hence C is continuous in each interval.

VII. CONCLUSION

In this work, we attempted to present mathematical modeling of transmission in a communication network, using game theoretical concept with multiple chances available to users. The cost function involves the number of packets to be routed and time variables, in a non-linear fashion. Penalty to the users are also introduced for smoothing the game making the cost finite. Each user is given the flexibility to route their data at different time slots. The examples also demonstrate the cost function when packets send by the users through a link at a given shot exceeds the capacity of the link. Despite the results accomplished so far, there is space for more detailed investigation for multiuser; complex network with non-symmetrical links (i.e. links with different speed). Furthermore, different demands and different source and destination seem to play a critical role in this packet transmission that has not been investigated in detail yet.

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