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A Century Of Growth In Brazilian Cities: A Spatial Approach (1910-2010)

Wellington Ribeiro Justo[1]
[1](Economy, Regional University Of Cariri, Federal University Of Pernambuco, Brazil)

Abstract:
Background*: This article aims to analyze the dynamics of growth in Brazilian cities over the past century, employing advanced econometric methods and spatial modeling. The research focuses on the evolution of population growth in 431 comparable minimal areas in Brazil from 1910 to 2010.*
Materials and Methods *The methodological approach includes estimating spatial models for Zipf's Law, revealing a divergent trend, albeit attenuated in the last decade. The population distribution is thoroughly characterized through non-parametric density function estimations. Additionally, the growth process of Brazilian cities is investigated through a first-order Markov Chain analysis, highlighting a stationary pattern.*
Results*: The results underscore low inter-class mobility and notable persistence over time. The probability of cities remaining in the same class over decades throughout the last century is substantially high.*
Conclusion: *These findings provide significant insights into the demographic evolution and urban structure of Brazil, contributing to a deeper understanding of city growth patterns in the economic and regional context.*
Key Word*: Convergence; Urban growth; Spatial Models; Markov chains; Zipf's law.*

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I. Introduction

All economic activity has a location, although dissimilar activities flourish in different areas. Regional and urban economics is concerned with considering the effect of location and distance on economic activity. Regional economists seek to identify where certain economic activities will thrive or why certain activities are more concentrated in certain areas and why certain households choose to locate in certain areas. Another focus of this field of research is to understand what makes people migrate, as this phenomenon affects the growth rates of cities. Thus, regional economists combine tools from micro and macroeconomics and international economics to analyze patterns and other components of local growth rates. In this context, concepts such as proximity and transportation costs, increasing returns to scale and externalities are employed. These components change the face of traditional economic theory. Urban economists, on the other hand, focus on the relationships between urban and peripheral areas, as well as the pattern of land use in cities (Edwards, 2007).

According to Edwards (2007), the economic analysis of the spatial distribution of economic activities and, consequently, of population, goes back to Cantilon's treatise in 1755 when he analyzed why cities developed close to the most productive areas. Between 1800 and 1950, the German school was at the forefront in emphasizing the spatial issue in economic activity.

The process of economic development is associated with the distribution of population among a country's municipalities. In this context, the question arises of how cities of different sizes grow during the development process. The distribution of city sizes can occur in the sense that the smallest cities grow faster than the largest, or at the opposite extreme, the largest cities could grow more than the smallest, further increasing population concentration and benefiting a few localities. Many factors are involved in the growth dynamics of cities. Public policy decisions can make certain municipalities attractive to entrepreneurs. Firms bring jobs and increase the purchase of local goods and services, influencing the attraction of more firms and people. In this way, attracting companies is one of the key factors in the development process, raising the standard of living of residents.

Firms seek to minimize transport costs by obtaining gains in scale because they are close to large markets, contributing to the agglomeration of firms and benefiting from agglomeration economies. Amenities can also favor the decision of entrepreneurs to invest in each location. On the other hand, there may be diseconomies of agglomeration limiting the undetermined growth of certain locations.

The existence of clusters of firms in an area can generate agglomeration economies by creating centripetal forces that reinforce the agglomeration process. However, clusters can grow too large and create diseconomies of agglomeration. These, in turn, create centrifugal forces that repel economic concentration (Edwards, 2007).

A difficulty for regional economics researchers for a long time was the lack of formalization of regional issues, which made it difficult to empirically test their hypotheses. On the other hand, there was a regularity that had been observed and tested with various sets of data, but which, on the other hand, lacked theoretical explanations, which is the rule of the order of the size of cities (Zipf's Law). This empirical regularity shows how the spatial distribution of cities occurs over time. Fujita, Krugman and Venables (2001) and Duranton (2006), however, sought to explain this regularity theoretically.

The analysis of city growth, in turn, allows us to understand how this growth affects cities of various sizes, making it possible to identify the effectiveness of local public policies in solving endemic problems in urban areas.

In this sense, this work aims to identify the growth dynamics of Brazilian cities in the last century, improving knowledge of the urban system in Brazil. More specifically, it will answer the following questions: What has been the spatial distribution of Brazilian cities over the last century? Is there any mobility of cities within the distribution?

This work advances the literature by using a database that covers a longer period available for the Brazilian economy disaggregated at city level. Another contribution is to incorporate spatial neighborhood effects into the analysis of the Pareto coefficient estimation. And finally, by analyzing the growth dynamics of cities with Markovian matrices to capture possible movements in the distribution.

This article is organized as follows. In addition to this introduction, the next section presents the models to be estimated. The third section presents and discusses the results. Finally, the last section presents the conclusions.

II. Material And Methods

The data used was obtained from IPEA (Institute for Applied Economic Research). The MCAs (Minimum Comparable Areas) were used, made compatible for the last century (19102010). This sample comprises 432 observations, including the territory of Fernando de Noronha. In this work, this observation was disregarded because it did not have data available for the entire series and because being an island causes some problems in the construction of neighborhood matrices. The advantage of this sample for the analysis of Zipf's law is that it leaves the arbitrary aspect of the researcher to make an ad hoc choice of the size of the cities to be considered, which in the Brazilian case becomes a problem due to the creation of municipalities between one decade and another. In the Brazilian case, Justo (2007) and Oliveira (2005) estimated the Pareto coefficient for various city sizes and the values obtained varied depending on this choice. Similarly, in the international case, this problem was also raised by Lanaspa et al. (2003), who considered cities of 50,000 inhabitants or more for the Spanish case, and Mella and Chasco (2006).

The size order rule or Zipf's law

In estimating the Pareto exponent or size order rule, the original model proposed by Zipf (1949) was followed. The author suggested that the distribution of cities followed the rule of (Pareto, 1897) according to the following model:

$$R = a.\, S^{-\beta} \qquad (1)$$

Where R is the ranking of the order of distribution of the population; S is the population of the city; and a and β are parameters, the latter being the Pareto exponent and positive by construction.

According to Le Gallo and Chasco (2009), the size-order rule initially arose from a finding of regularity in the observed data without any foundation in economic theory. Krugman (1996), Eaton and Eckstein (1997), Overman and Yoannides (2001), Gabaix and Ibragimov (2006), among others, estimated this model and found this regularity for Zipf's Law for cities. Duranton (2006), however, provided a theoretical foundation based on the endogenous growth model suggested by Grossman and Helpan (1991) in an urban structure and analyzed the effect of R&D investments on city growth rates.

Formally, in this structure, the size of the distribution of cities depends on the value of the Pareto exponent (β). In the limit, if β tends to infinity, then all the cities will have the same population. When β is equal to 1, we have what is known as the size order rule or Zipf's law.

According to this rule, the population between any group of cities in time is inversely proportional to the ranking of their population in the group. The Pareto exponent can be interpreted as an indicator of convergence. When the value of the coefficient falls over time, it indicates a greater relative importance for large cities. In addition, this causes a tendency towards divergence in the group of cities or greater concentration in the largest cities. On the other hand, a 1% increase in city size produces a smaller drop (in %) in rank when increases. Thus, an increase in the value of β represents a dynamic of convergence, or in other words, a greater dispersion of the population outside the large urban centers and a greater balance in the distribution of the population between urban centers of different sizes (Le Gallo and Chasco, 2009).

The empirical model derived from equation (1) can be estimated in the following functional form:

$$\ln R_{it} = \ln a_t - \beta_t . \ln S_{it} + \varepsilon_{it} \quad (2)$$

According to Gabaix and Ibragimov (2006) the Ordinary Least Squares (OLS) estimation of equation (2) is biased for small samples. We therefore followed the authors Le Gallo and Chasco (2009) and corrected for the bias by subtracting ½ from the rank and estimated the model: ln(Rank - ½)= α-β ln(size). The estimation without this correction was tested and the robustness of the suggested correction was confirmed, thus maintaining this correction.

The spatial effect

OLS estimators can be affected by the omission of spatial autocorrelation. If the process generating the spatial autocorrelation of the residuals is autoregressive, the OLS estimators are unbiased, but not efficient. Statistical inference is therefore biased in this case. If they are due to the omission of the spatial autocorrelation of the variables, the OLS estimators are biased.

In this way, this work aims to identify and incorporate models that capture the neighborhood effect in the estimation of the Pareto exponent, thus making a breakthrough in the national literature on the subject.

Spatial autocorrelation, also known as spatial dependence, spatial interaction or local interaction, is defined as a measure of similarity between two values of an attribute that are spatially close. According to Pacheco and Tirrel (2002), spatial autocorrelation can be measured by various indices, the best known of which is Moran's I, which measures the degree of linear association between an attribute (y) at a given location and the weighted average of the attributes at neighboring locations (Wy) and can be interpreted as the slope of the regression of (y) on (Wy). Spatial autocorrelation can be visually illustrated in a graph where (Wy) is plotted on the vertical axis and (y) on the horizontal axis.

This statistic follows the following expression according to Battisti and Vaio (2009):

$$I = \frac{n}{q} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij} x_i x_j}{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j} \quad (3)$$

Where, ω_{ij} is an element of the weight matrix W, xi is a specific variable for observation i, n is the number of observations, q is a scaling factor equaling the sum of all the elements of the matrix. In this article, we followed Justo et al. (2010) and used the standardized row binary matrix based on the neighborhood structure with a fixed number of close neighbors (k-nearest), in which the elements are:

$$\left\{ \begin{array}{l} \omega_{ij}(k) = 0 \text{ se i=j} \\ \omega_{ij}(k) = 1 \text{ se } \mathrm{d_{ij}} \le d_i(k) \\ \omega_{ij}(k) = 0 \text{ se } \mathrm{d_{ij}} > d_i(k) \end{array} \right\}$$

Where di is a critical value, defined for each observation i, ensuring that each municipality has the same number of neighbors.[1]

Spatial heterogeneity (also known as a spatial structure, non-stationarity, large-scale global trend of the data) refers to differences in the mean and/or variance, and/or covariance, including autocorrelation in a spatial region. Unlike spatial autocorrelation, it requires that the mean and variance of an attribute is constant in space, and the spatial autocorrelation of an attribute at any two locations depends on a lag of the distance between two locations, but not on the location itself (Justo, et al., 2010).

It is not always easy to distinguish spatial heterogeneity from spatial autocorrelation. The presence of clusters, for example, can induce spatial autocorrelation between neighbors, but it can also be a sign of different possibilities for spatial regimes (Anselin, 2001). Tests to determine spatial autocorrelation or heteroscedasticity can generate inconclusive results.

[1] In this work, various types of matrices were tested and k-nearest with (k = 6) was chosen because it gave the best results. 3 For more on these models, see LeSage and Pace (2009).

According to LeSage and Pace (2009), when considering spatial autocorrelation in a data set it is necessary to establish the neighborhood structure for each location by specifying which locations are considered neighbors. It is necessary to specify a matrix of weights corresponding to the neighborhood structure such that the variance-covariance matrix can be expressed as a function of a small number of estimable parameters compatible with the sample size (Anselin, 2002). The types of weight matrices used in spatial econometrics include, among others, the following types: tower, queen, contiguity matrix, spatial weight matrix by means of a limit distance but with a fixed number of close neighbors (k-nearest), distance weight matrix, and the inverse distance matrix. The weight matrix is usually defined exogenously and after comparing various types of matrices. According to Voss and Chi (2006) several types of weight matrices are created and the one with the highest statistical significance is chosen.

According to Chi and Zhu (2008) there are two problems associated with specifying spatial weights in practice. One problem is that the weight structure can be affected by the quality of the georeferenced data. The other problem is that the use of some distance weight matrix may require a threshold value, which can be difficult to determine, especially when there is strong spatial heterogeneity. A small threshold can produce too many islands, while a large threshold creates too many neighbors. One solution to this case proposed by Anselin (2002) is to structure the spatial weight matrix using a threshold distance, but with a fixed number of near neighbors (k-nearest). According to Chi and Zhu (2008), the commonly used spatial linear regression model includes, in addition to the usual coefficients of the explanatory variables (β) and the variance of the error term (σ^2), a spatial autoregressive coefficient (ρ), which measures the strength of spatial autocorrelation. It also includes a weight matrix (W) corresponding to the neighborhood structure and the weight matrix (D) which are pre-specified.

A spatial linear regression model will now be specified when the error terms are specified. Two of the most used models will be presented: the Spatial Lag Model whose structure is modeled in this way:

$$Y = X\beta + \rho WY + \varepsilon i \quad (4)$$

Where Y is the vector of dependent variables, X the matrix of explanatory variables, W the matrix of spatial weights, andε the vector of error terms that are independent but not necessarily identically distributed. The other model is the spatial error model specified as follows:

$$Y = X\beta + u,\ u = \rho Wu + \varepsilon i \quad (5)$$

Where the terms are defined as in the previous model.

In the spatial Lag model, spatial autocorrelation is modeled by a linear relationship between the dependent variable (y) and a spatially lagged variable (Wy). In the case of the spatial error model, spatial autocorrelation is modeled by a term (u) and the spatially lagged error term (Wu). In either model, the interpretation of a significant spatial autoregressive coefficient is not always straightforward. A significant spatial error term indicates spatial autocorrelation in the errors that may be due to important explanatory variables that were not included in the model (Anselin, 1995).

According to Chi and Zhu (2008) several regression models can be specified for a given data set. If the models are nested, a likelihood ratio (LR) test can be used to compare the models. If the models are not nested, AIC (Akaike's Information Criterion) and BIC (Schwartz's Bayesian Information Criterion) can be used. Models with lower BIC and AIC are considered better.

Mobility in the Brazilian Urban System

The density function and Zipf's Law allow us to characterize the evolution of the overall distribution, but they don't provide any information about the movement of cities in the distribution. For example, it is not known whether the municipalities in the lower tail of the distribution in 1910 are the same as those in 2010. Le Gallo and Chasco (2009) based themselves on the work of Kemeny and Snell (1976) and suggest that one way to deal with this issue is to check the evolution of the relative position of each city over the period analyzed by estimating the matrix of transition probabilities with Markov chains. Black and Hendesrson (2003) used this procedure for the American urban system.

The analysis of the evolution over time of the cross-section distribution, or in other words, the analysis of dynamics, is a methodology whose objective is to describe the Markovian stochastic process. In this sense, working with state-space has several advantages according to Bulli (2001).

Discrete probability distributions and transition matrices are easier to interpret than a Kernel stochastic process. Another advantage is that descriptive indices and the long-term ergodic distribution are easier to calculate. However, there is a problem with this methodology: stratification, i.e. the results can be conditioned on the choice of classes that divide the distribution.

Following Le Gallo and Chasco (2009), the formulation of the cross-section distribution of the population size of Brazilian cities at time t will be denoted by Ft. A set of K different class sizes is defined, which provide a discrete approximation of the population distribution. It is assumed that the frequency distribution follows a first-order Markovian process. In this case, the evolution of the city size distribution is represented by the probability transition matrix, M, in which each element (i, j) indicates the probability that a city that is in one class in period t will move to a higher class in period t+1. The vector Ft (K, 1) indicates the frequency of cities in each class at time t. It is then described by the following equation:

$$F_{t+1} = M\,Ft \tag{6}$$

Where M is the probability transition matrix (K,K) representing the two distributions as follows:

$$M = \begin{bmatrix} p_{11} & p_{12} & .. & p_{1K} \\ p_{21} & p_{22} & .. & p_{2K} \\ .... & .... & .... & .... \\ p_{K1} & p_{K2} & .. & p_{KK} \end{bmatrix} \tag{7}$$

Where each element $p_{ij} \geq 0, \sum_{j=1}^{K} p_{ij} = 1$. The stationary probability transition matrix pij captures the probability that each city in class t-1 will move to class j in t.

The elements of the M matrix can be estimated from the observed frequency of class changes from one period to the next. According to Amemiya (1985) and Hamilton (1994), the maximum likelihood estimator of p_{ij} is:

$$p_{ij} = n_{ij} / n_i \tag{8}$$

Where nij is the total number of cities moving from class i in decade t-1 to class j in the immediately following decade t in the ten transition matrices and ni is the sum of cities in i in the ten transition matrices.

If the transition probabilities are stationary, i.e. if the probability between two classes does not vary over time, then:

$$F_{t+s} = M^S F_t \tag{9}$$

In this structure, the ergodic distribution (also called steady state distribution) of Ft is characterized when s tends to infinity in equation (9). Since the changes represented by the Mmatrix are repeated an arbitrary number of times. The distribution exists if the Markov Chain is regular, i.e. only if, for one m, Mm has no entries with zero values. In this case, according to Le Gallo and Chasco (2009) the transition probability matrix converges in the limit to a matrix M* of rank equal to 1. The existence of the ergodic distribution, F*, is denoted by:

$$F^* M = F^* \tag{10}$$

This vector F* describes the future distribution of cities if the movements observed in the sample period are repeated infinitely. According to equation (10), the limit of the distribution is given by the eigenvector associated with the eigenvalue of M.

The assumption that the Markov Chain p_{ij} process is stationary requires the probability matrix, pij, to be of first order. If the Markovian matrices are of a higher order, they will contain only part of the information needed to describe the true evolution of the population distribution. In addition, the Markovian matrix property implicitly assumes that the transition probabilities, pij, , are not of order zero (Le Gallo and Chasco, 2009).

In this case, Bickenbach and Bode (2003) suggest testing this property, i.e. temporal independence. First, test order 0 against order 1; then order 1 against order 2 and so on. If the hypothesis of order 0 against the hypothesis of order 1 is rejected and if the hypothesis of order 1 against order 2 is not rejected, then the process is of order 1.

To test the order 0, following Le Gallo and Chasco (2009), the null hypothesis $H_o : \forall_i : p_{ij} = p_j (i = 1,...,K)$, is tested again following the alternative $H_a : \exists_i \setminus p_{ij} \neq p_j$. The likelihood ratio (LR) test then follows:

$$LR^{(O(0))} = 2\sum_{i=1}^{K} \sum_{j \in A_i} n_{ij}(t) \ln(\hat{p}_{ij} / \hat{p}_i) \sim asy\chi^2[(K-1)^2] \tag{11}$$

Assuming that $\hat{p} > 0, \forall_j (j = 1,...,K)$. $A_i = \{ j : \hat{p}_{ij} > 0 \}$ is the set of non-zero transition probabilities under the null hypothesis.

To test order 1 against order 2, the second-order Markov Chain is defined by considering the size of the population in class k (k=1, ..., K) in which the cities were at time t-2 and if the pair of successive classes k form a class. So, the probability of a city moving into a class j at time t, given that it was in k at time t-2 and in i at time t-1, is pkij . The corresponding absolute number of transitions is nij (t), with the marginal frequency being nki $n_{ki}(t-1)=\sum_j n_{kij}(t-1)$.

To test $H_o:\forall k: p_{kij}=p_{ij}(k=1,...,K)$ against the alternative hypothesis $H_a:\exists k: p_{kij}\neq p_{ij}$, the the probabilities pkij are estimated as $\hat{p}_{kij}=n_{kij}/n_{ki}$, onde $n_{kij}=\sum_{t=2}^{T} n_{kij}(t)$ e $n_{ki}=\sum_{t=2}^{T} n_{ki}(t-1)$. The likelihood ratio (LR) test is estimated as follows:

$$LR^{(O(1))}=2\sum_{k=1}^{K}\sum_{i=1}^{K}\sum_{j\in C_{hi}} n_{kij}\sum_{j\in A_i} n_{ij}\ln(\hat{p}_{kij}/\hat{p}_{ij}) \sim asy\chi^2[\sum_{i=1}^{K}(c_i-1)(d_i-1)] \quad (12)$$

Similarly for higher orders $C_i=\{j:\hat{p}_{ij}>0\}, c_i=\#C_i, C_{ki}=(j:\hat{p}_{kij}>0\}$ e $\mathrm{d_i}=D_i=\#\{k:n_{ki}>0\}$.

If it rejects that the Markov chain is of order 0 and 1, it proceeds to test for higher orders. However, since the number of parameters to be estimated grows exponentially with the number of lags and the number of observations decreases linearly for a given data set, the power of the test decreases rapidly. That said, an order to be tested must be defined a priori.

III. Result and Discussion

Table 1 shows the descriptive statistics for the distribution of Brazilian cities in the period 1910-2010. The accelerated growth of the means and medians over the hundred years analyzed indicates the rapid expansion of the Brazilian population in this period. Between 1910 and 2000 there is a monotonically increasing dispersion measured by the coefficient of variation, but the trend reverses in the last decade.

Table no 1 Descriptive Analysis

Year	Average	Standard deviation	Median	Coefficient of Variation
1910	54009.71	94149.76	26977	1.74
1920	70688.29	130965.9	33369	1.85
1940	95335.39	219925	41091	2.31
1950	120245.3	298493.8	51745	2.48
1960	163481.0	441705.1	61635	2.71
1970	215578.1	630807.8	73813	2.93
1980	275425.8	834924.9	87347	3.03
1991	339689.3	1014843	104553	2.97
2000	392667.3	1167194	120611	2.97
2010	439823.0	1296059	131271	2.95

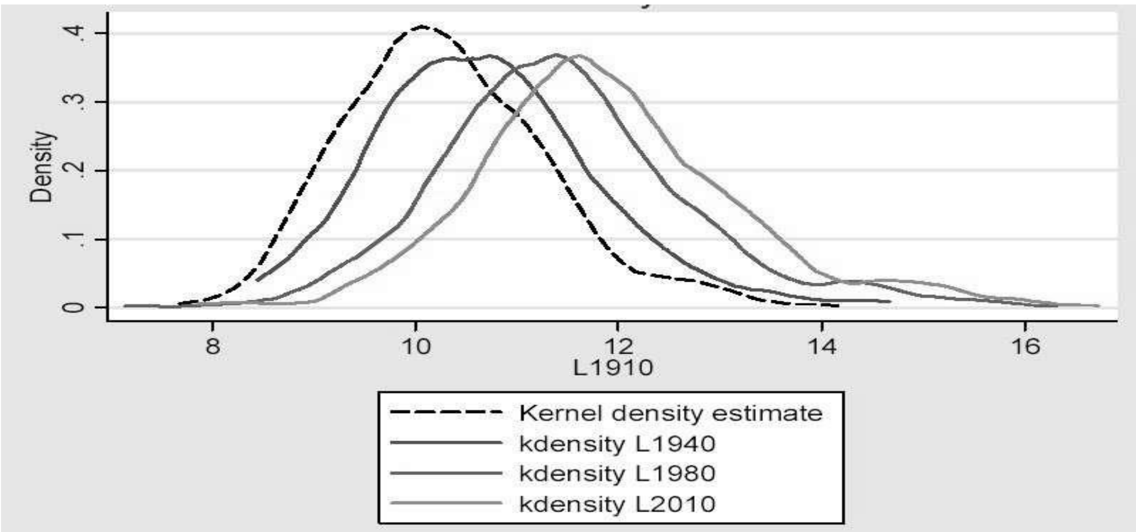
Source: Based on IPEADATA data.

The evolution of population distribution in Brazilian cities

Using the observations available for the period 1910-2010 of the population of Brazilian cities, the shape of the distribution of city size was analyzed. Figure 1 shows the estimates of the non-parametric Kernel density functions of the population of Brazilian cities for 1910, 1940, 1980 and 2010. There is a unimodality characteristic at the beginning of the century, until the 80s. This result suggests a divergence in the size of cities. In 2010, however, there is a slight indication of the formation of another mode among the cities located in the upper tail of the distribution, comprising the largest cities. In this way, density begins to capture a possible effect of population redistribution of Brazilian cities among the group of largest cities, which in the Brazilian case is mostly made up of state capitals. Justo et al. (2010) show, for example, that some northeastern cities, not just the capitals, but above all medium-sized cities, already have a larger inflow of migrants than outflows. When analyzing return migration, this behavior is even more evident. It is known, however, that migration is an important factor in the growth of cities in Brazil.

The shift in the density function shows the high population growth rates of Brazilian cities between 1910 and 2010. Le Gallo and Chasco (2010) in a similar study for the Spanish urban system found unimodal distribution until the 1980s and multimodality in recent decades.

Figure1 Kernel density function for the population of Brazilian cities: 1910-2010.



Source: Prepared with data from IPEADATA.

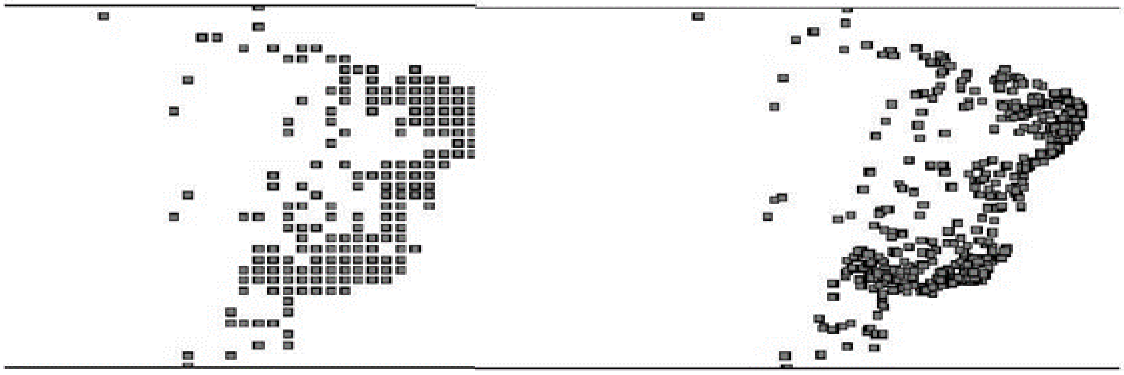
The size order rule: Zipf's law

Continuing to explore Brazil's urban evolution in the 1910-2010 century, Zipf's law is explored here. As previously mentioned, this paper works with MCAs which comprise 431 observations for the period analyzed. This is a departure from the random choice made by researchers to define a size of cities a priori. The literature shows that the results are conditioned by this choice. The works by Justo (2007), Oliveira (2005), Bowen et al. (2022) and De Marzo et al. (2023) highlight this problem.

Figure 2 shows the population distribution of Brazilian cities in 1910 (left) and 2010 (right). After a century there is a spatial concentration of the population of Brazilian cities and a tendency towards agglomeration in cities in the Southeast and the coast and interior of the Northeast. This result is in line with those indicated by the estimation of the Kernel density functions presented earlier.

On the other hand, this dynamic spatial distribution of the population of Brazilian cities makes it necessary to test and incorporate the neighborhood effect into econometric models, if the tests indicate it, to understand the growth trajectory of Brazilian cities between 1910 and 2010.

Figure 2 Spatial distribution of the population of Brazilian cities: 1910-2010.

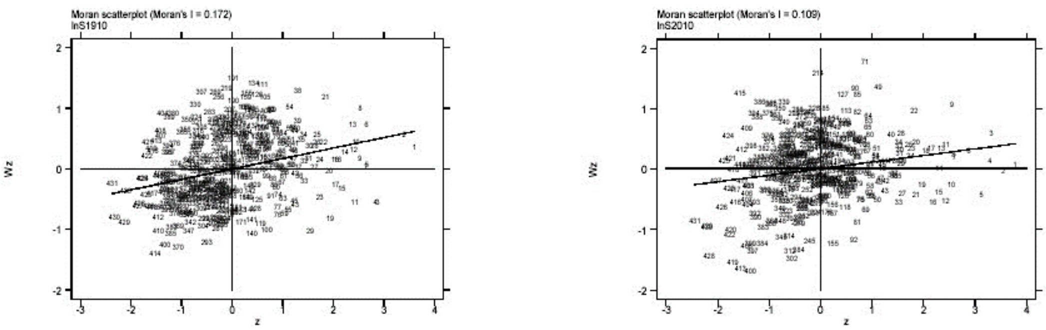


Source: Based on IPEADATA data.

Having said this, we move on to test the spatial effect using Moran's I statistic. Figure 3 shows the results of the Moran's I statistic test for equation 3 for 1910 and 2010. The value of the statistic confirms the need to incorporate the neighborhood effect when estimating models that seek to explain the growth dynamics of Brazilian cities.[2]

[2] Tests were carried out for all the decades. Only the first and last decades are presented due to lack of space. Other tests were also estimated to indicate the spatial effect, such as Moran's tests under the assumption of randomness, among others.

Figure 3 Moran 's Scatter Plot: 1910 and 2010



Source: Based on IPEADATA data.

Table 2 shows the results of the estimations of the model that tests the order of the size of Brazilian cities in each decade in the period 1910-2010. The results of the estimations of the equation by (OLS) of equation (2) indicate the non-normality of the residuals and exhibit heteroscedasticity as shown by the Jarque-Bera, Koenker-Basset tests, respectively. Thus, it can be inferred that both effects are present in the 10 estimations.

We then move on to estimating the spatial models. The results in the (SEM) and (SLM) columns are the estimates of equation (2) modified by equations (4) and (5). That is, the Spatial Error Model and the Spatial Lag Model.

The LM-Error and LM-Lag statistics are used to choose the spatial model best suited to the data set. In this case, the results indicate that the spatial error model is the most appropriate. The higher value of the LM test statistic confirms the superiority of the Spatial Error Model. The value of the AIC and BIC statistics also confirms this choice. The corrected Pareto exponents in the spatial model are smaller than those estimated by MQO. In other words, the divergence in the distribution of Brazilian cities over the last century is even more intense after the spatial correction. Le Gallo and Chasco (2009) found similar results, but with different magnitudes for the Spanish urban system. There the Pareto exponents are smaller and showed a significant spatial effect for Spanish cities.

Table 2 Regressions of the rank of the size of Brazilian cities: 1910-2010

Ano	MQO					SEM			SLM		
	$\hat{\beta}_1$	$\hat{\beta}_2$	KB	JB	$\overline{R}^2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\rho}$
1910	14.21	0.93	25	904	0.91	13.76	0.89	0.25	16.2	0.98	0.32
1920	14.04	0.90	24	828	0.90	13.61	0.86	0.26	16.08	0.95	0.31
1940	13.70	0.85	20	572	0.90	13.34	0.82	0.27	15.59	0.89	0.31
1950	13.54	0.82	19	623	0.91	13.11	0.79	0.27	15.57	0.87	0.34
1960	13.42	0.80	18	726	0.91	12.98	0.76	0.26	15.31	0.84	0.31
1970	13.24	0.77	17	704	0.92	12.79	0.73	0.26	15.10	0.81	0.30
1980	13.16	0.75	16	1146	0.91	12.69	0.71	0.24	15.02	0.78	0.32
1991	13.12	0.73	15	1636	0.92	12.61	0.69	0.24	15.00	0.77	0.31
2000	13.15	0.73	14	1843	0.92	12.63	0.68	0.25	15.03	0.76	0.30
2010	13.16	0.72	12	2024	0.91	12.62	0.67	0.24	15.09	0.75	0.31
LR						23.691			10.118		
AIC						132			147		
BIC						140			159		

Source: Based on IPEADATA data. Prepared by the author.

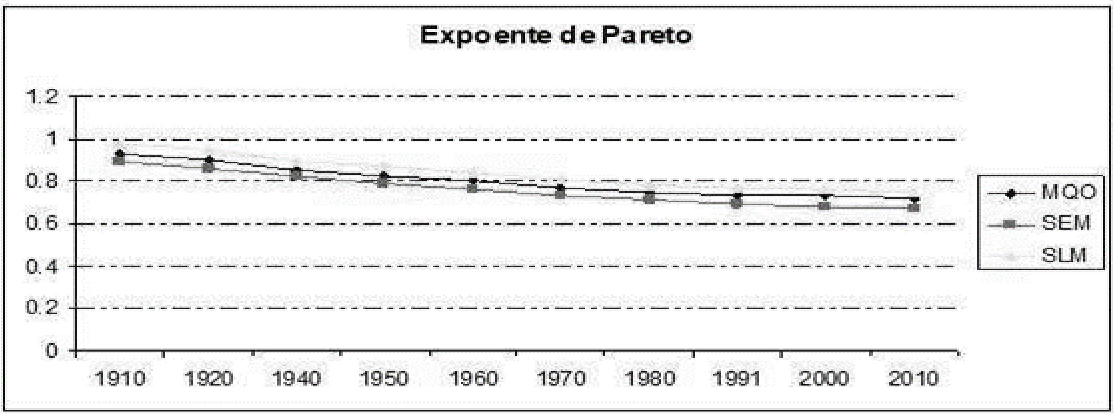
* All coefficients are significant at 1%. OLS (Ordinary Least Squares estimation).JB - Jarque-Bera tests for normality of residuals. KB - Koenjer-Basset test for heteroscedasticity. AIC and BIC - (Akaike Information Criterion and Schwarz Bayesian Information Criterion).

Graph 1 shows the evolution over time of the three estimates of the Pareto exponent over the last 100 years. The largest estimates are found by the Spatial Lag Model, followed in order by the Ordinary Least Squares estimates and the Spatial Error Model. There is also a monotonically decreasing trend in the estimates of the three models. However, there has been a slowdown in recent decades. This last result indicates a possible population deconcentration in Brazil's large metropolises, either due to a reduction in the growth rate of the largest cities and/or an increase in the growth rates of medium-sized cities, which have been attracting more migrants, as shown by Justo (2010). These results are corroborated by the estimates of the Kernel density functions presented above.

Le Gallo and Chasco (2009) also analyzed the behavior of population distribution over the last 100 years among municipalities in Spain and found a result in this direction, with an inflection at the end of the period.

An analysis of Zipf 's Law shows that until the 1980s, the increase in urban concentration in the largest cities was more rapid and then lost momentum. The values of the coefficients of variation already indicated this behavior.

Graph 1 Evolution of the estimates of the Pareto exponent (N= 431). Estimation by OLS (Ordinary Least Squares), Spatial Error Model (SEM) and Spatial Lag Model (SLM).



Source: Based on the data in table 1.

Mobility in the Brazilian Urban System: 1910-2010.

The results obtained so far by estimating the Kernel density functions and analyzing Zipf's Law, although considering the spatial effect, do not provide any information about the possibility of movements in the distribution over time. We therefore moved on to the analysis of the Markovian stochastic process.

Using the methodology explored in the previous section, however, can lead to some inaccuracies depending on the division of the distribution into a given set of classes used to find the probability transition matrix. We therefore tested various possibilities and opted for the discrete distribution that was closest to the continuous distribution.

The results of the probability transition matrix are shown in table 3[3]. The probabilities on the diagonal show low inter-class mobility, i.e. high persistence of cities remaining in their own class between one decade and another over the last hundred years. If the elements of the main diagonal were all equal to 1, they could be interpreted as parallel growth, as suggested by Eaton and Eckstein (1997)[4]. However, as they are not exactly equal to 1, it is interpreted as the propensity of each city to move to another cell. In particular, the results show that the largest and smallest cities (class 6 and 1) show greater persistence, while medium-sized cities are likely to move n the distribution (classes 3, 4 and 5). However, in addition to the low mobility found in the results, when there is movement between classes it is only to the next class.[5]

[3] The matrix comparison test using equation 11 indicates that the process is of order 1. The value of the LR statistic = 15.000 and Prob = 0.0000.

[4] The authors' idea is that the cities would start out with different sizes but would grow at a similar rate.

[5] Le Gallo and Chasco (2009) also found low mobility and high persistence for Spanish cities, but in different magnitudes. There, there is mobility in the intermediate classes beyond the immediate upper class and also to lower classes. However, in the dynamics of Spanish urban growth at the end of the last century, the behavior of the Pareto exponent reverses the trend.

Table no 4 Initial distribution versus ergodic distribution of the population of Brazilian cities: 19102010

	1 <20%	2 <50%	3 <80%	4 <135%	5 <185%	6 >185%
Initial distribution	0.47.48	0.3846	0.0626	0.0510	0.0244	0.0476
Ergotic Distribution	0.4567	0.3834	0.0615	0.0499	0.0232	0.0476

Source: Prepared from IPEADATA data.

As previously mentioned, the ergodic distribution can be affected by the way the distribution is divided into classes. Other types of stratification and different numbers of city distribution classes were tested. The results of slight convergence, however, were robust to these choices.

IV. Conclusion

Throughout the past century, the growth process of Brazilian cities has been propelled by industrialization and the expansion of the agricultural frontier. Industrialization exhibited a significant initial concentration in the Southeast region, with recent indications of deconcentration. The agricultural frontier, in turn, spans the North and Midwest regions, encompassing the occupation of the cerrado, and more recently extending into areas of the Northeast. In this study, we employed IPEA's AMC data to analyze the dynamics of Brazilian city growth from 1910 to 2010. The advantage of this database lies in its coverage over the longest available period for Brazil at this level of aggregation, mitigating issues associated with the arbitrary selection of city sizes by researchers.

The density function of the distribution of Brazilian cities over the last century reveals divergence, with a slight indication of the formation of a bimodal distribution in the last decade. Zipf's Law supports this divergence process but exhibits a loss of intensity in the last analyzed decade. In other words, larger cities were growing more rapidly than smaller ones, but in the last decade, medium-sized cities show signs of accelerated growth.

Neighborhood effects prove to be crucial in the analysis of the distribution of Brazilian cities, as tests indicate spatial dependence in the distribution. Therefore, Zipf's Law was analyzed incorporating spatial effects, thereby contributing to the literature in this area.

Finally, the Markov Chain analysis reveals low mobility and high persistence. In other words, the probability of a city transitioning between size classes in successive decades is very low. Conversely, the probability of cities remaining within the same size class is high throughout the period from 1910 to 2010. Only cities positioned in the middle of the distribution show the ability to advance to higher classes. These results remained robust across different methods used to divide the distribution into classes.

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