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Evaluation, Modeling and Forecasting Volatility of Daily and Weekly Returns on Nairobi Securities Exchange Using Garch Models

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Abstract: *This paper evaluates the forecasting ability of Nairobi Securities Exchange share prices at different time pointsusing Generalized Autoregressive Conditional Heteroskedasticity (GARCH).A five-year period wasused and appropriate models were determined for each time point for the companies chosen from amongst the lower order GARCH models that is GARCH (1, 1), GARCH (1, 2), GARCH (2, 1) and GARCH (2, 2). The best fitting GARCH models were chosen based on Akaike Information Criterion and Bayesian Information Criterion. Adequacy of the chosen models was done using Ljung Box and Lagrange Multiplier Autoregressive Conditional Heteroskedasticity (ARCH LM) tests. Parameter estimation was done by Bollerslev-Woodridge's Quasi Maximum Likelihood Estimator (QMLE. The intervals with the least mean errors were considered to have the best predictive ability. The results revealed that GARCH (1, 1) models performed well in modeling most return series for companies investigated especially for daily when compared to weekly returns. GARCH (2, 1) seemed better for KQ weekly data while GARCH (2, 2) performed poorly for all the data sets. The forecasting performance of each time point based on the selected models, daily returns gave better prediction than weekly returns and the models generally performed well when modeled with higher frequency data.*
Key words:*Volatility, GARCH model, MSE, Ljung Box Test*

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I. Introduction

Stock market, also termed as equity market is a place where securities, shares and bonds of publicly held companies are issued and traded either through exchanges or over-the-counter markets. The main purpose of a stock market is to provide a platform where investors can buy and sell shares without necessarily having to move from one place to the other looking for prospective buyers. The stock market plays a pivotal role in the growth of the sectors listed in the Securities exchange and the commerce of the country through the mobilization of resources. To trade in the stock market, the buyers and sellers agree on a price of a product, in this case, the transaction of "shares" which represent an equity or ownership interest in a particular company. The NSE is an institution that deals in exchange of securities issued by publicly quoted companies and the Government of Kenya. It is part of the broader market referred to as financial market. The prices of financial securities which are traded in the financial markets as well as interest rates and foreign exchange rates are subject to constant variability. The current price is where their mutual interests intersect. That intersection is a moving target so at one moment there is more supply than demand.

Forecasting volatility is a crucial and demanding financial matter which has attained extensive attention in the past few decades. It is widely accepted that though returns of financial securities prices are more or less unpredictable on daily as well as weekly basis, return volatility is forecastable along with vital inference for financial economies and risk management [1]. Good forecasts therefore become extremely important in making financial decisions. With the recent advancements in technology and communication, and subsequent automation of trading activities, real-time stock market information on the listed securities facilitates price discovery for the interested persons at whatever times of interest. Moreover, future prices being uncertain, they must be described by probability distributions, thus statistical methods become a natural way to investigate prices. Time series methods have also been found to be able to predict many financial time series. Swarchz, further argues that time series models are the ultimate tool for letting the "data speak for themselves" because all inferences are based on the observed series[2].

1.1 Time Series Analysis

Time series analysis is a form of statistical data analysis on a series of sequential data points that are usually measured at equal time intervals over a period of time. The most common characteristics or patterns of a time series are increasing or decreasing trend, cyclic, seasonality, and irregular fluctuations. Time series Analysis concerns the analysis of data collected over time for instance daily, weekly, monthly and so forth.

Models for time series data can have many forms and represent different stochastic processes. When modeling variations in the level of a process, there are three broad classes of practical importance these are; AR, I, MA models. These three classes depend linearly on previous data points. A combination of the three ideas produces Autoregressive Moving Averages (ARMA) and Autoregressive Integrated Moving Averages (ARIMA) models [3]. The notation AR (p) indicates an autoregressive model of order p. The AR (p) model is defined as:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t \qquad (1)$$

where $\varphi_1, \ldots, \varphi_P$ are the parameters of the model, c is a constant, and ε_t is white noise.

The Moving Average (MA) model is a common approach for modelling univariate time series models. MA(q) refers to the moving average model of order q, it is defined as:

$$X_t = \mu + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (2)$$

where μ is the mean of the series, the $\theta_1, \ldots, \theta_q$ are the parameters of the model and the ε_t, ε_{t-1},... are white noise error terms.Therefore a MA model is conceptually a linear regression of the current value of the series against current and previous (unobserved) white noise error terms.They are used in time series to describe stationary events. The Autoregressive Moving Average models(ARMA) consists of two parts, the Autoregressive part and the Moving Average part..The model is then referred to as ARMA(p,q) where p is the order of the autoregressive part and q is the order of the moving average part i.e. it contains the AR(p) and MA(q) models. ARMA models are sometimes called Box Jenkins models after Box and Jenkins who expounded an iterative method for choosing and estimating them [4]. ARMA model allowed greater flexibility in fitting of actual time series.

Therefore, ARMA(p,q) is as follows:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \qquad (3)$$

and when ARMA is differenced then it yields ARIMA (p.q) model.

Among other types of non-linear time series models, there are models to represent the changes of variance over time (heteroskedasticity). These models represent autoregressive conditional heteroskedasticity (ARCH) and among others the GARCH model.In the empirical application of the ARCH model a relatively long lag in the conditional variance equation called for, and to avoid problems with negative variance parameters a fixed lag structure is typically imposed.If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a generalized autoregressive conditional heteroskedasticity (GARCH) model [5].

1.2 GARCH Models

The GARCH model was introduced by Bollerslev to overcome the ARCH limitation. It generalized ARCH to make it more realistic i.e. to allow for both a longer memory and a much more flexible lag structure. The extension of the ARCH process to GARCH process bears much resemblance to the extension of the standard time series AR process to the general ARMA process and, this allows a more parsimonious description in many situations.GARCH models have been found to perform well with stock market returns, exchange rates, Consumer price indices and many other variables. Existing literature suggests that GARCH models are better in describing returns series that have the changing variance level. They have been extensively researched on and tested statistically and empirically.The Gaussian GARCH (1, 1) process, in particular, is widely used and highly regarded in practice as well as in the academic discourse. It is often preferred by financial modeling professionals because it provides a more real-world context than other forms when trying to predict the prices and rates of financial instruments. The model is the most celebrated among the ARCH family.
The GARCH (p, q) is given as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \quad (4)$$

Such that the standard GARCH (1, 1) model is defined as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (5)$$

where

- ε_t are returns with zero mean and unit variance
- ω, α_1 and β_1 are model coefficients, ω> 0, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$ and

$$\varepsilon_t = u_t\sqrt{\sigma_t^2}\ ;\ u_t \approx \text{i.i.d}\quad N(0,1)$$

The GARCH (1, 1) model is equivalent to an infinite ARCH model with exponentially declining weights.

In practice, the standard GARCH (1, 1) has been found to be sufficient to capture the volatility clustering in the data. And according to most researches, it is rarely necessary to use more than a GARCH (1, 1) model for financial applications. In particular, GARCH (1, 1) successfully captures thick tailed returns, and volatility clustering, and can readily be modified to allow for several other stylized facts, such as non-trading periods and predictable information releases[6]. In addition to the standard GARCH (1, 1), other lower order GARCH models have been found to fit well to stock returns, for instance GARCH (1, 2), GARCH (2, 1) and GARCH (2, 2).

II. Methodology

2.1 Study Area

The study evaluated the three selected companies namely, National Bank of Kenya (NBK), East Africa Portland cement and Kenya Airways (KQ) which are among the listed companies in Nairobi Security exchange (NSE).The NSE is part of the African Stock Exchange Association (ASEA) founded in early nineties to create a way for all the stock exchanges in Africa. Today there are over 50 business and companies listed in NSE and trading with more than a 100 million shares per month. These companies have been sub divided into varioussectors; Agriculture, Manufacturing& Allied segments, Finance & Investments and Commercial Services. The three companies were selected to represent the following sectors; Finance and Investment, Industrial & Allied and Commercial services respectively.

2.2 Research Methods

The three data sets from three companies as mention in section 3.1 were considered. The data set for daily and weekly closing stock priceswere collected from the NSE historical price database for the period 3rd June, 2006 to 31st Jan, 2012. The five-year study period was chosen to ensure that likelihood function is well defined and that the models properly converge, a few years of data are needed but not too many that current market conditions are not reflected. If we take a too short a period data, then parameter estimates may not be robust [7].

In the preliminary, the descriptive statistics; mean, maximum, minimum and standard deviation, Skewness, Kurtosis, Jarque-Bera, were computed using the standard formulae [8]. The data analysis was done using the Box and Jenkins approach of ARIMA model building, that is, model identification, estimation and diagnostic testing [3]. In the model identification stage, exploratory analyses were done to determine the characteristics of the data sets but model order was however not considered because the models used were already predetermined as the lower GARCH models, then the data was fit to the respective models identified by estimation and finally diagnostic testing was done to rule out any model misspecification that could have occurred.The three stages of performing time series analysis as follows [3]

2.1.1 Identification

Augmented Dickey–Fuller test (ADF) is a test of unit roots in time series. ADF is to confirm the stationarity of the returns by testing the presence of unit roots. ADF tests the hypothesis that the series is non-stationary against an alternative hypothesis that the series is stationary. It is a negative number. The more negative it is, the stronger the rejection of the hypothesisthat there are no unit roots at some level of confidence.The hypothesis is also rejected if the p values are found to be smaller than the level of significance used in the investigation.Acceptance of the null hypothesis would imply that the series is non-stationary, thus further differencing would be needed to make the data sets stationary.Sample autocorrelation function (ACF) plots were then used to ascertained the serial dependence for observations $x_1, x_2, \ldots x_n$ at varying time lags. It is rarely necessary to test correlations to lags greater than 20, so they were tested at lags 10, 15 and 20.ACF is given by

$$\text{ACF}(h) = \rho(h) = \frac{\gamma(h)}{\gamma(0)}, \text{ for time lags h=0,1,…,n-1} \qquad (6)$$

The autocorrelations should be near zero for all the time lags if the time series is an outcome of a completely random phenomenon, otherwise, one or more of the autocorrelations will be significantly non-zero.Transformation of the series into returns was done using the operation;

$$r_t = \ln\left[\frac{p_t}{p_{t-1}}\right] \quad (7)$$

where P_t and P_{t-1} are current and previous closing share prices for times t= 1, 2, 3 · · ·

2.1.2 Model Estimation

GARCH (1, 1), GARCH (1, 2), GARCH (2, 1) and GARCH (2, 2) parameters for the three companies for all the returns were estimated using robust method of Bollerslev-Woodridge's Quasi Maximum Likelihood Estimator (QMLE) assuming the Gaussian standard normal distribution.

2.1.3 Diagnostic testing

In model selection, the two procedures AIC and BIC were used and values of AIC and BIC werecomputed and compared such that the models with smaller AIC and BIC were preferred. Akaike Information Criterion (AIC) is a measure of the relative quality of a statistical model for a given set of data; as such it provides a model for model selection. For any statistical model, AIC is given by

$$\text{AIC} = 2k - 2\,\text{In}(L) \quad (8)$$

where k is the number of parameters in the model and L is the maximized value of the likelihood function for the model.

Bayesian Information Criterion (BIC), on the other hand is also a criterion for model selection among a finite set of models. It is based on the likelihood function and is closely related to AIC. BIC is given by

$$\text{BIC} = 2\,\text{In}(L) + k\text{In}(n) \quad (9)$$

where k is the number of parameters in the model, L is the maximized value of the likelihood for the model and n is the sample size. In addition, the Ljung-Box text was formally applied on the returns series to check for the presence of GARCH effects. The Ljung-Box modified Q^* statistic is computed as:

$$Q^*(m) = n(n+2)\sum_{i=1}^{m}\frac{\hat{p}_j^2}{n-1} \quad (10)$$

where:

- m is the maximum number of lags included in the ARCH effect test,
- $\hat{p}_j$ is the sample autocorrelation at lag j for the squared residuals

$Q^*(m)$ has an asymptotic chi-square distribution with m degrees of freedom.

$$Q^*(m) \sim \chi^2(m)$$

where $\chi^2(m)$is a Chi-square probability distribution function and m is the degrees of freedom for the Chi-square distribution.

The computed p-value is compared with the significance level used in the investigation, and the hypothesis of no GARCH effects was rejected if the p value is found to be less than 0.05.

2.2 Forecasting evaluation

In time series, forecasting is a mathematical way of estimating future values using present and historical values of the series [9]. Forecasting of future share prices was done using the fitted models of each data set and the forecasting performance of each time points evaluated and compared using common statistical error functions which measure forecasting accuracy. The intention was to find a time point that gives the best forecasting power. These measures are Mean Absolute Percentage error (MAPE), Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) which are given by:

$$\text{MAPE} = \frac{100}{K}\sum_{t=n+1}^{n+k}\left|\frac{\hat{\sigma}_t - \sigma_t}{n-1}\right| (11)$$

$$\text{MAE} = \frac{\sum_{t=n+1}^{n+k}|\hat{\sigma}_t - \sigma_t|}{K} (12)$$

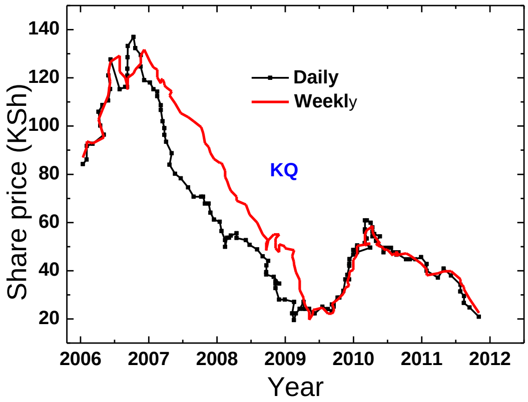
$$\text{RSME} = \sqrt{\frac{\sum_{t=n+1}^{n+k}(\hat{\sigma}_t - \sigma_t)^2}{K}} \qquad (13)$$

where n is the sample size, K is the number of steps ahead, $\hat{\sigma}_t$ and σ_t are the square root of the conditional forecasted volatility and the realized volatility respectively.The time point which yields the lowest mean error values of the forecast evaluation statistics is considered better than the rest

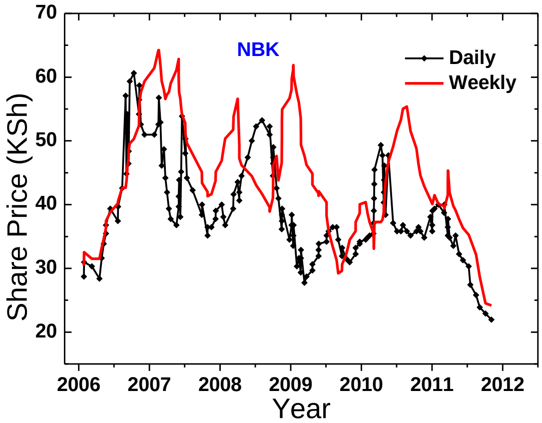
III. Results and Discussion

3.1 Time plots for Daily and Weekly closing prices

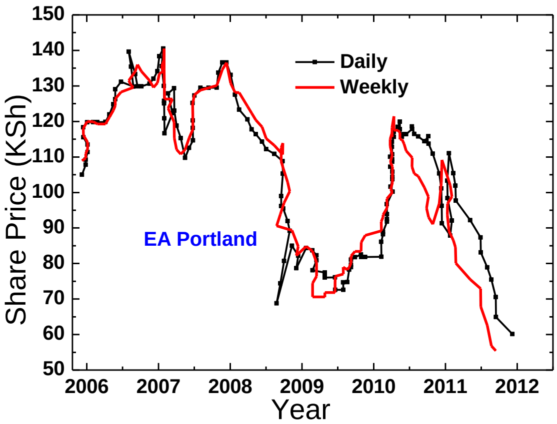
The time plots of the daily and weekly closing prices of the companies whose share prices were analyzed are presented in Fig. 1 below.



(a)



(b)



(c)

Figure1: Time plots for the Daily and Weekly series for (a) KQ (b) NBK (c) EA Portland

From the time plots in Fig. 1 (a), (b) and (c) for the daily series, the swings are evident and its clear that the stock prices are very irregular with varied degree of flactuations. The mean and variance are not constant impling that the series are non stationary. In comparison, the visual inspection of Fig.1 (a), (b) and (c) above, also shows that the weekly prices are quite irregular and that, fluctuations are frequent, suggesting that the mean and variance are not constant and hence non stationary.

3.2 The time plots of Daily returns

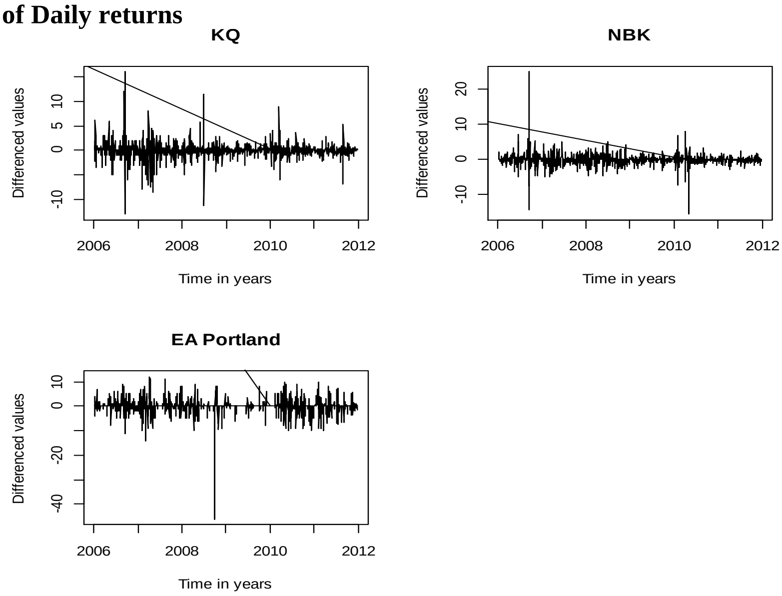


Figure2: Time plots of daily returns for KQ, NBK and EA Portland

Transforming the raw series as shown in Fig. 1resulted into the daily returns presented in Fig.s 2 above. Unlike the time plots for the raw series, the plots for returns are trendless and their amplitudes vary over time. They tend to flactuate around zero, implying a constant mean and stabilized variance. This is because of the presence of ARCH effects [10]. The plots are marked by periods of calmness interposed with turmoil. This phenomenon is referred as volatility clustering because the returns appear in form of cluster and it is very conspicuous in all the returns plots.

3.3 The time plots of Weekly returns

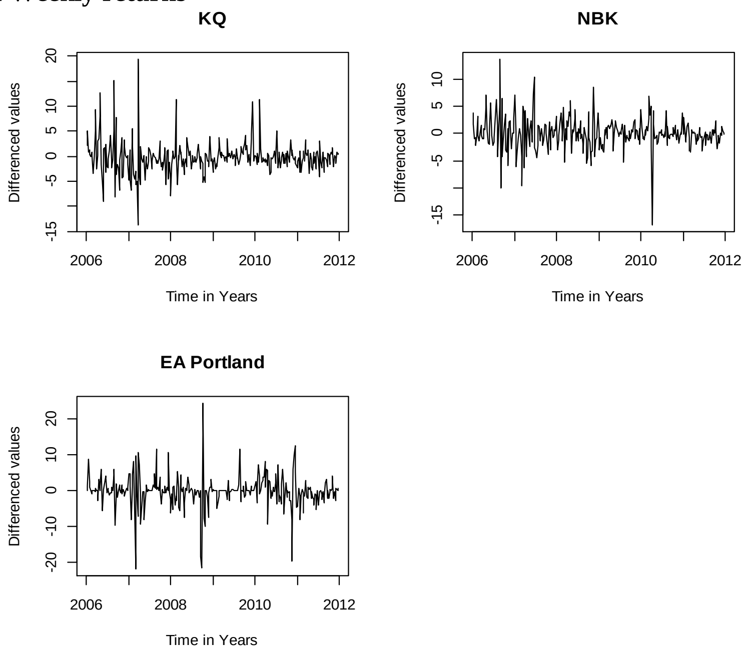


Figure 3: Time plots of weekly returns for KQ, NBK and EA Portland

The weekly returns in Fig. 3 appear in clusters, with varying amplitude but generally vary around zero. This suggests that the returns are now stationary. The plots are marked by periods of relative calmness interposed with turmoil. This phenomenon is referred as volatility clustering and it is very conspicuous in all the returns plots.

3.4 Statistics on data sets for the daily returns

Descriptive statistics on the data sets were also carried out and presented in Tables 1.

Table 1: Descriptive statistics for the daily returns

Company	Min	Max	Mean	Stdv	Skew	Kurt	Jarque/Berra
KQ	-0.255	0.209	-0.0009	0.028	-0.196	13.459	11383.880(<2.2e-16)
NBK	-0.346	0.476	-0.0002	0.034	0.873	37.324	87741.240(<2.2e-16)
EA Port	-0.555	0.108	-0.0005	0.027	-5.874	121.509	936435.500(<2.2e-16)

Generally, the difference between the maximum and minimum returns was large, which is a common feature of index returns, and as expected for time series of returns, the mean is quite close to zero for all the returns series. The standard deviation for all the returns are also high indicating a high level of fluctuations of returns, for instance, for the daily series, NBK was the most volatile with a standard deviation of 0.034 while East African Portland Cement was the least volatile with a standard deviation of 0.027. The kurtosis of all the three data sets exceeded the normal value of 3, indicating evidence of fat tails (leptokurtic) and sharp peaks around the mean. This implies that their distributions were quite close to normal distributions. This is in line with the literature available. Positive and negative skewness were also observable in all the returns, this means that the right and the left tail is particularly extreme respectively, and an indication of lack of symmetry. The Jarque-Berra test also led to the same rejection of normality in all the return series at 5% level of significance. In order to test whether the returns are stationary or not, ADF tests were done and the results are shown in Table 2 below.

Table 2: Unit root testing for daily return and closing prices

Company	Daily returns		Prices	
	ADF	P-value	ADF	P-value
KQ	-11.1087	0.01	-1.7900	0.6671
NBK	-34.0284	0.01	-3.6687	0.0607
EA Port	-40.1553	0.01	-3.0446	0.1361

Table 2 presents the stationary checks for the raw data and the returns using ADF test statistics. The ADF values are more negative for returns series than for closing prices, p values are also conspicuously smaller for the returns series than for the raw series.

3.5 Statistics on data sets for the Weekly returns

Descriptive statistics were carried out to further explore the distribution characteristics of weekly returns and the results are presented in Table 3.

Table 3: Descriptive statistics for weekly returns

Company	Min	Max	Mean	Stdv	Skew	Kurt	Jarque/Berra
KQ	-0.1693	0.2331	-0.0045	0.0556	0.8504	3.5984	210.9605 (<2.2e-16)
NBK	-0.3637	0.2244	-0.0011	0.0624	-0.2168	4.3222	251.4412(<2.2e-16)
EA Port	-0.2758	0.2554	-0.0021	0.0445	-0.3579	11.7112	1824.407(<2.2e-16)

The means for the three sets of returns are all negative and the difference between the minimum and maximum returns are high. NBK exhibited the highest volatility with standard deviation of 0.0624 while East African Portland cement was the least volatile with a standard deviation of 0.0445. KQ has positive skewness while NBK and EA Portland both have a negative skewness. Kurtosis for all the returns are greater than three thus clearly indicating deviation from the normal distribution. Moreover, the Jarque-Bera tests rejects null hypothesis for all the returns in the three cases because of the small p values as indicated in brackets. These tests confirm that the returns are not normally distributed. In order to test whether the returns are stationary or not, ADF tests were done and the results are shown in Table 4 below.

Table 4: ADF test for weekly series and returns

Company	Weekly returns		Weekly prices	
	ADF	P-value	ADF	P-value
KQ	-6.5658	0.01	-1.6314	0.7318
NBK	-6.2064	0.01	-3.2398	0.0816
EA Port	-7.3272	0.01	-2.4041	0.4060

In table 4 above, the ADF values are more negative for weekly returns series than for the weekly prices. The p values were also significantly smaller for the returns series than for the raw series. These p values are also less than 0.05, suggesting the rejection of non-stationary null hypothesis at 95% confidence level.To check for stationarity of the series, before and after differencing, ADF tests were used, and the results summarized in Tables 2, and 4. From the results, the unit roots were found to be more negative for the raw series than the returns series, and the p values were less than 5% significance level for all the returns series, which led to the rejection of the no unit roots null hypothesis. This implied that unit roots were detected in returns series but failed significantly for the closing prices. This further, showed that unlike the raw series, the returns were stationary and could be used for time series modeling in order to examine volatility of share prices over time.

3.6 Autocorrelation of daily and weekly returns and squared returns

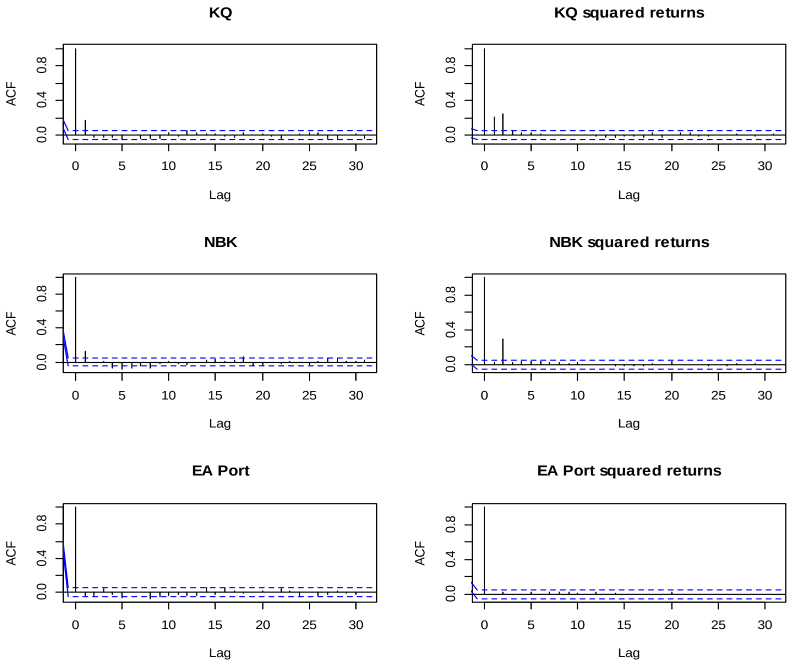


Figure4: ACF of returns and squared returns for the daily returns

Autocorrelation plots were further inspected to ascertain the presence of autocorrelation and from Fig. 4 and 5. ACF plots show no indication of correlation characteristics of returns because some time lags had non-zero values (except for lag 0, which is always 1). The autocorrelations should be near zero for all the time lags if the time series is an outcome of a completely random phenomenon, otherwise, one or more of the autocorrelations will be significantly non-zero [11]. The ACF of squared returns, however, show significant correlation and die out slowly indicating that the variance of returns is conditional on its past history and may change over time.

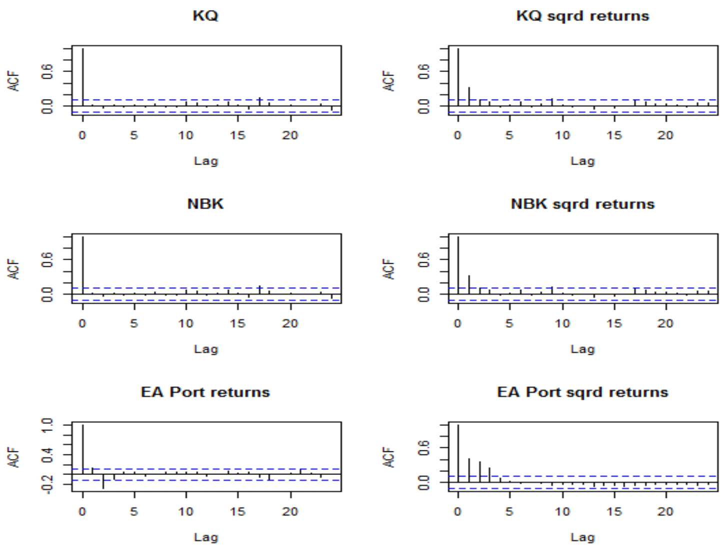


Figure 5: ACF plots of weekly returns and squared returns.

3.7 Ljung Box test for daily and Weekly returns

The Ljung Box test statistics for daily and weekly returns for lags 10, 15 and 20 are provided in Table 5 and 6

Table 5: Ljung Box test for daily returns

Company	**Lag**					
	10		**15**		**20**	
	Statistic	**p value**	**Statistic**	**p value**	**Statistic**	**p value**
KQ	**52.765**	<2.2e-16)	60.064	<2.2e-16	62.257	<2.2e-16
NBK	**61.189**	<2.2e-16)	65.534	<2.2e-16	76.844	<2.2e-16
EA Port	**28.378**	<2.2e-16)	38.489	<2.2e-16	43.14	<2.2e-16

Table 6: Ljung Box test for weekly returns

Company	**Lag**					
	10		**15**		**20**	
	Statistic	**P value**	**Statistic**	**P value**	**Statistic**	**P value**
KQ	12.0948	0.2788	19.3479	0.1984	23.2288	0.2777
NBK	31.3988	0.0005	35.3701	0.0022	40.7585	0.0040
EA Port	19.7058	0.0322	25.2052	0.0473	3.4914	0.9989

The Ljung-Box test were then utilized to ascertain the presence of GARCH effects and from Tables 5 and 6, the test rejects the null hypothesis of no GARCH effects in the returns series at 5% level of significance as evidenced by the small p values for daily and weekly returns. This suggests the presence of GARCH effects in the daily and weekly returns.

3.8 Model Estimation and Evaluation.

Table 7: AIC & BIC values for the GARCH Models

	Models	**KQ**		**NBK**		**EA PORT**	
Criteria		**AIC**	**BIC**	**AIC**	**BIC**	**AIC**	**BIC**
Daily Returns series	GARCH (1, 1)	-4.5905	-4.5764	-4.3071	-4.2929	-4.4169	-4.4028
	GARCH (1, 2)	-4.5891	-4.5714	-4.3058	-4.2881	-4.4159	-4.3982
	GARCH (2, 1)	-4.5895	-4.5718	-4.3073	-4.2996	-4.4156	-4.3975
	GARCH (2, 2)	-4.5881	-4.5669	-1.5025	-1.3113	-4.4145	-4.3933
Weekly returns series	GARCH (1, 1)	-2.9680	-2.9707	-2.7888	-2.7409	-3.7978	-3.7499
	GARCH (1, 2)	-2.9771	-2.9772	-2.7847	-2.7249	-3.7915	-3.7317
	GARCH (2, 1)	-2.9609	-2.9711	-2.7905	-2.7307	-3.7915	-3.7317
	GARCH (2, 2)	-2.9707	-2.8989	-2.7845	-2.7127	-3.7851	-3.7134

The results of AIC and BIC parameter estimation for the models under consideration were summarized in Table 7. From the comparison of AIC and BIC models in Table 7, GARCH (1, 1) model most returns series had minimum values like KQ and EA Portland daily and weekly returns, whileNBKweekly returns for AIC and BIC had the smallestvalues for GARCH (2,1), suggesting a better fit than other competing models. KQ weekly returns had the smallest AIC and BIC values for GARCH (1,2). Another candidate model fitted to the data and tested was the GARCH (2, 2). The analysis showed that the GARCH (2, 2) was less preferred for any data set, this implies that it does not capture well the volatility clustering and leptokurtic characteristics of the stock returns as compared to the other competing models. This model was therefore not fitted to any series.

3.9 Parameter Estimation

The Parameter Estimation for the chosen models was done and summarized in table 8, 9, 10 and 11.

Table 8: Estimation of GARCH (1, 1) models

Daily returns				
Parameters	**KQ**		**EA Port**	
	Estimate	**P value**	**Estimate**	**P value**
ω	0.0002	0.0059	0.0002	0.0553
α_1	0.4016	0.0042	0.0425	0.0452
β_1	0.3795	0.0004	0.6871	0.0003
Weekly returns				
EA Portland				
	Estimate			**P value**
ω	0.0007			0.0101
α_1	0.5048			0.0403
β_1	0.1115			0.3848

The parameter estimates for each of the GARCH (1, 1) models in Table 8 show that the coefficients of the conditional variance equation α_1 and β_1 are all positive and significant at 5%, levels, except for some weekly. This implies a strong support for GARCH models. The sum of α_1 and β_1 are quite close to unity for most series. The sum $\alpha_1+\beta_1$ is an indication of volatility persistence. A high persistence implies that volatility is likely to die slowly; new shock will affect the returns for a longer period. In such markets, old information is more important than recent information and such information decays very slowly [11].

Table 9: GARCH (1, 2) model for KQ weekly

Parameters	Estimates	P values
ω	0.0006	0.4987
α_1	0.2365	0.0584
β_1	0.2027	0.4585
β_2	0.3955	0.0234

The p values were greater than 0.05 for all the parameters except for β_2

GARCH (1, 2) for KQ weekly returns was favoured by AIC and BIC, thus it can be considered a better model for KQ weekly returns as compared to other competing models. The parameters of the model were all positive and insignificant at 5% level of significance except for β_2.

Table 10: GARCH (2, 1) models for NBK daily and weekly returns

	NBK Daily		NBK weekly	
Parameters	Estimate	P value	Estimate	P value
ω	0.0001	0.0016	0.0019	0.811
α_1	0.3914	0.0018	0.3164	0.3144
α_2	0.0921	0.5232	0.2834	0.8650
β_1	0.5413	<2e-16	0.0000	1.0000

In table 10, Parameters estimates for GARCH (2, 1) for NBK daily and weekly are all positive and significant at the given levels of significance. Although GARCH (2, 1) for NBK daily has been favoured by AIC, the sum of parameters is greater than one (1.0248), violating the variance stationarity condition. GARCH (1, 1) also violates this condition but comparing their log likelihood, 3244.053 verses 3242.947 respectively. GARCH (1, 1) is more preferred because it has smaller log likelihood and it is also simpler; a simpler model requires less parameters.

Table 11: GARCH (1, 1) model for NBK daily

Parameters	estimates	P values
ω	0.0001	0.0149
α_1	0.4461	0.0000
β_1	0.5885	<2e-16

In table 11. the GARCH (1, 1) parameter estimates for NBK daily are all positive and significant at 5% level of significance. The sum $\alpha_1+\beta_1$ is greater than one, suggesting an explosive volatility. This implies that the daily share prices for NBK were highly volatile. High volatility implies that, if there is a new shock it will have implication on returns for a longer period.

3.10 Diagnostic testing.

Adequacy of the models was checked to ensure detection of possible model misspecification. This was done by analyzing the residuals of the fitted models. Ljung Box and ARCH-LM tests were carried out on squared residuals for all the returns series up to lag 20 and the results summarized in Tables 12, 13 and 14.

Table 12: LJUNG-BOX & ARCH LM tests for KQ returns

Returns	Q(m)	Statistic	P value
Daily	Q(10)	0.45827	0.99956
	Q(15)	1.11349	0.99995
	Q(20)	3.79738	0.99997
	ARCH LM Test	0.47587	0.99999
Weekly	Q(10)	8.2182	0.6075
	Q(15)	16.6508	0.7769
	Q(20)	13.2146	0.8679
	ARCH LM Test	8.3747	0.7552

The p values for Ljung Box and ARCH LM are all greater than 0.05.

Table 13: LJUNG-BOX & ARCH LM tests for NBK returns

Returns	Q(m)	Statistic	P value
Daily	Q(10)	3.1416	0.9779
	Q(15)	3.8849	0.9981
	Q(20)	65.1961	0.0001
	ARCH LM Test	3.35022	0.9925
Weekly	Q(10)	3.4039	0.9703
	Q(15)	5.6408	0.9852
	Q(20)	9.1819	0.9807
	ARCH LM Test	3.5118	0.9907

The p values for Ljung Box and ARCH LM are all greater than 0.05 except for NBK which is 0.0001 at lag 20.

Table 14: LJUNG & ARCH LM tests EA Port returns

Returns	Q(m)	Statistic	P value
Daily	Q(10)	0.1401	0.9999
	Q(15)	0.23639	0.9999
	Q(20)	1.11025	0.9999
	ARCH LM Test	0.2044	0.9999
Weekly	Q(10)	2.7882	0.9859
	Q(15)	3.4914	0.9989
	Q(20)	5.6806	0.9993
	ARCH LM Test	3.1972	0.9939

The p values for Ljung Box and ARCH LM tests are all greater than 0.05.

Using squared residuals based on the estimated models of KQ, NBK and EA Portland daily and weekly data sets, the Ljung Box test and the ARCH tests in Tables 12, 13 and 14 indicate acceptance of the null hypothesis because of the large p-values (they are all greater than 0.05, except for EA Port squared residuals at lag 20). The ARCH LM test fails to reject the no GARCH effects in the residuals (no heteroscedasticity), and the Ljung Box test also fails to reject the null hypothesis of no correlation for all the data sets. This implies that there is no autocorrelation left in the residuals and that there is no heteroskedasticity in the fitted models. This suggests that the GARCH models considered were all adequate and fit for describing the volatility of NSE and thus appropriate to forecast future volatilities for the companies under investigation.

3.11 Forecasting and Evaluation

Forecasting was done and the time points were evaluated in terms of their forecasting ability of future returns. The mean errors measures for each of the data set were calculated and summarized in Table 15 below.

Table 15: Forecasting performance based on MAE, RMSE and MAPE

	KQ			NBK			EA Port		
	MAE	**RMSE**	**MAPE**	**MAE**	**RMSE**	**MAPE**	**MAE**	**RMSE**	**MAPE**
Daily	0.8980	1.6488	1.6663	0.8629	1.5628	2.0205	1.1273	2.6702	1.0810
Weekly	1.9778	3.1753	3.7798	1.8918	2.8350	4.4067	2.4821	4.3726	2.4729

Forecasting performance of the different sampling intervals was established by ranking the mean errors with respect to the time points for all the companies under investigation. The time point that gave the lowest values of the error measurements was considered the best one. The results in Table 15 show that the daily returns outperformed the other time point, this is because its smallest error measurements for all the measures utilized as compared to the weekly returns i.e. daily series for each of the data sets gave the smallest mean squares, followed by weekly series. This implies that the higher the frequency of data used (smaller sampling intervals), the better the forecasts produced. Better forecasts translate to better risk management and better option pricing for the stock market products.

IV. Conclusion

This paper examined and evaluated forecasting performance of share prices for Kenya Airways, National Bank of Kenya and East African Portland cement at different time points sampled intervals for daily and weekly using GARCH models, with the aim of finding out which of the two provides better forecasts in order to guide trading operations of the Nairobi Securities exchange.GARCH models were estimated and fitted because the exploratory analyses confirmed the leptokurtic, volatility clustering and asymmetric properties of financial time series in the NSE data and GARCH effects were confirmed to be present. GARCH (1, 1) models performed better for most series, particularly the daily series for the companies considered while, GARCH (1, 2) and GARCH (2, 1) were favoured by KQ and NBK weekly series respectively. This supports several other researches which confirmed that the simplest GARCH (1, 1) model captures all the stylized characteristics of financial time series and can consequently be used to estimate and describe the characteristics of financial time series. GARCH (2, 2) performed poorly for all the returns and was therefore not utilized to fit any data set.

The study revealed that there is no clear difference between the sectors investigated as shown by the fact that no model was particularly favoured by one sector as compared to the others. While considering the represented companies, volatilities were quite different, for instance KQ was highly volatile around 2007/ 2008, EA Portland was highly volatile in 2008/2009 while NBK was around 2010/2011. The difference in volatility is probably because of the varied extraneous factors that affect different sectors independently.

Comparing the different time points examined for each company, the study found out that there is a strong evidence of data sampled daily performing better than weekly intervals. The outcome of the study therefore suggests that in order to obtain accurate volatility forecasts for the sectors investigated, investors and other stock market participants ought to closely watch the share prices, at a higher frequency. This will enable them make better investment decisions and hence increased gains not only for individuals but also for the country.

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