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Metric Dimensions in the Fuzzy Cartesian Product Of Two Fuzzy Graphs

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Abstract
Let G = (V, E, µ) be a fuzzy graph. Let M˜ be a subset of V . M˜ is said to be a fuzzy metric basis of G if for every pair of vertices x, y ∈ V − M˜ there exists a vertex w ∈ M˜ such that d˜(w, x) 6= d˜(w, y) . The number of elements in M˜ is called the fuzzy metric dimension (FMD) of G and is denoted by β˜(G). In this article, we will find the exact values of the fuzzy metric dimension of the fuzzy Cartesian product of two fuzzy paths, the fuzzy metric dimension of the fuzzy Cartesian product of a fuzzy path and a fuzzy cycle, and the fuzzy metric dimension of the fuzzy. Cartesian product of two fuzzy cycles.
Keywords: Fuzzy metric dimension, fuzzy path, fuzzy cycle etc.

I. Introduction

Maps can be assigned to study various concepts of space navigation. All graphs considered here are finite, connected, undirected, and without multiple edges. We use standard terminology. Terms not defined here can be found in The metric dimension was first studied by Harary and Melter and independently by Slater. In 2012 they introduced B. Prabu, P. Venugopala. and N. Padmapriya concept of finding fuzzy metric dimension in graphs.

Fuzzy metric dimension of the fuzzy Cartesian product of two fuzzy paths, fuzzy paths and fuzzy cycles.

In this section, we determine the fuzzy metric dimension of the fuzzy Cartesian product of two fuzzy paths and the fuzzy Cartesian product of two fuzzy paths and fuzzy cycles.

Fuzzy Metric Dimension of fuzzy Cartesian product of two fuzzy paths P_n and P_m.

If $G = P_n \times P_m$ is a Cartesian product of two fuzzy paths P_n and P_m then

$\beta(G) \leq 2$, when n is two and m is two.

$\beta(G) = 2$, when $n \geq 3$ and $m \geq 3$.

Proof:

Case i: Let P_n $(n = 2)$ be a fuzzy path with two vertices u_1, u_2 and P_m $(m = 2)$ be a fuzzy path with two vertices v_1, v_2. Let $G = P_2 \times P_2$ be a Cartesian product of two fuzzy paths with four vertices u_1v_1, u_1v_2, u_2v_1, u_2v_2. By the above Theorem 1.3.59 metric dimension of G is less than or equal to two.

Case ii: Let P_n $(n \geq 3)$ be a fuzzy path with n vertices $u_1, u_2, \ldots, u_n$ and P_m $(m \geq 3)$ be a fuzzy path with m vertices $v_1, v_2, \ldots, v_m$. Let $G = P_n \times P_m$ be a Cartesian product of two fuzzy paths

P_n and P_m with nm vertices $u_1v_1, u_1v_2, \ldots, u_1v_m, u_2v_1, u_2v_2, \ldots, u_2v_m, \ldots, u_nv_1, u_nv_2, \ldots, u_nv_m$.

We will separate G as the union of fuzzy paths by following different sub cases, that

is $G = P_1 \cup P_2$.

Sub case i: If n is odd, m is odd.

P_1: u_1v_1 u_2v_1 … $u_{n-1}v_1$ u_nv_1 u_nv_2 $u_{n-1}v_2$ … u_1v_2 u_1v_3 u_2v_3 … u_nv_3 u_nv_4 … u_1v_4 u_1v_5 … u_nv_5 u_nv_6 un−1v6 … u1v6 … u1v7 u2v7 … unv7 … unvm−3 un-1vm−3 … u1vm−3 u1vm−2 u2vm−2 unvm−2 unvm−1 un1vm−1 … u1vm−1 u1vm u2vm … unvm.

P_2: u_1v_m u_1v_{m-1} … u_1v_1 u_2v_1 … u_2v_m u_3v_m … u_3v_1 u_4v_1 … u_4v_m u_5v_m … u_5v_1 u_6v_1 … u_6v_m u_7v_m … u_7v_1 … $u_{n-1}v_1$ … $u_{n-1}v_m$ u_nv_m … u_nv_1.

Sub case ii: If n is odd, m is even. (or) If n is even, m is odd.

P1: u1v1 u2v1 u3v1… un−1v1 unv1 unv2 un−1v2 … u1v2 u1v3 u2v3 … unv3 unv4 un−1v4… u1v4 u1v5 u2v5 … unv5 unv6 un−1v6 … u1v6 … unvm−2 un−1vm−2 … u1vm−2 u1vm−1 u2vm−1 … unvm−1 unvm un− $_1v_m$ … u_1v_m.

P2: u1vm u1vm−1 u1vm−2 … u1v1 u2v1 u2v2 … u2vm u3vm u3vm−1 … u3v1 u4v1 u4v2 … u4vm u5vm u5vm−1 … u5v1 … un−1v1 un−1v2 … un−1vm unvm … unv1.

Sub case iii: If n is even, m is even.

P1: u1v1 u2v1 u3v1… un−1v1 unv1 unv2 un−1v2 … u1v2 u1v3 u2v3 … unv3 unv4 un−1v4… u1v4 u1v5 u2v5 … unv5 unv6 un−1v6 … u1v6 … u1vm−1 u2vm−1 … un−1vm−1 unvm−1 un−1vm … u1vm.
P2: u1vm u1vm−1 u1vm−2 … u1v1 u2v1 u2v2 … u2vm u3vm u3vm−1 … u3v1 u4v1 u4v2 … u4vm … un−1vm un−1vm−1 … un−1v1 unv1 unvm.

Suppose u_1v_1 is fixed as a source vertex. If two vertices $u_iv_j \in P_1$ and $u_kv_l \in P_2$ such that FSP (fuzzy shortest path) for u_iv_j from u_1v_1 is through P_2 and FSP for u_kv_l from u_1v_1 is

through P_1 then $d(u_1v_1, u_iv_j) = d(u_1v_1, u_kv_l)$ if and only if $N(u_1v_1, u_iv_j) = N(u_1v_1, u_kv_l)$. This

implies that, $\beta(G) \neq 1$. Include u_1v_m as another source vertex so that $N(u_1v_m, u_iv_j) \neq N(u_1v_m,$

$u_kv_l)$, $d(u_1v_m, u_iv_j) \neq d(u_1v_m, u_kv_l)$. Thus,

$$M = \{u_1v_1, u_1v_m\}.$$

Hence $\beta(G) = 2$.

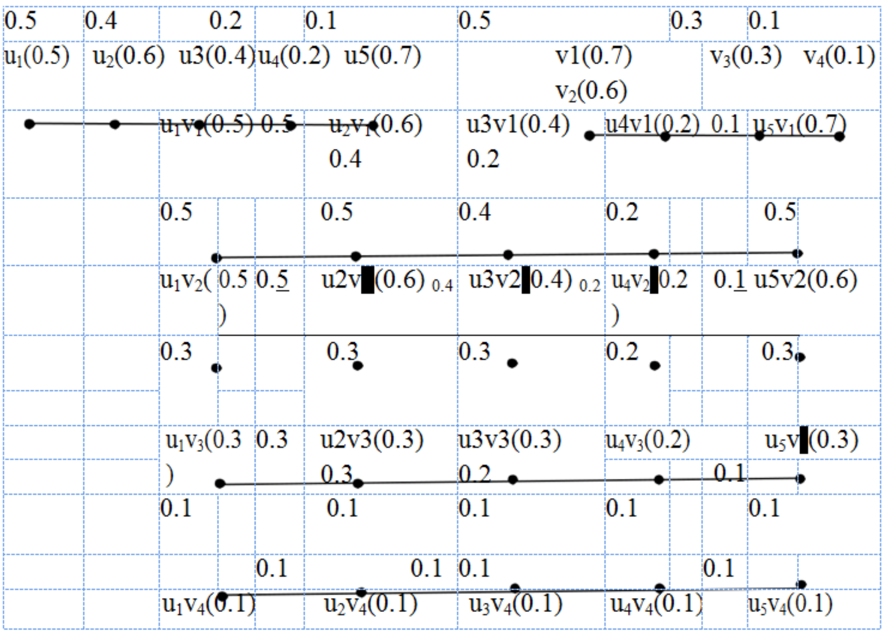

Figure: 1 fuzzy Cartesian product of two fuzzy paths.

Fuzzy Metric Dimension of Cartesian product of fuzzy path P_n and fuzzy cycle C_m. Theorem: If $G = P_n \times C_m$ be a Cartesian product of fuzzy path P_n ($n \geq 2$) and fuzzy cycle C_m ($m \geq 3$) then $\beta(G) = 2$.

Proof: Let $u_1, u_2, \ldots, u_n$ be vertices of fuzzy path P_n and $v_1, v_2, \ldots, v_m$ be vertices of fuzzy

cycle C_m. Let $G = P_n \times C_m$ be a Cartesian product of fuzzy path P_n ($n \geq 2$) and fuzzy cycle C_m

($m \geq 3$) with nm vertices $u_1v_1, u_1v_2, \ldots, u_1v_m, u_2v_1, u_2v_2, \ldots, u_2v_m, \ldots, u_nv_1, u_nv_2, \ldots, u_nv_m$.

We will write G as the union of fuzzy paths, that is $G = P_1 \cup P_2$, where

Sub case i: If n is odd, m is odd.

P1: u1vm–1 u1vm–2 … u1v1 u1vm u2vm u2v1 u2v2 … u2vm–1 u3vm–1 u3vm–2 … u3vm u4vm u4v1… u4vm–1 u5vm–1 u5vm–2 … u5vm … unvm–1 unvm–2 … unvm.
P2: u1vm u1vm–1 u2vm–1 u2vm u3vm u3vm–1 u4vm–1 u4vm … unvm unvm–1 unvm unv1 un–1v1… u1v1 u_1v_2 u_2v_2 … u_nv_2 u_nv_3 $u_{n-1}v_3$ … u_1v_3 u_1v_4 … u_nv_4 u_nv_{n-1} … u_nv_m $u_{n-1}v_m$ … u_1v_m.

Sub case ii: If n is odd, m is even.

P1: u1vm–1 u1vm–2 … u1v1 u1vm u2vm u2v1 u2v2 … u2vm–1 u3vm–1 u3vm–2 … u3vm u4vm u4v1… u4vm–1 u5vm–1 u5vm–2 … u5vm … unvm unv1 … unvm–1.
P2: u1vm u1vm–1 u2vm–1 u2vm u3vm u3vm–1 u4vm–1 u4vm … unvm unvm–1 … u1v1 u1v2 u2v2 … unv2 u_nv_3 $u_{n-1}v_3$ … u_1v_3 u_1v_4 … u_nv_4 u_nv_{n-1} … u_1v_m u_2v_m … u_nv_m.

Sub case iii: If n is even, m is even.

P1: u1vm–1 u1vm–2 … u1v1 u1vm u2vm u2v1 u2v2 … u2vm–1 u3vm–1 u3vm–2 … u3vm u4vm u4v1… u4vm–1 u5vm–1 u5vm–2 … u5vm … unvm unv1 … unvm–1.

P2: u1vm u1vm–1 u2vm–1 u2vm u3vm u3vm–1 u4vm–1 u4vm … unvm–1 unvm … u1v1 u1v2 u2v2 … unv2 u_nv_3 $u_{n-1}v_3$ … u_1v_3 u_1v_4 … u_nv_4 u_nv_{n-1} … u_1v_m u_2v_m … u_nv_m.

Sub case iv: If n is even, m is odd.

P1: u1vm–1 u1vm–2 … u1v1 u1vm u2vm u2v1 u2v2 … u2vm–1 u3vm–1 u3vm–2 … u3vm u4vm u4v1…

u4vm–1 u5vm–1 u5vm–2 … u5vm … unvm–1 unvm–2 … unvm.

P2: u1vm u1vm–1 u2vm–1 u2vm u3vm u3vm–1 u4vm–1 u4vm … unvm–1 unvm unvm … u1v1 u1v2 u2v2 …

unv2 unv3 un–1v3 … u1v3 u1v4 … unv4 unvn–1 … unvm un–1vm … u1vm.

Take u_1v_m as a source vertex. If two vertices $u_iv_j \in P_1$ and $u_kv_l \in P_2$ such that FSP (fuzzy shortest path) for u_iv_j from u_1v_m is through P_2 and FSP for u_kv_l from u_1v_m is through P_1 then $d(u_1v_m, u_iv_j) = d(u_1v_m, u_kv_l)$ if and only if $N(u_1v_m, u_iv_j) = N(u_1v_m, u_kv_l)$. This implies that, $\beta(G) \neq 1$. Include u_1v_{m-1} as another source vertex so that $N(u_1v_{m-1}, u_iv_j) \neq N(u_1v_{m-1},$

$u_kv_l)$ and $d(u_1v_{m-1}, u_iv_j) \neq d(u_1v_{m-1}, u_kv_l)$. Thus $M = \{u_1v_m, u_1v_{m-1}\}$.
Hence $\beta(G) = 2$.

Fuzzy Metric Dimension of fuzzy Cartesian product of Two Fuzzy Cycles.

In this section, we find the values of fuzzy metric dimension of fuzzy Cartesian

product of two fuzzy cycles.

Fuzzy Metric Dimension of Cartesian Product of odd cycles.

Lemma: If $m \geq 3$ and $n \geq 3$ are odd positive integer then $\beta(C_m \times C_n) = A$ where A = 2, 3 or 4.

Proof: Let $m \geq 3$ and $n \geq 3$ are odd positive integer and consider the graph $C_m \times C_n$ with $\{u_iv_j$

$i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$ vertices and 2nm edges which admits cycle decomposition if its edge set can be partitioned into cycles. Consider the following two Hamiltonian cycles of $C_m \times C_n$ in three different ways, In general

(i) $C_m \times C_3$ has two Hamiltonian odd cycles in the form of

Cn1: u1v1 u2v1 u3v1 u3v2 u4v2 u5v2 … um−1v2 umv2 u1v2 u2v2 u2v3 u3v3 u4v3 u4v1 u5v1 u5v3 u6v3 u6v1… um−2v1 um−2v3 um−1v3 um−1v1 umv1 umv3 u1v3 u1v3 u1v1.

Cn2: umv1 umv2 umv3 um−1v3 um−1v2 um−1v1 um−2v1 um−2v2 um−2v3 … u6v3 u6v2 u6v1 u5v1 u5v2 u5v3

u_4v_3 u_4v_2 u_4v_1 u_3v_1 u_3v_3 u_3v_2 u_2v_2 u_2v_1 u_2v_3 u_1v_3 u_1v_2 u_1v_1 u_mv_1.

We will write $C_m \times C_3$ as the union of two Hamiltonian odd cycles, that is $C_m \times C_3$ =

Cn1 ∪ Cn2

In C_{n1}, fix u_1v_1 as a source vertex. If nm is odd then $((nm+1)/2)^{th}$ (u_iv_j) and $((nm+3)/2)^{th}$ (u_iv_j) are the two diametrically opposite vertices of u_1v_1. In C_{n2}, fix u_mv_1 as a source vertex C_{n2} which also have the same characterization which mentioned above. In C_{n1}, take u_1v_1 as a source vertex, let P_1 be the path u_1v_1 u_2v_1 u_3v_1 u_3v_2 … $((nm+1)/2)^{th}(u_iv_j)$ and P_2

be the path u_1v_1 u_1v_3 u_mv_3 u_mv_1 $u_{m-1}v_1$ … $((nm+3)/2)^{th}(u_iv_j)$ $((nm+1)/2)^{th}$ (u_iv_j). In C_{n2}, take

u_mv_1 as a source vertex. Let P_3 be the path u_mv_1 u_mv_2 u_mv_3 u_mv_4 … $((nm+1)/2)^{th}(u_iv_j)$ and P_4 be the path u_mv_1 u_1v_1 u_1v_2 u_1v_3 u_2v_3 … $((nm+3)/2)^{th}(u_iv_j)$ $((nm+1)/2)^{th}(u_iv_j)$

(ii) $C_m \times C_5$ has two Hamiltonian odd cycles in the form of

Cn1: u1v1 u2v1 u3v1 u3v2 u4v2 u5v2 … um−1v2 umv2 u1v2 u2v2 u2v3 u3v3 u4v3 u4v4 u4v5 u4v1 u5v1 u5v5 u5v4 u5v3 u6v3 u6v4 u6v5 u6v1 … um−2v1 um−2v5 um−2v4 um−2v3 um−1v3 um−1v4 um−1v5 um−1v1 umv1

u_mv_5 u_mv_4 u_mv_3 u_1v_3 u_1v_4 u_2v_4 u_3v_4 u_3v_5 u_2v_5 u_1v_5 u_1v_1.

Cn2: umv1 umv2 umv3 um−1v3 um−1v2 um−1v1 um−2v1 um−2v2 um−2v3 … u6v3 u6v2 u6v1 u5v1 u5v2 u5v3

u4v3 u4v2 u4v1 u3v1 u3v5 u4v5 u5v5 u6v5 … um−2v5 um−1v5 umv5 u1v5 u1v4 umv4 um−1v4 um−2v4 …

u_6v_4 u_5v_4 u_4v_4 u_3v_4 u_3v_3 u_3v_2 u_2v_2 u_2v_1 u_2v_5 u_2v_4 u_2v_3 u_1v_3 u_1v_2 u_1v_1 u_mv_1.
We will write $C_m \times C_5$ as the union of two Hamiltonian odd cycles, that is $C_m \times C_5$ =

Cn1 ∪ Cn2
In C_{n1}, u_1v_1 is fixed as a source vertex, let P_1 be the path u_1v_1 u_2v_1 u_3v_1 u_3v_2 …

$((nm+1)/2)^{th}(u_iv_j)$ and P_2 be the path u_1v_1 u_1v_5 u_2v_5 u_3v_5 … $((nm+3)/2)^{th}(u_iv_j)$

$((nm+1)/2)^{th}(u_iv_j)$. In C_{n2}, take u_mv_1 as a source vertex. Let P_3 be the path u_mv_1 u_mv_2 u_mv_3 …

$((nm+1)/2)^{th}(u_iv_j)$ and P_4 be the path u_mv_1 u_1v_1 u_1v_2 u_1v_3 u_2v_3 … $((nm+3)/2)^{th}(u_iv_j)$

$((nm+1)/2)^{th}(u_iv_j)$.

(iii) $C_m \times C_n$ (n = 7, 9, …) has two Hamiltonian odd cycles in the form of

Cn1: u1v1 u2v1 u3v1 u3v2 u4v2 u5v2 … um−1v2 umv2 u1v2 u2v2 u2v3 u3v3 u4v3 … u4vn−1 u4vn u4v1 u5v1 u5vn u5vn−1 … u5v3 u6v3 … u6vn−1 u6vn u6v1 … um−2v1 um−2vn um−2vn−1 … um−2v4 um−2v3 um−

1v3 … um−1vn−1 um−1vn um−1v1 umv1 umvn umvn−1 ... umv3 u1v3 u1v4 u2v4 u3v4 u3v5 u2v5 u1v5

u1v6 u2v6 u3v6 ... u3vn−2 u2vn−2 u1vn−2 u1vn−1 u2vn−1 u3vn−1 u3vn u2vn u1vn u1v1.

Cn2: umv1 umv2 umv3 um−1v3 um−1v2 um−1v1 um−2v1 um−2v2 um−2v3 … u6v3 u6v2 u6v1 u5v1 u5v2 u5v3

u4v3 u4v2 u4v1 u3v1 u3vn … um−1vn umvn u1vn u1vn−1 umvn−1 um−1vn−1 … u4vn−1 u3vn−1 u3vn−2 … um−

1vn−2 umvn−2 u1vn−2 … u1v6 umv6 um−1v6 … u5v6 u4v6 u3v6 u3v5 u4v5 u5v5 … um−1v5 umv5 u1v5

u_1v_4 u_mv_4 $u_{m-1}v_4$ ... u_4v_4 u_3v_4 u_3v_3 u_3v_2 u_2v_2 u_2v_1 u_2v_{n-1} … u_2v_5 u_2v_4 u_2v_3 u_1v_3 u_1v_2 u_1v_1 u_mv_1. We will write $C_m \times C_n$ as the union of two Hamiltonian odd cycles, that is $C_m \times C_n$ =

Cn1 ∪ Cn2

In C_{n1}, u_1v_1 is fixed as a source vertex, let P_1 be the path u_1v_1 u_2v_1 u_3v_1 u_3v_2 …

$((nm+1)/2)^{th}(u_iv_j)$ and P_2 be the path u_1v_1 u_1v_n u_2v_n u_3v_n … $((nm+3)/2)^{th}(u_iv_j)$ $((nm+1)/2)^{th}$

(u_iv_j). In C_{n2}, take u_mv_1 as a source vertex. Let P_3 be the path u_mv_1 u_mv_2 u_mv_3 …

$((nm+1)/2)^{th}(u_iv_j)$ and P_4 be the path u_mv_1 u_1v_1 u_1v_2 u_1v_3 u_2v_3 … $((nm+3)/2)^{th}(u_iv_j)$

$((nm+1)/2)^{th}(u_iv_j)$.

Here we calculate the metric dimension of $C_m \times C_n$ (n = 3, 5, 7, 9, …)

Case i:
In C_{n1}, let $u_{i1}v_{j1}$ and $u_{i2}v_{j2}$ be two vertices on C_{n1} such that both $u_{i1}v_{j1}$ and $u_{i2}v_{j2} \in P_1$ or

P_2. If both $u_{i1}v_{j1}$ and $u_{i2}v_{j2}$ have the same FSP (fuzzy shortest path) from u_1v_1 then u_1v_1, $u_{i1}v_{j1}$

and $u_{i2}v_{j2}$ will be in same path then $\beta(C_{n1}) = 1$. In C_{n2}, Let $u_{i3}v_{j3}$ and $u_{i4}v_{j4}$ be two vertices on

C_{n2} such that both $u_{i3}v_{j3}$ and $u_{i4}v_{j4} \in P_3$ (or P_4) and If both $u_{i3}v_{j3}$ and $u_{i4}v_{j4}$ have the same FSP

(fuzzy shortest path) from u_mv_1 then u_mv_1, $u_{i3}v_{j3}$ and $u_{i4}v_{j4}$ will be in same path then $\beta(C_{n2}) = 1$.

$\beta(C_m \times C_n) = \beta(C_{n1} \cup C_{n2})$ and $M = \{u_1v_1, u_mv_1\}$. Therefore, $\beta(C_m \times C_n) = 2$.

Case ii:

In C_{n1}, if the two vertices $u_{i1}v_{j1}$ and $u_{i2}v_{j2}$ belongs to either P_1 (or P_2), then by case (i).

We get, $\beta(C_{n1}) = 1$. In C_{n2}, if $u_{i3}v_{j3}$ and $u_{i4}v_{j4}$ such that the FSP for $u_{i3}v_{j3}$ from u_mv_1 is through

P_4 and FSP for $u_{i4}v_{j4}$ from u_mv_1 is through P_3 then $d(u_mv_1, u_{i3}v_{j3}) = d(u_mv_1, u_{i4}v_{j4})$ if and only if $N(u_mv_1, u_{i3}v_{j3}) = N(u_mv_1, u_{i4}v_{j4})$. This implies that, $\beta(C_{n2}) \neq 1$. Include u_mv_2 as

another source vertex so that $N(u_mv_2, u_{i3}v_{j3}) \neq N(u_mv_2, u_{i4}v_{j4})$, $d(u_mv_2, u_{i3}v_{j3}) \neq d(u_mv_2, u_{i4}v_{j4})$. Then metric basis of C_{n2} is u_mv_1 and u_mv_2. Hence $\beta(C_{n2})=2$.

$\beta(C_m \times C_n) = \beta(C_{n1} \cup C_{n2})$. Hence $M = \{u_1v_1, u_mv_1, u_mv_2\}$. Therefore, $\beta(C_m \times C_n) = 3$.

Case iii:

In C_{n1}, if $u_{i1}v_{j1} \in P_1$ and $u_{i2}v_{j2} \in P_2$ such that the FSP for $u_{i1}v_{j1}$ from source vertex

u_1v_1 is through P_2 and FSP for $u_{i2}v_{j2}$ from source vertex u_1v_1 is through P_1 then $d(u_1v_1, u_{i1}v_{j1})$

$d(u_1v_1, u_{i2}v_{j2})$ if and only if $N(u_1v_1, u_{i1}v_{j1}) = N(u_1v_1, u_{i2}v_{j2})$. This implies that, $\beta(C_{n1}) \neq 1$. Include u_2v_1 as another source vertex so that $N(u_2v_1, u_{i1}v_{j1}) \neq N(u_2v_1, u_{i2}v_{j2})$, $d(u_2v_1, u_{i1}v_{j1}) \neq$

$d(u_2v_1, u_{i2}v_{j2})$. Then metric basis of C_{n1} is u_1v_1 and u_2v_1. Hence $\beta(C_{n1}) = 2$.

Similarly, we get the metric basis of C_{n2} as $\{u_mv_1, u_mv_2\}$. $\beta(C_m \times C_n) = \beta(C_{n1} \cup C_{n2})$.

Hence $M = \{u_1v_1, u_2v_1, u_mv_1, u_mv_2\}$. Therefore, $\beta(C_m \times C_n) = 4$.

Fuzzy Metric Dimension of Cartesian Product of odd and even cycles.

Lemma: If $m \geq 3$ be odd positive integer and $n \geq 4$ be even positive integer (or m is

even positive integer and n is odd positive integer) then $\beta(C_m \times C_n) = A$ where A = 2, 3, or 4.

Proof: Let $m \geq 3$ be odd positive integer and $n \geq 4$ be even positive integer. Consider the

following sequences of vertices of the graph $C_m \times C_n$: $u_1v_1, u_1v_2, \ldots, u_1v_n, u_2v_1, u_2v_2 \ldots, u_2v_n,$

$\ldots, u_mv_1, u_mv_2, \ldots, u_mv_n$ which admits a cycle decomposition. That is, these sequences

constitute edge-disjoint Hamiltonian cycles of the graph $C_m \times C_n$ in consider the following

two edge-disjoint Hamiltonian cycles in two different ways.
In general, (i) $C_m \times C_4$ have two Hamiltonian even cycles in the form of

Cn1: u1v1 u2v1 u2v4 u2v3 u3v3 u3v4 u3v1 u4v1 u4v4 u4v3 u5v3 u5v4 u5v1 u6v1 u6v4 u6v3 … um−1v2 umv3 u1v3 u1v4 umv4 umv1 umv2 um−1v2 um−1v1 um−1v4 um–2v4 um–2v1 um–2v2 … u4v2 u3v2 u2v2 u1v2 u1v1.

Cn2: umv1 um−1v1 um−2v1 … u6v1 u6v2 u6v3 u5v3 u5v2 u5v1 u4v1 u4v2 u4v3 u3v3 u3v2 u3v1 u2v1 u2v2 u2v3 u1v3 u1v2 umv2 umv3 umv4 um−1v4 um−1v3 um−1v2 um–2v2 um–2v3 um–2v4 … u5v4 u4v4 u3v4 u2v4

u_1v_4 u_1v_1 u_mv_1.

We will write $C_m \times C_4$ as the union of two Hamiltonian even cycles, that is

$C_m \times C_4 = C_{n1} \cup C_{n2}$

In C_{n1}, fix u_1v_1 as a source vertex. If nm is even then $((nm/2)+1)^{th}(u_iv_j)$ is a diametrically opposite vertices of u_1v_1. In C_{n2}, fix u_mv_1 as a source vertex. C_{n2}, which also have the same characterization which mentioned above. In C_{n1}, let P_1 be the path u_1v_1 u_2v_1 u_2v_4 u_2v_3 u_3v_3 … $((nm/2)+1)^{th}(u_iv_j)$ and P_2 be the path u_1v_1 u_1v_2 u_2v_2 u_3v_2 …

$((nm/2)+1)^{th}(u_iv_j)$. In C_{n2}, Let P_3 be the path u_mv_1 $u_{m-1}v_1$ $u_{m-2}v_1$ … $((nm/2)+1)^{th}(u_iv_j)$ and P_4 be the path u_mv_1 u_1v_1 u_1v_4 u_2v_4 u_3v_4 u_4v_4 u_5v_4 … $((nm/2)+1)^{th}(u_iv_j)$.

$C_m \times C_n$ (n = 6, 8, 10, …) has two Hamiltonian even cycles in the form of

C_{n1}: u_1v_1 u_2v_1 u_2v_n … u_2v_4 u_2v_3 u_3v_3 … u_3v_{n-1} u_3v_n u_3v_1 u_4v_1 u_4v_n … u_4v_4 u_4v_3 u_5v_3 … u_5v_{n-1} u5vn u5v1 u6v1 u6vn … u6v4 u6v3 … um−1v3 umv3 u1v3 u1v4 umv4 … umvn−3 u1vn−3 u1vn−2 umvn−2 umvn−1 u1vn−1 u1vn umvn umv1 umv2 um−1v2 um−1v1 um−1vn um−1vn−1 … um−1v5 um−1v4 um−2v4 um−2vn− 1 um−2vn um−2v1 um−2v2 … u2v2 u1v2 u1v1.

Cn2: umv1 um−1v1 um−2v1 … u6v1 u6v2 u6v3 u5v3 u5v2 u5v1 u4v1 u4v2 u4v3 u3v3 u3v2 u3v1 u2v1 u2v2 u2v3 u1v3 u1v2 umv2 umv3 umv4 um−1v4 um−1v3 um−1v2 um−2v2 um−2v3 um−2v4 … u5v4 u4v4 u3v4 u2v4 u1v4 u1vn−3 … umvn−3 umvn−2 … u1vn−2 u1vn−1 … umvn−1 … umvn … u2vn u1vn u1v1 umv1.

We will write $C_m \times C_n$ as the union of two Hamiltonian even cycles, that is $C_m \times C_n$ =

Cn1 ∪ Cn2

In C_{n1}, take u_1v_1 as a source vertex, let P_1 be the path u_1v_1 u_2v_1 u_2v_n … u_2v_4 u_2v_3 u_3v_3

… $((nm/2)+1)^{th}(u_iv_j)$ and P_2 be the path u_1v_1 u_1v_2 u_2v_2 … $u_{m-2}v_2$ $u_{m-2}v_1$ $u_{m-2}v_n$ $u_{m-2}v_{n-1}$ … $((nm/2)+1)^{th}(u_iv_j)$. In C_{n2}, u_mv_1 is fixed as a source vertex. Let P_3 be the path u_mv_1 $u_{m-1}v_1$ $u_{m-2}v_1$ … $((nm+1)/2)^{th}(u_iv_j)$ and P_4 be the path u_mv_1 u_1v_1 u_1v_n u_2v_n … u_mv_n u_mv_{n-1} …

$((nm/2)+1)^{th}(u_iv_j)$.

Here three cases are arising for calculating the metric dimension of $C_m \times C_n$ (n = 4, 6, …)

Case i:

In C_{n1}, let $u_{i1}v_{j1}$ and $u_{i2}v_{j2}$ are two vertices on C_{n1} such that both $u_{i1}v_{j1}$ and $u_{i2}v_{j2} \in P_1$

(or P_2). If both $u_{i1}v_{j1}$ and $u_{i2}v_{j2}$ have the same FSP (fuzzy shortest path) from u_1v_1 then u_1v_1,

$u_{i1}v_{j1}$ and $u_{i2}v_{j2}$ will be in same path then $\beta(C_{n1}) = 1$. Similarly, we get metric basis of C_{n2} as $\{u_1v_1, u_mv_1\}$. Therefore, $\beta(C_m \times C_n) = 2$.

Case ii:

As in case (i), we get metric basis of C_{n1} as $\{u_1v_1\}$. Therefore, $\beta(C_{n1}) = 1$.

In C_{n2}, if $u_{i3}v_{j3}$ and $u_{i4}v_{j4}$ such that the FSP for $u_{i3}v_{j3}$ from u_mv_1 is through P_4 and FSP

for $u_{i4}v_{j4}$ from u_mv_1 is through P_3 then $d(u_mv_1, u_{i3}v_{j3}) = d(u_mv_1, u_{i4}v_{j4})$ if and only if $N(u_mv_1, u_{i3}v_{j3}) = N(u_mv_1, u_{i4}v_{j4})$. This implies that, $\beta(C_{n2}) \neq 1$. Include $u_{m-1}v_1$ as a another source vertex so that $N(u_{m-1}v_1, u_{i3}v_{j3}) \neq N(u_{m-1}v_1, u_{i4}v_{j4})$, $d(u_{m-1}v_1, u_{i3}v_{j3}) \neq d(u_{m-1}v_1, u_{i4}v_{j4})$,

therefore metric basis of C_{n2} is u_mv_1 and $u_{m-1}v_1$. Hence $\beta(C_{n2}) = 2$.

$\beta(C_m \times C_n) = \beta(C_{n1} \cup C_{n2})$. Hence $M = \{u_1v_1, u_mv_1, u_{m-1}v_1\}$ and $\beta(C_m \times C_n) = 3$.

Case iii:

Similarly, we get metric basis of C_{n1} as $\{u_1v_1, u_2v_1\}$. Therefore, $\beta(C_{n1}) = 2$.

Similarly, we get metric basis of C_{n2} as $\{u_mv_1, u_{m-1}v_1\}$. Therefore, $\beta(C_{n2}) = 2$.

$\beta(C_m \times C_n) = \beta(C_{n1} \cup C_{n2})$.

Hence $M = \{u_1v_1, u_2v_1, u_mv_1, u_{m-1}v_1\}$ and $\beta(C_m \times C_n) = 4$.

Fuzzy Metric Dimension of Cartesian Product of even cycles.

Lemma: If $m \geq 4$ and $n \geq 4$ are two even positive integers then $\beta(C_m \times C_n) = A$ where

$A = 2, 3$, or 4.

Proof: Let $m \geq 4$ and $n \geq 4$ are two even positive integer. Consider the graph $C_m \times C_n$ with nm vertices and 2nm edges whose edge set can be partitioned into Hamiltonian cycles. Consider the following two Hamiltonian even cycles in two different ways. In general,

$C_m \times C_4$ has two edge-disjoint Hamiltonian even cycles in the form of

Cn1: u1v1 u2v1 u3v1 u4v1 u4v2 u5v2 u6v2 … umv2 u1v2 u2v2 u3v2 u3v3 u4v3 u5v3 u6v3 … um−1v3 umv3

u1v3 u2v3 u2v4 u3v4 u4v4 u4u5 u5v4 u5v1 u6v1 u6v4 … um−1v4 um−1v1 umv1 umv4 u1v4 u1v1.

Cn2: umv1 umv2 umv3 umv4 um−1v4 um−1v3 um−1v2 um−1v1 um−2v1 um−2v2 um−2v3 um−2v4 … u5v4 u5v3

$u_5v_2\ u_5v_1\ u_4v_1\ u_4v_4\ u_4v_3\ u_4v_2\ u_3v_2\ u_3v_1\ u_3v_4\ u_3v_3\ u_2v_3\ u_2v_2\ u_2v_1\ u_2v_4\ u_1v_4\ u_1v_3\ u_1v_2\ u_1v_1\ u_mv_1$.

We will write $C_m \times C_4$ as the union of two Hamiltonian even cycles, that is $C_m \times C_4 = C_{n1} \cup$

Cn2

In C_{n1}, take u_1v_1 as a source vertex. If nm is even then $((nm/2)+1)^{th}(u_iv_j)$ is a diametrically opposite vertices of u_1v_1. In C_{n2}, u_mv_1 is fixed as a source vertex C_{n2} which also have the same characterization. In C_{n1}, let P_1 be the path $u_1v_1\ u_2v_1\ u_3v_1\ u_4v_1$ …

$((nm/2)+1)^{th}(u_iv_j)$ and P_2 be the path u_1v_1 u_1v_4 u_mv_4 u_mv_1 $u_{m-1}v_1$ $u_{m-1}v_4$ … $((nm/2)+1)^{th}(u_iv_j)$.

In C_{n2}, Let P_3 be the path u_mv_1 u_mv_2 u_mv_3 … $((nm/2)+1)^{th}(u_iv_j)$ and P_4 be the path u_mv_1 u_1v_1 u_1v_2 u_1v_3 u_1v_4 u_2v_4 u_2v_1 … $((nm/2)+1)^{th}(u_iv_j)$.
(i)$C_m \times C_n$ (n = 6, 8, 10, …) has two edge-disjoint Hamiltonian even cycles in the form of

C_{n1}: u_1v_1 u_2v_1 u_3v_1 u_4v_1 u_4v_2 u_5v_2 … u_mv_2 u_1v_2 u_2v_2 u_3v_2 u_3v_3 u_4v_3 … u_mv_3 u_1v_3 u_2v_3 u_2v_4 u_3v_4 … u5v4 u5v5 u5vn–1 u5vn u5v1 … u6v1 u6vn u6vn–1 … u6v4 … um–1v4 um–1v5 … um–1vn–1 um–1vn um–1v1 umv1 umvn umvn–1 umv4 u1v4 u1v5 u2v5 u3v5 u4v5 … u4vn–2 u3vn–2 u2vn–2 u1vn–2 u1vn–1 u2vn–1 u3vn–1

u4vn–1 u4vn u3vn u2vn u1vn u1v1.

Cn2: umv1 umv2 umv3 umv4 um–1v4 um–1v3 um–1v2 um–1v1 … u5v1 u4v1 u4vn u5vn … umvn u1vn u1vn–1 umvn–1 um–1vn–1 … u4vn–1 u4vn–2 u5vn–2 … umvn–2 u1vn–2 … u1v5 umv5 um–1v5 um–2v5 … u4v5 u4v4 u4v3 u4v2 u3v2 u3v1 u3vn u3vn–1 … u3v3 u2v3 u2v2 u2v1 u2vn u2vn–1 u2vn–2 … u2v4 u1v4 u1v3 u1v2

u_1v_1 u_mv_1.

We will write $C_m \times C_n$ (n = 6, 8, 10, …) as the union of two edge-disjoint Hamiltonian even cycles, that is $C_m \times C_n = C_{n1} \cup C_{n2}$

In C_{n1}, take u_1v_1 as a source vertex, let P_1 be the path u_1v_1 u_2v_1 u_3v_1 u_4v_1 … $((nm/2)+1)^{th}(u_iv_j)$ and P_2 be the path u_1v_1 u_1v_n u_2v_n u_3v_n u_4v_n … $((nm/2)+1)^{th}(u_iv_j)$. In C_{n2},

u_mv_1 is fixed as a source vertex. Let P_3 be the path u_mv_1 u_mv_2 u_mv_3 … $((nm+1)/2)^{th}(u_iv_j)$ and P_4 be the path u_mv_1 u_1v_1 u_1v_2 u_1v_3 u_1v_4 u_2v_4 … $((nm/2)+1)^{th}(u_iv_j)$.

Here we calculate the metric dimension of $C_m \times C_n$ where m, n are two even positive

integer in the following three cases. Case (i), (ii) and (iii) are similar to the cases of $C_m \times C_n$

where m and n are odd positive integer we get, the metric basis are

= {u v , u v } for case (i), M 1 1 m 1

= {u v , u v , u v } for case (ii) and M 1 1 m 1 m 2

= {u v , M 1 1

u2v1, umv1, umv2} for case (iii)

Hence

β(Cm × Cn) = 2, 3 or 4.

Theorem: If $C_m \times C_n$ be a Cartesian product of two fuzzy cycles then $\beta(C_m \times C_n) = A$,

where A = 2, 3 or 4.

Proof: The proof follows from Lemma 4.2.1, Lemma 4.2.2 and Lemma 4.2.3.

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