

Violation of the Focus Axiom in Fuzzy Multidimensional Poverty Measurement

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Abstract

Sen (1976) challenged the way in which a measure of poverty is constructed by recommending that a poverty index should satisfy certain axiomatic properties to ensure its reliability, stability and, most importantly, effectiveness. Since then, many authors have developed research based on this new methodology. Thus, the axiomatic approach has been adopted in both one-dimensional and multi-dimensional measures. For the latter, some authors, such as Bourguignon and Chakravarty (2006), have extended the analysis by considering the interrelationships between the dimensions of poverty in the multidimensional measure. These two authors postulate for a substitutability relationship between dimensions. For us, we have extended this assumption to assume a cause-and-effect relationship between dimensions. However, this type of relationship (cause and effect), leads to an attack on the focus axiom in the measurement of poverty. Our research has succeeded in proposing a measurement index that addresses this problem. The measure we propose makes it possible to assess the impact of a dimension on the level of poverty, even if that dimension exceeds its reference poverty threshold. Therefore, we can call this index a relative measure. The latter comprises parameters whose values may also influence the other axioms, in particular the transfer axiom. In order to assess the results, an application to urban households in Madagasikara was carried out between 2002 and 2012.

Keywords: *poverty, multidimensional poverty, fuzzy measurement, axiomatic measurement, multidimensional measurement*

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I. Introduction

It is recognized by all (researchers, international organizations and sovereign states) that poverty is a multidimensional phenomenon. This multidimensionality has led to major philosophical, economic, political and practical debates, with the ultimate objective of identifying the multiple facets of the phenomenon in order to make poverty reduction strategies as effective as possible.

Given the extent of the dimensions of the phenomenon, researchers are divided on how to capture all of them and especially on the methods of effective, reliable and robust synthetic measurements. Thus, for the sake of loss of information, some authors prefer not to develop synthetic indices and present all dimensions separately and in a single document; this is the case for reports on the millennium development goals. Others have found a way to reduce them to synthetic information such as for composite indices, statistical approaches, counting approaches. However, the latter do not comply with certain principles even though the counting approach proposed by Alkire and Foster (2007) was formally adopted by UNDP in 2010. Indeed, Sen (1976) advocates the possibility for a poverty index to identify the poor and also to take into account the joint distribution of dimensions. Added to this, and in order to unintentionally exclude certain groups of individuals in the synthetic measure, the fuzzy measures were proposed.

The research then continued in order to associate the fuzzy measurements with axiomatic properties and complement them: identification of structurally poor groups, aggregation of dimensions to account for joint distributions, characterization of stability in time and space and invariance to small state change (robustness). Hence the emergence of several multidimensional axiomatic measures whose variants in fuzzy multidimensional measurement.

Each measure has its own specificities and no measure is universally accepted, but the literature shows that the most appropriate measure will be a fuzzy and axiomatic one: (i) Fuzzy, as it would be difficult to observe the change in intra-group situation with dichotomous measurements; (ii) Axiomatic, since axioms would guide certain properties essential for a measure of poverty Current trends in measures of poverty seem to

point towards a fuzzy, axiomatic and multidimensional measure of poverty. The authors thus offer clues synthesizing this multidimensional aspect of the phenomenon. However, this multidimensionality could be translated differently in measurement methods. Some methods include all dimensions in a simple aggregation, or at the limit assign different weights to variables, and/or specify shape parameters to each variable. Other methods consider the interrelationships that may exist between variables. Aggregating an index is simply a generalization of one-dimensional measurement, but what we will try to explore in this work is the consideration of the interrelationships between variables in the way we measure poverty. These interrelations could be the cause of the violation of the axiom of focusing in terms of poverty.

1. Interrelationships between dimensions in measuring poverty

Referring to Sen's (1985) capacities approach, the interrelationships between the different dimensions of poverty can take many forms, and the very definition of the approach leaves an opening to all kinds of combinations of dimensions (according to this approach, there are several possibilities to achieve well-being). According to the classical theory and the World Bank theory (2000), income contributes to reducing poverty over time. In interpreting this theory, income will be seen as the fulcrum of all dimensions of poverty. Other authors such as Watts (1968), D'Ambrosio et al. (2005) and Mussard and Alperin (2005), reconcile themselves to this classical theory by arguing that income has a transversal dimension.

The integration of these inter-relationships, between dimensions, in the measurement of poverty has been subject to a variety of methods, but its application in fuzzy and axiomatic measures is very limited. Only the works of Bourguignon and Chakravarty (2003) and Chakravarty (2006) have dealt with some forms of interdimensional linkages by demonstrating their applications in the measurement of poverty. We will resume some of that work and then use it to build a new proposal for legislation.

1.1. From iso-utility curve (indifference curve) to iso-poverty curve

Bourguignon and Chakravarty (2003) and Chakravarty (2006) have postulated three forms of relationships between poverty variables: substitutability, complementarity and independence. In this sense, the authors drew inspiration from the theory of the consumer by making the household, or poor individual, play the role of the consumer. Thus, the individual or household is faced with several attributes or variables and seeks a combination of attributes that will allow him to reduce his poverty situation as much as possible.

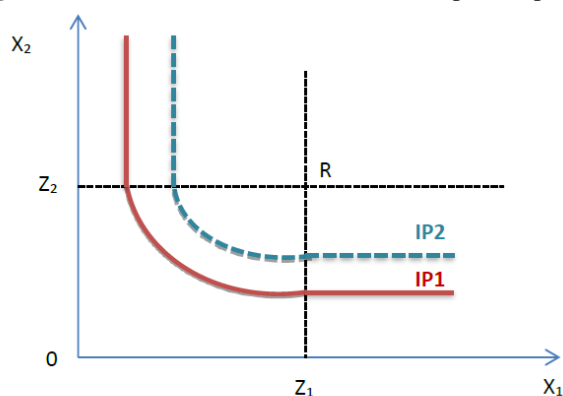
The construction of the iso-poverty curve follows the same mechanism as the consumer indifference curve. This assumes that the attributes of poor households are substitutable, as are those of the consumer. With this consideration, iso-poverty curves would be the inverse of the iso-utility curve. Indeed, utility increases (well-being improves) along with an increase in variable values.

By analogy, the level of poverty decreases after an increase in variables (for example, an increase in income associated with access to electricity would only increase the well-being of the poor household, which means at the same time that its level of poverty has decreased).

In this case, if the iso-utility curve moves to the right and shows an increase in the level of utility, then the iso-poverty curve moves in the opposite direction to the left to show the decrease in the level of poverty induced by a simultaneous improvement of the two variables.

Moreover, according to Bourguignon and Chakravarty's reasoning (2003, page 38), iso-poverty is concave in relation to the meeting point of the two deprivation thresholds. For further clarification, we will present this observation graphically, note that this presentation was also taken up by Lugo and Maasoumi (2008, page 10).

Figure 1 : Concave of wellness curve to completion point



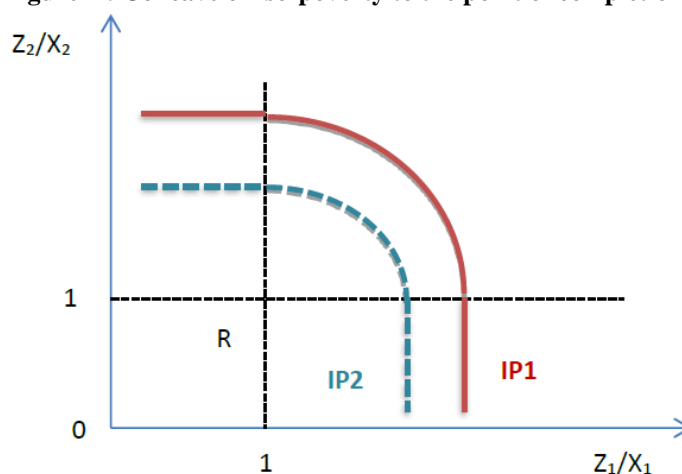
Source : Bourguignon and Chakravarty (2003, page 34)

In this graph, Z_1 and Z_2 represent the respective poverty thresholds in relation to X_1 and X_2 . An improvement in these variables is reflected in their shifts to their respective thresholds, and an upward shift in the curve is reflected in an improvement in well-being. Here, the curve represents well-being, not iso-poverty as such. The latter is concave with respect to the completion point R . In this case, the poverty function, reflecting the utility function, would be concave; However, the authors who proposed this presentation advanced functions of convex memberships (for example, the membership function proposed by Bourguignon and Chakravarty, 2003 and that of Chakravarty et al., 2006).

Returning to the graph above, the well-being curve (and indirectly the iso-poverty curve (IP1)) is concave with respect to the meeting point of the poverty thresholds R (or completion point). It is increasing with respect to the two variables (X_1 and X_2), and is represented by the transition from IP1 to IP2, which would lead to the concavity of the characteristic function of poverty despite the imposition made for the respect of certain axioms of poverty.

However, there is a contradiction when it is said that the iso-poverty curve is an increasing function of poverty. The authors have replaced the iso-poverty curve with a well-being curve, assuming that moving the curve to the right (IP1 to IP2) is synonymous with improving well-being and thus reducing poverty. However, in the opposite direction, the presentation of this illusion is lifted while taking the relative values Z_1/X_1 and Z_2/X_2 instead of X_1 and X_2 . Thus, we have been able to obtain the following presentation and we find that the iso-poverty curve is indeed concave with respect to the point R , and is also decreasing with respect to the improvement of the two variables X_1 and X_2 .

Figure 2 : Concave of Iso-poverty to the point of completion



Source : Author

This concavity of the iso-poverty curve relative to the completion point results in the concavity of the membership function. However, the latter is decreasing while the utility function is concave increasing. Indeed, the concavity of the membership function is naturally deduced from the concavity of the utility function, but the two take opposite directions. Thus, the membership function is decreasing and the utility function is increasing. In addition, authors such as Bourguignon and Chakravarty (2003, page 39) and Aaberge and Brandolini (2014, page 40) do not rule out the possibility of the concavity of the membership function.

The main specificity of the utility function is the decrease in its marginal utility. In this case, the surplus of goods generates less satisfaction for the consumer.

By analogy, marginal poverty is also falling. This means that poverty reduction is slow. A surplus of goods would lead to a reduction in poverty, but at a lower level than one would wish, in the belief that a certain level of goods should provide some satisfaction.

All this information confirms the properties of the function we are looking for below:

- Decreasing: the greater the quantity of a good, the lower the poverty.
- Concave or quasi-concave: Marginal poverty is low (the decline in poverty is slow), therefore rigid in the decline,
- Continue,
- Monotonous,
- Either the membership function $f(x)$, then:
 - Zero homogeneous i.e. $f(ax) = f(x)$, (scale invariance property)
 - $x \in [0, m]$ with m the maximum of x ,
 - $f(x) \in [0, 1]$ with $f(0) = 1$ and $f(m) = 0$ (maximality and normalization properties)
 - $f: \mathbb{R}^2_+ \rightarrow \mathbb{R}$
 - $(x_1, x_2) \rightarrow f(x_1, x_2) = f(x_1) + f(x_2)$
 - The decrease in poverty relative to x_1 is measured by $f'(x_1) < 0$ and
 - The decrease in poverty relative to x_2 is measured by $f'(x_2) < 0$
 - Marginal poverty rigidity assumption $\Rightarrow f'(x_1) < 0$ and $f'(x_2) < 0$

All these results assume that substitutability or complementarity of variables exists. In view of this situation, the question arises whether other types of interrelationships exist between the variables. In response to this question, we will propose the cause-and-effect relationship that could also exist between the variables.

1.2. The cause effect relationship of the dimension of poverty

a) Cause and effect variables

Bourguignon and Chakravarty (2003) considered the goods to be substitutable or complementary. In this context, the consumer's choice between several baskets of goods is anticipated by the author in the context of the choice of poverty variables. However, in the analysis of poverty, there are variables independent of household choice, such as access or time to public infrastructure, which are a variable of poverty.

The substitutability of poverty variables still raises questions. Bourguignon and Chakravarty (2003) have taken the example of income and education, arguing that the decline in income can be offset by the improvement in education. This leads us to say that the state could decide to increase the allocation to education expenses in exchange for the reduction of income through the reduction of the wage bill. When two variables are substitutable, the level of household poverty remains unchanged or tends to decline.

Using data on rural households in Brazil over two years (1981 and 1987), it was found somewhere that two-dimensional poverty reduction was achieved when the decline in educational poverty appeared in the face of an increase in poverty relative to income. In this context, the final decision as to the choice to be made rests with the State and not with the household.

Positioning themselves in place of the household, these two variables are variables of cause and effect, that is to say, to have a high-income level, one must have a high education level. Conversely, a household with high incomes would be able to finance the education of its children so that they could have a high level of education. Returning to the classical reasoning, rising incomes will reduce poverty.

Watts (1968) gave a definition of poverty based on classical theory, which stipulated maximization of utility under the constraint of scarce resources. As in previous analyzes, the poor individual took the place of the consumer.

By analogy, each individual faces an income constraint to acquire a certain quantity of goods and services that could lift him out of poverty.

Our approach starts from this Watt theory, assuming that income is the main cause of poverty, and other variables are effect variables. Moreover, according to D'Ambrosio, Deutsch and Silber (2005) and Mussard and Alperin (2005), household income is an indispensable element because of its transversal nature with regard to the other dimensions of poverty.

But we also kept the assumption that these other variables are substitutable. For example, income is the only causal variable, housing assets and characteristics are effect variables, and both are considered substitutable. This means that with an increase in income, the household can choose between buying a durable good or improving the condition of its home, but its choice would give it the same level of poverty.

b) The poor household's budgetary constraint

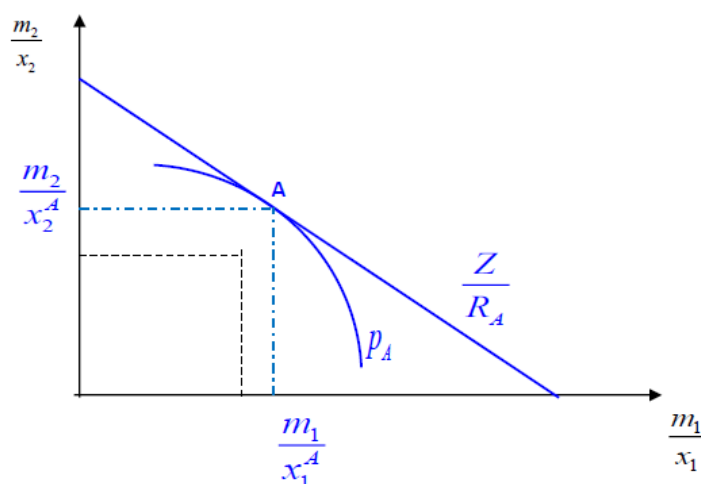
In this theory, the consumer must take his budget into account and compare his income and his expenses. Total expenditures cannot exceed income, as shown by the following formula: $p_1x_1 + p_2x_2 \leq R$ (when two properties are involved). With R the income, x_1 and x_2 the quantities of the two goods and p_1 and p_2 the respective prices of the two goods. The concept of price comes into play, but we are not going to detail it in order to facilitate understanding.

The term budgetary constraint (or income constraint or budget line) is used to refer to the whole $\{(x_1, x_2) \in \mathbb{R}^2, p_1x_1 + p_2x_2 = R\}$.

The same consumer theory holds true for poor households. In the example above, household income would limit the basket of variables it chooses. Income is the main variable that causes household poverty. For this purpose, it could not be taken as a substitutable variable with another. Income is very important for the purchase of certain goods (refrigerator, car...) or for the construction of other goods (house, water supply...), etc.

We will consider two substitutable variables x_1 and x_2 , and to have a more explicit presentation of the iso-poverty curve, take the ratios m_1/x_1 and m_2/x_2 . We will also introduce a budgetary constraint determined by income R , but instead of taking R we will consider Z/R where Z is the poverty threshold relative to income. It will also be a declining function of income, implying a decline in income-related poverty. Therefore, we have the graphic presentation below.

Figure 3 : Iso-poverty and fiscal constraint



Source : Author

In this graph, the budget line for the poor household is represented by $\frac{Z}{R_A}$, the quantities of goods that the household can acquire with this budget constraint are represented by x_1^A and x_2^A . In addition, the ratios $\frac{m_1}{x_1^A}$ and $\frac{m_2}{x_2^A}$ will be considered to show the decrease in poverty level following the improvement of x_1 and x_2 .

This means that the balance of the poor household will be at point A, where the budgetary constraint and the combination of the two goods meet. This also represents an optimal level of poverty that relieves the poor household. Following an improvement or deterioration in income, the reasoning is analogous to that presented in consumer theory.

The introduction of this budgetary constraint does not therefore leave the axioms of poverty unblemished, so the focus axiom will be called into question as we will develop later.

1.3. Infringement of the FOCUS AXIOM

Since some variables are considered to be the cause of poverty, such as income, which is also a budget constraint, and others as the effect of poverty, then improving the cause variables to a level above their thresholds could always lead to an improvement in the effect variables, as long as the latter remain relatively deprived. This implies a breach of the focus principle since an attribute/variable even above the threshold (the poverty level) can be touched and improved.

a) Income as a source of violation of the focusing axiom

i. Focus axiom reminder

In a one-dimensional measure, the axiom is that the rich do not affect the poor as a whole, and that if the rich get richer again then this will have no influence on the level of poverty as a whole. In the multidimensional case, if an individual is poor in relation to one dimension while he is rich in relation to another, and if he is given a surplus in relation to the dimension to which he is rich, then this will not affect the level of poverty.

In formal terms, the axiom is as follows in terms of multidimensional index of measurement:

Focus Axiom (FOC) :

For any $n \in \mathbb{N}$; $X, Y \in M^n$, $\mu \in A$

If

- (i) For any individual i , we have $x_{ij} \geq m_j$, $y_{ij} = x_{ij} + \delta$ with $\delta > 0$;
- (ii) $y_{ij} = x_{ij}$ for any $t \neq i$; and
- (iii) $y_{is} = x_{is}$ for any $s \neq j$ whatever i ;

Then $P^n(X; \mu) = P^n(Y; \mu)$

This axiom indicates that if a person is non-poor in relation to an attribute j , the intensity of

poverty will not be affected if he is given a plus () for this attribute, even though he is poor in relation to the other attributes.

This axiom assumes that the attributes are independent of each other, but what would happen if the attributes were interdependent? If, for example, attribute j represents income and the other attributes (s) are represented by the characteristics of the dwelling and the possession of household goods. Assuming that the household is above the poverty line (the household is non-poor) in relation to income but below (poor) in relation to the other two attributes; how can we be sure that, by increasing its income, the household will always be at the same level of poverty? By obtaining a surplus of income, would he not be tempted to improve his living conditions?

Setting a poverty line for income does not mean that the excess income over that line would no longer affect other spending items. And now, in the case of a multidimensionally poor household, even in a state of wealth relative to its income and in a state of poverty relative to the other variables, it will indeed be able to improve its situation of deprivation relative to these other variables with the little surplus income that it has.

In this case, the focus axiom is called into question in a multidimensional measure, since the surplus of income above the poverty line would affect other expenditure items; the level of poverty would improve. Ultimately, the interrelationships between dimensions would influence the measures of poverty. But how is this really going to happen?

ii. The budget constraint and the iso-poverty curve

Starting from the hypothesis of substitutability between effect variables, we will pose income as a variable of budgetary constraint, resume the iso-poverty curve based on the presentation of Bourguignon and Chakravarty (2006), and show how this scenario will unfold.

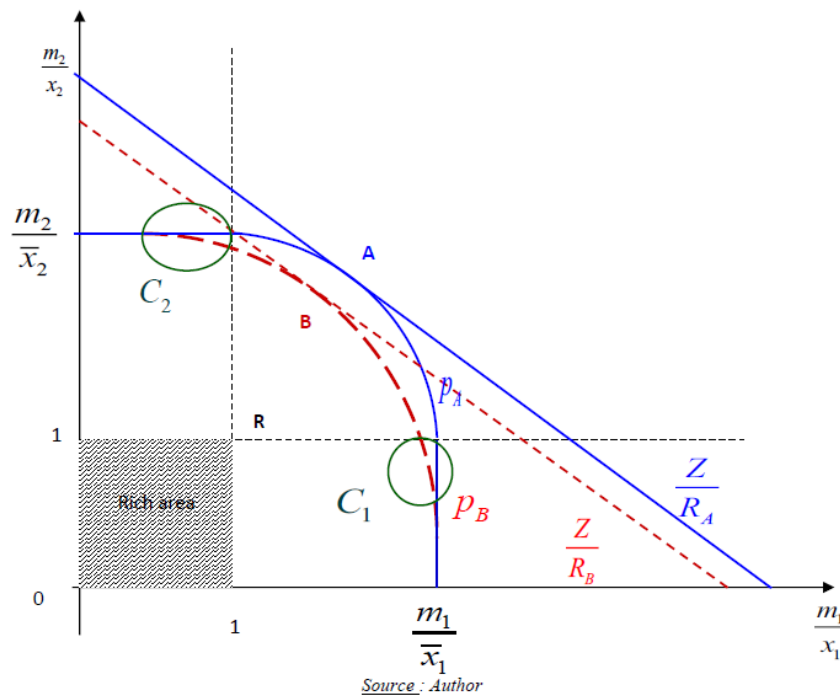
Two effect variables x_1 and x_2 and a cause variable (income) R . In this graph, the budget line is represented by $\frac{Z}{R_A}$ at the beginning, with Z the poverty line in relation to income. This ratio declines as income rises and moves the right side of the budget down, which in turn pushes the iso-poverty curve down. Thus, at the beginning the iso-poverty curve is represented by p_A . The tangency of the latter to income is represented by point A. The quantities of variables x_1 and x_2 that the household can acquire with income R_A thus coincide with point A.

The richness area is included in the hatched frame. For $\frac{m_1}{x_1} \geq 1$ the household is poor in relation to the variable x_1 and for $0 \leq \frac{m_1}{x_1} < 1$ the household is wealthy (that is, x_1 exceeds the poverty line m_1). Similar reasoning applies to x_2 .

According to Bourguignon and Chakravarty (2003), when $\frac{m_1}{x_1} \in [0;1[$ the area of wealth, then the iso-poverty curve p_A becomes a constant line at the point $\frac{m_2}{\bar{x}_2}$. To say that, whatever the value taken by the variable x_1 in its zone of wealth, the iso-poverty curve no longer moves and the improvement in the poverty level is no longer possible, moreover the value corresponding to x_2 remains at the point $\frac{m_2}{\bar{x}_2}$. The same is true when the wealth zone

corresponding to x_2 , the iso-poverty curve P_A remains constant and immobile, while the variable x_1 will take a value corresponding to the point $\frac{m_1}{\bar{x}_1}$.

Figure 4 : Iso-poverty, budgetary constraint and focus axiom



The aim of all these explanations is to show that above the poverty line, no modification of the iso-poverty curve is possible, which is in line with the axiom of focusing. However, this presentation by Bourguignon and Chakravarty (2003) did not introduce budgetary constraints. Now, we're going to continue with the analysis by looking at income.

Because the budget line is represented by $\frac{Z}{R_A}$, then, as income improves, the budget line changes from $\frac{Z}{R_A}$ to $\frac{Z}{R_B}$. The latter pushes the iso-poverty curve down, which in turn goes from p_A to p_B . The tangency is now represented by point B. This shift of the iso-poverty curve towards p_B will change the shape of the curve beyond the respective poverty lines. This change is represented by the two circles C_1 and C_2 . As a result, an increase in income (R) would improve the quantity of poverty effect variables (x_1 and x_2) and thus push the poverty level down (p_A to p_B).

This movement results in a change in the iso-poverty curve beyond the thresholds. This shift in the iso-poverty curve remains valid regardless of the income level (above or below its own threshold).

By extension, the change in the pattern of the iso-poverty curve beyond the threshold would be possible if income is both a cause and an effect variable. As a cause variable, with high income, one can invest in activities that could generate surplus income, which will affect the other variables. As an effect variable, income is a substitute for the other variable and surplus income can fill the gap in the other variable. Ultimately, all of this argues that the "axiom of focus" may be exceeded in multidimensional measures of poverty, when variables may be both cause and effect variables rather than substitutable variables only.

Some authors have already announced the possibility of this case, that is, an improvement in the level of poverty when some variables exceed their thresholds while others cannot. We'll determine which ones.

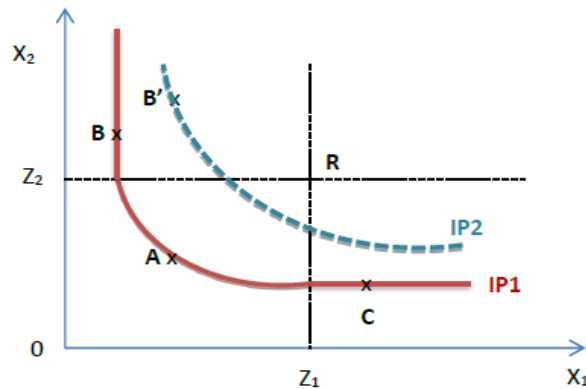
b) The violation of the focus axiom according to some authors

Bourguignon and Chakravarty (2003) have announced the possibility of this situation, but according to them it remains to find the characteristic function that fulfills this condition. Lugo and Maasoumi (2008) also suggested this possibility by presenting it graphically as follows, only he repeated the reasoning of Bourguignon and Chakravarty (2003) considering a curve of well-being instead of a curve of iso-poverty. In this case, the dial $(0, Z_1, Z_2, R)$ represents the poverty area and the well-being curve is convex.

In the poverty zone $(0, Z_1, Z_2, R)$, the intersection criterion is used, i.e., at this point, the individual is poor in relation to the two indicators (X_1 et X_2). It could be at point A, for example. Also in this dial, substitutability between the two variables is possible and an improvement in one could fill the gap compared to the other, which would improve poverty.

In the part below Z_1 and above Z_2 , the union criterion is used, i.e. the individual is poor in relation to X_1 but rich in relation to X_2 (case of point B).

Figure 5 : Wellness curve and focusing axiom



Source : Lugo and Maasoumi (2008, page 10)

At point B, the iso-poverty (well-being) curve never changes, regardless of the surplus added to variable X_2 even though X_1 is extremely poor. As a result, poverty does not improve, even if the variable X_2 is improved above its threshold. A similar situation is observed when the individual is above Z_1 for X_1 and below Z_2 for X_2 (case of point C).

However, by adopting an intermediate criterion between intersection and union, the displacement from point B to point B' will be observed since the surplus with respect to X_2 will cause the displacement of the well-being curve upwards. And since both variables are substitutable, this surplus of X_2 can replace a lack of X_1 (if 1 orange juice is 1 sandwich, adding 1 other orange juice is equivalent to adding 1 other sandwich). In other words, if X_2 exceeds the threshold and X_1 does not, then the average of the two will fill the gap X_1 and improve well-being. We can apply the same reasoning to the iso-poverty curve as in the previous graph.

However, Lugo and Maasoumi have not proposed a measure that takes this situation into account. They just stopped at this graphical presentation.

Alkire and Foster (2007) and Alkire et al. (2015) shared the same view as those previous authors and proposed the use of the Alkire and Foster (AF) measure using the deprivation score.

Indeed, the measure AF consisted, for a given individual, in identifying his situation with respect to each variable. In this case, a threshold of deprivation is set for each dimension and it will be concluded that the individual in question is private for some dimensions but rich for others.

The intermediate criterion combines all these situations by issuing a (average) score for this individual, then setting a threshold for this (average) score and finally deciding on the individual's poverty situation.

In the same vein as Alkire and al. (2015), we will propose a new measure that takes account of this violation of the focus axiom. This new measure will be axiomatic in nature, unlike the one proposed by Alkire and Foster (2007), which was not axiomatic in essence. Moreover, this AF measure is not used in fuzzy measures.

In this case, it is possible to violate the focus axiom. It remains only to identify the measure which makes it possible to capture this aspect without violating the other axioms.

The question is therefore how to measure poverty at a threshold that is exceeded; and a poverty index between $[0 ; 1]$. Not all measures available in the literature verify this property, except the Relative Total Fuzzy (RTF) measure by Cheli and Lemmi (1995)¹. However, there is a problem with this measure; it does not satisfy many axioms.

c) The characteristic function as being representative of the violation of the focus axiom: to a relative measure

After performing a multitude of shape-combining exercises, we were able to identify a function. This was inspired by this idea of Cheli and Lemmi (1995) combined with our own

¹ D'Ambrosio, Deutsch, Silber (2005)

research, which is also based on the functional forms of density functions and probability distribution functions, such as exponential law, gamma law, Khi-two law, and many other mathematical functions. This function is referred to as a "relative function" because its values remain in the range 0 to 1, even if the interest variable exceeds the poverty (non-poor) threshold. We will see below what this function is. As we have announced, we have looked for a function that retains above all the rigidity criterion and that considers the violation of the focus axiom while keeping the other properties.

➤ *The functional form of a relative measure*

This will be a relative measure of fuzzy poverty, as it will still move even if the interest variable is above the poverty line, with a more resilient downward trend. Therefore, the membership function for this measure must have the following properties (familiar properties are always retained):

1. Continue,
2. Decreasing,
3. Level zero homogeneous i.e. $f(ax) = f(x)$, (scale invariance)
4. $x \in [0; +\infty[$,

$f(x) \in [0, 1]$ (maximality and standardization)

with $f(0) = 1$ or $\lim_{x \rightarrow 0} f(x) = 1$ and $\lim_{x \rightarrow +\infty} f(x) = 0$ when x tends towards $+\infty$.

The shape of the curve ensuring the decrease can be convex or concave. We have opted for a concave form in order to imply resistance to the decline in poverty, having seen the persistence of the phenomenon despite the various actions taken to combat it. However, given the relativity with respect to the value of x , that is to say $\lim_{x \rightarrow +\infty} f(x) = 0$ when x tends towards $+\infty$, a change of inflection will be inevitable in the course of the function. This led us to choose a shape that looks like a sigmoid or logistic shape². Thus, the initial concavity will ensure

² Chiappero and Martinetti (2008)

resistance (especially when poverty is very deep). In addition, the change of gait from a certain point of the inflection will ensure that the value of x is considered when it exceeds its threshold.

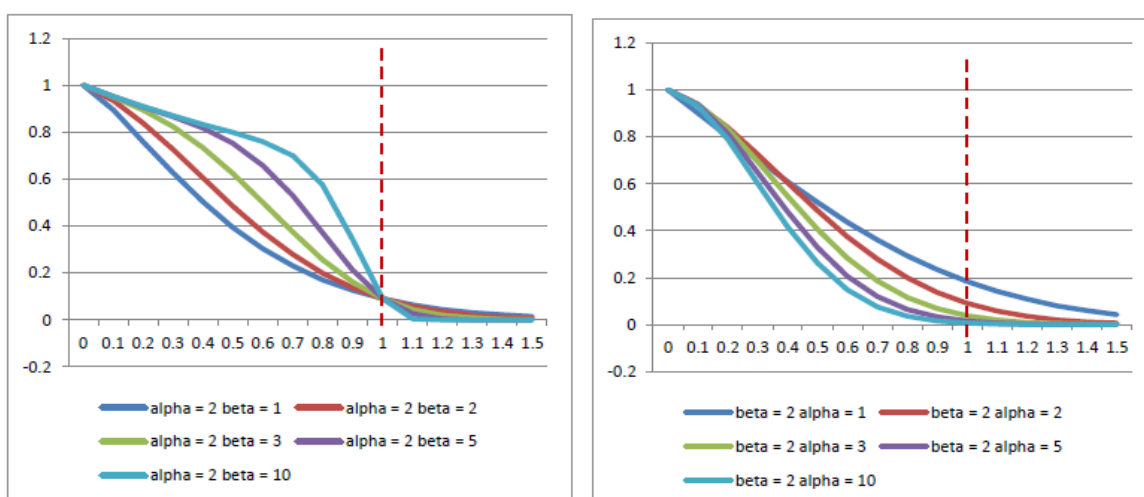
After having looked at the mathematical functions available in the literature, and after having been inspired by the functions of densities and distribution of exponential laws, gamma law, Laplace-gauss law, Khi-deux law, etc., we were able to consider that the best function that

best meets these criteria is the following:
$$f(x) = \frac{\alpha e^{-\alpha(\frac{x}{m})^\beta}}{\frac{x}{m} + \alpha} \quad (01)$$

Where α , a function shape parameter, ensures concavity at the start (when x is low and poverty is still high). Thus $\alpha > 1$ ensures this shape. β is an amplifier parameter of the form proposed by α . Also, the shape is amplified from $\beta > 1$.

Simulations of the curve shape of this function are presented as follows:

Figure 6 : Différentes formes fonctionnelles d'une mesure relative



Source : Author

In these graphs, the poverty line is represented by the hatched vertical line corresponding to the value 1, where $x/m=1$, i.e. x has reached its threshold.

We note that, even exceeding this threshold, the value of $f(x)$ remains between 0 and 1 (relativity criterion). It takes the value 1 when $x=0$ (maximality criterion) and tends towards 0 when x becomes larger and larger (normalization principle).

➤ *The generalization of the relative measure*

The writing of the relative function is similar to that of Tsui (2002), which can be broken down into subgroups. It may therefore take the following general form:

$$R_{\alpha,\beta}^n(X; \mu) = \frac{1}{n} \sum_{j=1}^h \sum_{i \in S_{\mu_j}} \delta_j \frac{\alpha_j e^{-\alpha_j (\frac{x_{ij}}{m_j})^\beta}}{\frac{x_{ij}}{m_j} + \alpha_j} \quad (02)$$

Where α_j and β_j take values that depend on the assessments of the gears that are attributed to each variable.

Conclusion of the first chapter

The inclusion of multidimensionality in the measurement of poverty is not limited to simple aggregation. It can be extended to consider the interrelationships that may exist between variables. Bourguignon and Chakravarty (2003) suggested that these interrelationships may take the form of substitutability or complementarity between dimensions. By analogy with the consumer theory, in which the utility function resulting from the substitutability of goods is increasing concave, this analysis of substitutability between dimensions of poverty led us to suspect a decreasing concave membership function.

In trying to introduce a constraint variable (budgetary constraint in consumer theory), the analysis changes facets. The analysis of the interrelationships between the dimensions of poverty, in terms of substitutability between variables, was indeed supplemented by the addition of another variable playing the role of budgetary constraint, as seen in consumer theory.

From another perspective, the analysis focused on the characteristics of poverty variables by identifying whether there is a causal relationship between them, by identifying which are the cause variables (budgetary constraint) and the effect variables (substitutable variables). With this new approach, the focus axiom is also violated.

This violation of the focus axiom indicates that even above the poverty line, a variable could influence the measurement of poverty. Thus, the measure which fulfills such a condition could not be called otherwise than relative, as could the total and relative fuzzy measure of Cheli and Lemmi (1995). In order to keep the other axioms intact, we came across a relative measure that meets our needs, that is, a measure that influences the level of poverty, even if the variable exceeds its poverty threshold (relativity).

The shape of this relative measure is concave at the beginning, changes inflection and is almost convex at the end to ensure relativity. In order to highlight the difference between the existing measures and those we propose in this work, it would be interesting to apply them and compare the results obtained.

2. The implication of the measure violating the focus axiom

2.1. Implications on axioms

a) The implications of the relative measure on the axiom of transfer in Madagascar

In relation to the transfer axiom, the results depend on the values taken by the parameters α and β provided that the two parameters take the values most representative of the most representative form of the function, that is to say $\alpha, \beta > 1$. Let us take the example of the transfer of income between two poor people, one of whom is less poor than the other, to highlight it.

Considering the relative measure:
$$R_{\alpha, \beta} = \frac{\alpha e^{-\alpha(\frac{x}{m})^\beta}}{\frac{x}{m} + \alpha} \quad (03)$$

1st case: $\alpha < \beta$

In this case, the transfer from the least poor to the poorest only increases poverty as shown in the following table:

Table 1 : Implication of the transfer principle measure ($\alpha < \beta$)

individual	Revenu	Relative Mesure <i>alpha=2 and beta=3</i>	Revenu after transfer	Relative Mesure <i>alpha=2 and beta=3</i>	Deviation
1	80	25,7%	60	49,9%	24,3
2	25	86,2%	45	68,0%	-18,1
<i>Global poverty index</i>		<i>55,9%</i>		<i>59,0%</i>	<i>3,1</i>

Source : Author

2nd case: $\alpha \geq \beta$

In this case, the transfer from the least to the poorest reduces poverty.

Table 2 : Implication of the transfer principle measure ($\alpha \geq \beta$)

Individual	Revenu	Relative Mesure <i>alpha=3 and beta=2</i>	Revenu after transfer	Relative Mesure after transfer <i>alpha=3 and beta=2</i>	Deviation
1	80	11,6%	60	28,3%	16,7
2	25	76,5%	45	47,4%	-29,2
<i>Global poverty index</i>		<i>44,0%</i>		<i>37,8%</i>	<i>-6,2</i>

Source : Author

According to these results, the transfer principle changes as the parameters take their values. It is therefore up to researchers to use the parameters that are appropriate for their research.

b) The implications of the relative measure on other axioms

As already shown above, this index violates the focus axiom given the properties of the characteristic function. In addition, it checks the axioms of Monotonicity (MON), Continuity, and Invariance with respect to scales (SCI) which also depend on the characteristic function. The monotonicity property (MON) of the membership function implies its μ -monotonicity property.

This index also checks the Normalization Axiom (NOM) since its membership function allows a limit that tends towards zero when the variable x is sufficiently large.

Considering its general form and following the implications of the measure on the principle of progressive transfer, the index $R_{\alpha,\beta}$ check the axiom "Increasing poverty under correlation increasing switch (IPC)" when $\alpha < \beta$. It verifies the axiom of Decreasing poverty under correlation increasing switch (DPC) when $\alpha \geq \beta$.

Moreover, this index checks the Population axiom (POP), because the form of membership function remains the same, regardless of the variable. It also checks the Symmetry principle (SYM), because the characteristic function depends only on x .

The axiom of non-poverty growth (NPG) is respected in view of the decline in the membership function. In fact, if we have $x_1 < x_2$ then $\mu(x_1) > \mu(x_2)$ and $\frac{\mu(x_1) + \mu(x_2)}{2} < \mu(x_1)$.

For instance, $\mu(x_1) = 0,4$ and $\mu(x_2) = 0$ so $\frac{\mu(x_1) + \mu(x_2)}{2} = 0,2 < 0,4$.

2.2. Sample practice

Given the different functional forms proposed for a fuzzy measure of poverty, the results will certainly not be the same. Some measures respect certain axioms and others violate them. We will try to show in this section the possible impacts of the cause variables on the effect variables as well as the results obtained using a measurement that would violate the focus axiom.

a) Urban household data and sample

We considered data from periodic household surveys in Madagascar, known as EPM. These surveys were administered by the National Institute of Statistics (INSTAT). To do this, we have targeted data for 2002 and 2012 for the following reasons. Between these two dates, the incidence of poverty fell significantly by 9.2 percentage points. Indeed, the one-dimensional poverty-to-expenditure ratio per adult equivalent estimated from 80.7% in 2002 to 71.5% in 2012 (Source: EPM2002 and National Millennium Development Goals Monitoring Survey (ENSOMD 2012/2013)). A decline in poverty relative to expenditure thus implies an increase in income between these two years. Next, we will show the effects of such an improvement on the other poverty indicators we call here the poverty effect variables.

For ease of understanding, we will consider only three dimensions/variables: expenditure, assets and housing characteristics.

We assumed that the variables included in both dimensions, and considered to be effect variables here, depend largely on income level. But we also note that these variables are more likely to be found in urban areas than in rural areas, for a variety of reasons. By taking only variables such as television or hifi, they are conditioned by the existence of electricity, whereas in rural areas access to electricity is very limited, and it is obvious that rural households do not care about these types of goods even if they have the means. Cars or motorcycles are not a priority for rural households, but what interests them are tractors, carts, etc.

As for the characteristics of housing, some rural households do not care about the quality of their place of residence. Indeed, they refer to their entourage or limit themselves to their degrees of civility. It may also be that the materials they would like to have are not available in the places where they live, etc. As regards access to drinking water, it depends on the installation of appropriate infrastructure in the places. The administrative authorities are responsible for carrying out these tasks. Without this infrastructure, households would not have access to clean drinking water.

This aspect will therefore distort our hypothesis, since we are looking not only for the possession of these goods, but above all for the variability of this possession following a change in income. However, in urban areas, no exogenous factors will prevent access to these various goods.

Only the means available to urban households would condition their purchases. As a result, we will use only urban household data for most analyzes. However, to take a national view, we will calculate some poverty indices towards the end of this chapter.

Thus, the sample size is forged from urban households, which according to the EPM, is 2,316 for 2002 and 4,079 for 2012.

The description of the study variables is as follows:

(i). Expenses

This is exactly consumer spending. Since 2001, the consumption basket has been made up of the following three components: food consumption, non-food consumption and consumption of durable goods.

(ii). Assets :

According to Townsend (1979b), the absence of certain assets for each individual/household can even be considered an expression of poverty. Durable goods often illustrate the variables in this dimension of poverty. For reasons of intertemporal comparability, only few durable goods were selected and are also considered to be the most representative of their categories.

(iii). Characteristic of the accomodation

As stated by Orshansky (1965) [cited by Gabsi, 2016], the cost of housing is a significant part of the minimum cost of living. The cost of housing, as imputed rent, has already been included in the expenditure components. In this case, from another perspective, the characteristics of housing, housing or housing may describe poverty or social status, and may also influence the health status of each individual.

The type of materials used indicates the degree of comfort the accommodation can provide. It allows you to appreciate its luxury and cost of ownership. We have chosen materials that are easily observable and that allow us to differentiate Malagasy households. These materials can be found on the walls, floor or ceiling.

We have also introduced the source of water supply as a characteristic variable of the household's housing, assuming that mobilization of this indicator signals the magnitude of the improvement in housing conditions or, indirectly, the household's financial situation.

b) Appreciation of income allocations on effect variables

Let us recall the wording of the following relative measure :

$$R_{\alpha,\beta}^n(X; \mu) = \frac{1}{n} \sum_{j=1}^h \sum_{i \in S_{\mu_j}} \delta_j \frac{\alpha e^{-\alpha(\frac{x_{ij}}{m_j})^\beta}}{\frac{x_{ij}}{m_j} + \alpha} \quad (02)$$

Where α and β take the same values for each dimension, for the sake of simplification.

To show the specificity of this measure, we have considered the values of the parameters that give the most representative form to the individual function. Alpha = 2 and beta = 3 were used.

The measure needs to establish a poverty line above which the household is considered completely non-poor. Therefore, these thresholds are set arbitrarily. In our study, we opted for a poverty line that is the ninth decile, presenting itself as the line separating the bottom 90% of the data from the top 10%. Excluded are the richest 10% for each dimension and the threshold for the value between the richest 10% and the remaining 90%.

At present, we are only taking into account values below the poverty thresholds set. And we will come back to the consideration of exceeding the threshold later.

The table below shows that the largest difference is in expenditures, followed by assets, and ultimately the characteristics of housing. We can say that the allocation of urban household income is much more oriented to the acquisition of durable goods (furniture, household appliances, audiovisual equipment, car, motorcycle), as well as the rehabilitation, repair or change of house for more pleasure. The difference is in the order of -8.4 percentage points for expenditure. The decline led to a decline of -4.9 percentage points for assets and -0.7 percentage points for housing characteristics.

Table 3 : Relative Measure index, alpha = 2 and beta = 2

	<i>2002</i>	<i>2012</i>	<i>Deviation 2012/2002</i>
Expenses	0,751	0,667	-0,084
Assets	0,183	0,135	-0,049
Characteristics of accomodations	0,558	0,551	-0,007
Total	0,497	0,451	-0,047

Source : Author's calculation from INSTAT's EPM – 2002 and ENSOMD 2012/2013 data

The first remark that comes out of these results indicates that the deviations from each dimension are less than the expenditure difference, although the expenditure difference is quite considerable. This could mean several things such as other assignments taking income, the price of these goods hindering their acquisition...

The second observation is that the gap in housing characteristics is practically negligible compared to the gap in assets, while poverty in housing is much higher than in assets.

Compared to the security needs that exist for each individual, reducing housing poverty would be a priority, whereas here it is not. This is because households have opted for the low-hanging fruit to supplement their standard of well-being. For example, with the same financial means, buying a car is easier than repairing or rehabilitating your home. Even buying a house presents difficulties, because it implies a cumbersome procedure or several risks (scam, falsification of paperwork, etc.).

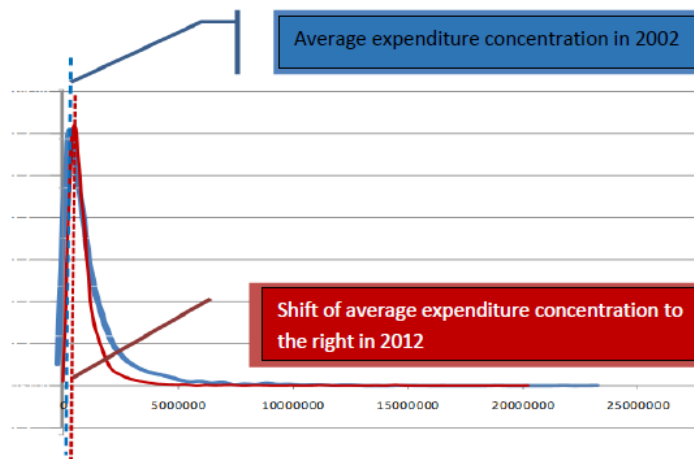
Substitutability could exist, but to a very low degree, so poverty reduction is not as sensitive. Thus, the choice of households is not indifferent, otherwise they could have chosen to improve the characteristics of their dwellings.

For one-dimensional poverty, each effect variable depends in part on the cause variable and also on other factors. Indeed, the decline in poverty relative to expenditure is 8.4 percentage points and could thus lead to an improvement in poverty relative to the other two variables. We can argue that this gap is quite large, because looking at the distribution of expenditures as shown in the chart below, there is an improvement in the level of concentration of expenditures.

Indeed, there has been a shift in the concentration of spending to the right, even if it could be subsidiary. The relative measure did, however, point to an improvement in living standards by referring to the reduction in income-related poverty of 8.4 percentage points on average

between the two years in question. This situation could be responsible for the reduction of three-dimensional poverty by -4.7 points.

Figure 7 : Kernel density estimation



Source : Author's calculation from INSTAT's EPM-2002 and ENSOMD 2012/2013 data

The previous results fail to take into account the overshooting of the poverty threshold, which is the object of the specificity of the relative measure. Taking into account this specificity, what will be the results?

c) The relative measurement and the focus axiom

According to the Focus axiom, if a person is non-poor in relation to an attribute/variable, the intensity of poverty will not be affected if he is given a bonus for that attribute, even if he is poor in relation to the other attributes. However, we have shown using the relative measure index that it is possible to affect the proportion of poverty when an attribute or variable exceeds the poverty line.

Moreover, we have introduced the concept of cause and effect variables. And according to consumer theory adapted to poor households, this cause variable may affect the intensity of poverty, regardless of whether it is below or above its threshold.

➤ *The intensity of poverty by violating the focus axiom*

Like the cause variables, the expenditure variable plays this role in this analysis and the other variables appear as an effect variable. To do this, we have freely allowed spending to exceed the poverty line. However, the other two dimensions are kept below their respective thresholds. So households might be rich or poor relative to spending, but they are certainly structurally poor relative to the other two dimensions.

The following table relates the calculation results with the relative measure. In the latter, the fuzzy proportion of poverty in relation to expenditure is 66.7% when these are increased by their poverty threshold. This proportion becomes 63.1% by allowing spending freely to exceed the threshold. This implies that spending (income) above the poverty line has influenced poverty intensity and reduced it by -3.7 percentage points. Keeping the other two dimensions below their respective thresholds, the three-dimensional poverty rate fell by -1.2 percentage points in 2012.

Table 4 : Impact of wealth on poverty

	Increased expenditure	Unincreased expenditure	Deviation
Expenses	0,667	0,631	-0,037
Assets	0,131	0,131	-
Characteristics of accomodations	0,551	0,551	-
Total	0,450	0,438	-0,012

Source : Author's calculation from INSTAT's ENSOMD 2012/2013 data

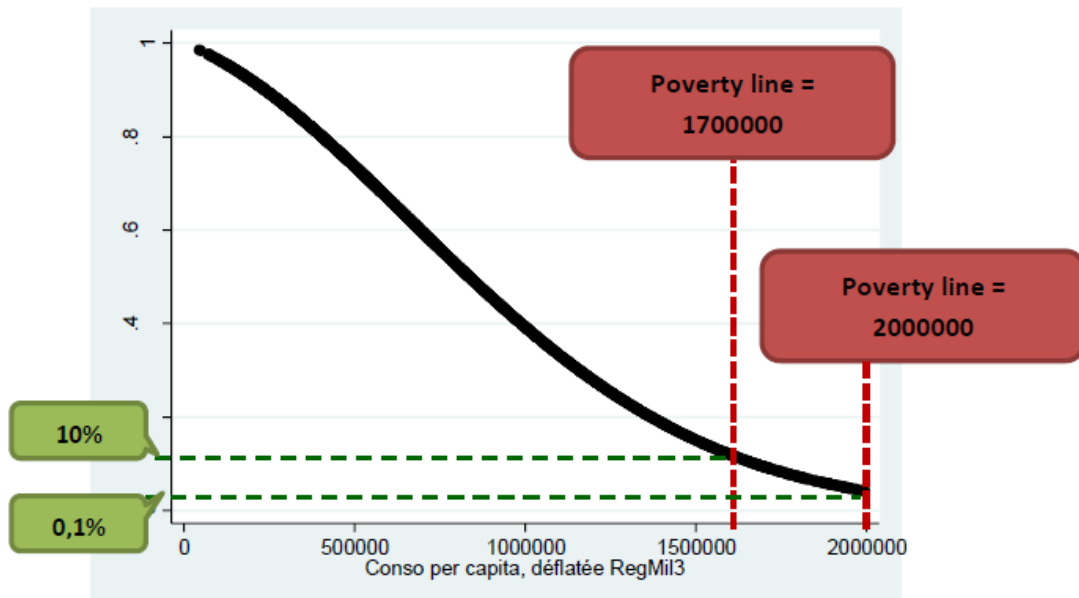
This phenomenon can be observed in everyday life, since it seems that income will never be sufficient to meet all needs. There will always be a new object of expenditure. This hypothesis is so true that if with my current income the purchase of a car is easy for me, then the next time it increases I would buy a house, after a space for leisure, then a hotel, then a beach... The consumer theory even states that the consumption of the good increases faster than the wealth when faced with so-called "superior goods"³.

³ Marshalian demand

A simulation of this relative measure is presented in the following graph to show the reduction in poverty intensity in the event that the household appears to be getting richer.

Indeed, taking as poverty threshold the value of 1,700,000 AR, this graph shows that poverty is high with a rate of 10%. On the other hand, the poverty ratio is in the order of 0.1% with a spending level of the order of 2,000,000 AR

Figure 8 : Impact of wealth on poverty



Source : Author's calculation from INSTAT's ENSOMD 2012/2013 data

We took the case with income above the threshold. It may also be that the other cases in which the other dimensions exceed their thresholds are taken, but the examples given should suffice for the time being to prove that a measurement method which violates the focus axiom exists without thereby violating the other axioms, particularly the normalization axiom and the maximality axiom.

The analysis could also be extended by introducing the effect of the effect variable threshold overshoot on the cause variables, but the availability of data does not allow this since cohort or panel data are required to do this.

➤ *Poverty intensity according to the value of the parameters*

We mentioned above that the level of poverty also changes according to the value of the parameters. It should be recalled that the relative measurement has two parameters, namely α and β . With α a shape parameter of the function that ensures the concavity at the start (when x is low and poverty is still high, so $\alpha > 1$ ensures this shape), and β an amplifier parameter of the form proposed by α . Also, the shape is amplified from $\beta > 1$.

Let us also recall the general form of the following relative measure:

$$R_{\alpha,\beta}^n(X; \mu) = \frac{1}{n} \sum_{j=1}^h \sum_{i \in S_{\mu_j}} \delta_j \frac{\alpha_j e^{-\alpha_j (\frac{x_{ij}}{m_j})^{\beta_j}}}{\frac{x_{ij}}{m_j} + \alpha_j} \quad (02)$$

Based on the results above, households are rather quick to allocate less of their income to the acquisition of durable goods at the expense of rehabilitation, repair or change of dwelling (for example, if the household previously rented a tin house, after having had income increases during these ten years, it decided to rent a brick house). We are even tempted to say that, faced with a galloping increase in income, Malagasy households prefer to buy a car to build a house. By the way, building a house takes a lot of time compared to the time it takes to buy a car. The same is true for repairs: repairing a car is faster than repairing a house. As a result, the values of the parameters may differ from dimension to dimension, and in this case, differences may arise in terms of assets and housing characteristics.

With this presentation, we will be tempted to opt for a more pronounced concavity for housing characteristics than for assets. Assets are much more flexible on the downside compared to housing characteristics. Thus, we can choose a beta value of 5 for housing and a value of 3 for assets. As we have seen in previous results, spending is more flexible than other dimensions, allowing us to choose a value of 2 for the beta parameter. On the other hand, we are going to let spending freely exceed the poverty line. For the representativeness of the desired shape for this relative measure, we will keep the alpha value at 2.

With these conditions, the proportion of three-dimensional poverty is therefore of the order of 48.2% overall, if it is 43.8% in the previous result. Indeed, with the beta value equal to 5, poverty is estimated at 65.9%, with a more pronounced concavity for housing characteristics. Similarly, for assets, with a beta value of 3, the poverty rate is slightly higher than it was with a beta value of 2.

Table 5 : Proportion of poverty by parameters (alpha = 2 everywhere)

Dimensions	Proportion of poverty	with beta =	Proportion of poverty	with beta=
Expenses	0,631	2	0,631	2
Assets	0,157	3	0,131	2
Characteristics of accomodations	0,659	5	0,551	2
Total	0,482		0,438	

Source : Author's calculation from INSTAT's ENSOMD 2012/2013 data

These results show the importance of parameter values. It is up to the user to decide on these, based on his or her assessments based on theories on this subject. Out of curiosity, we will try to verify the impact of the inclusion of certain categories of households in the analyzes, on the level of poverty.

2.3. Extension of analyzes at national level

To enrich the analysis a little, we will produce results with respect to household characteristics. This will be done by cross-checking the fuzzy proportions of poverty by dimension with certain categorical variables including gender, the level of education of the head of household and his socio-professional group.

a) Fuzzy poverty by gender, education and socio-professional group

- **Distribution by gender**

Looking at the gender-specific manifestation of the phenomenon, we find that there is no significant difference in the state of poverty between the two genders. Indeed, the fuzzy proportion of three-dimensional poverty is estimated at 48.1% for men and 48.5% for women.

Table 6 : Proportion of poverty by gender

	Expenses	Assets	Characteristics of accomodation	Three dimensional poverty
Men	0,630	0,155	0,657	0,481
Women	0,631	0,154	0,668	0,485
Total	0,631	0,157	0,659	0,482

Source : Author's calculation from INSTAT's EPM-2002 and ENSOMD 2012/2013 data

- **Fuzzy poverty by education level**

The distribution of the fuzzy proportion of three-dimensional poverty in relation to education indicates that households headed by a head with no education are the most deprived at 50.8%, in contrast to households headed by a head with a higher education (47.3%). The intermediate situation is practically no exception between households headed by a primary head and households headed by a secondary head, with the same proportion of around 48%.

Table 7 : Proportion of poverty by education level

	Expenses	Assets	Characteristics of accomodation	Three dimensional poverty
Uneducated	0,667	0,171	0,698	0,508
Primary	0,634	0,154	0,660	0,483
Secondary	0,626	0,154	0,688	0,489
Superior	0,623	0,148	0,638	0,473
Total	0,631	0,157	0,659	0,482

Source : Author's calculation from INSTAT's EPM-2002 and ENSOMD 2012/2013 data

We note, however, that compared to housing characteristics, households with primary-level heads have a lower level of poverty (66%) compared to those with a secondary-level head (68.8%).

• **Distribution by socio-professional group**

A breakdown by socio-occupational group shows that households headed by a senior manager are the most spared. The fuzzy poverty rate across the three dimensions is only 26.8%. These households minimize poverty in relation to expenditure and housing characteristics, unlike households headed by apprentices and self-employed workers who are the most disadvantaged.

These findings are the same as those found in the report of the National Survey on the Monitoring of the Millennium Development Goals in Madagascar (INSTAT ENSOMD 2012-2013).

Table 8 : Proportion of poverty by socio-professional group

	Expenses	Assets	Characteristics of accomodation	Three dimensional poverty
Senior Executive	0,293	0,158	0,354	0,268
Middle executive or supervisor	0,397	0,153	0,481	0,344
Skilled worker or employee	0,511	0,148	0,451	0,370
Unskilled worker	0,635	0,145	0,575	0,452
Maneuver	0,695	0,151	0,558	0,468
Paid trainee	0,634	0,142	0,457	0,411
Boss with staff	0,559	0,148	0,566	0,424
Independent	0,696	0,158	0,713	0,522
Trainee	0,896	0,118	0,747	0,587
Family helper	0,577	0,144	0,635	0,452
Total	0,631	0,157	0,659	0,482

Source : Author's calculation from INSTAT's ENSOMD 2012/2013 data

Table 9 is an excerpt from this report showing the incidence of poverty according to the Foster-Greer-Thorbecke (FGT) index for urban households. This shows that the incidence of poverty by sex is not differentiated. Similarly, for the distribution by educational attainment, households with uneducated heads are always the poorest and the wealthiest are those with heads who have completed higher education. Compared to the socio-professional group, it is shown that senior managers are in the least poor category, and apprentices and self-employed are among the most deprived.

These results indicate in part the fidelity of the urban poverty structure in Madagascar, regardless of the measurement method used.

Table 9 : Poverty incidence (P0) by gender, education and SPG

Categorie	Poverty incidence (P0)
Male	0,486
Female	0,488
Uneducated	0,527
Primary	0,493
Secondary	0,470
Superior	0,444
Senior Executive	0,095
Middle executive or agent	0,118
Skilled worker or employee	0,256
Unskilled worker	0,470
Maneuver	0,524
Paid trainee	0,340
Boss with staff	0,287
Independent	0,602
Trainee	1,000
Family helper	0,420

Source : INSTAT's ENSOMD 2012/2013 data

The results presented so far only concern urban households, taking into account the poverty variables considered. However, we will present results at the national level, taking into account the urban and rural environment.

b) Fuzzy poverty in Madagascar by relative measure

Despite the fact that the variables taken are not representative of rural households, the calculation was nevertheless carried out in order to get an idea of the extent of fuzzy poverty in Madagascar through the relative measure.

• **Distribution by areas**

When looking at the classification by areas, the table below shows that poverty is more pronounced in rural areas than in urban areas, with respectively a fuzzy proportion of 53.2% in rural areas and 48.2% in urban areas. Overall, three-dimensional fuzzy poverty is on the order of 50.7% in Madagascar. Whether in urban or rural areas, it is always expenditure-related poverty that is the most important determinant of housing characteristics.

Table 10 : Proportion of poverty by environment

	Expenses	Assets	Characteristics of accomodation	Three dimensional poverty
Urban	0,631	0,157	0,659	0,482
Rural	0,784	0,17	0,643	0,532
Total	0,708	0,163	0,651	0,507

Source : Author's calculation from INSTAT's ENSOMD 2012/2013 data

Distribution by region

The regional distribution of fuzzy poverty tells us that the Androy and Atsimo-Atsinanana regions are the poorest, with the same proportion of 59.4%. By contrast, the wealthiest region is Analamanga, with 42.9% of the population living in poverty. These findings are also observable with the FGT index presented in the INSTAT ENSOMD 2012-2013 report.

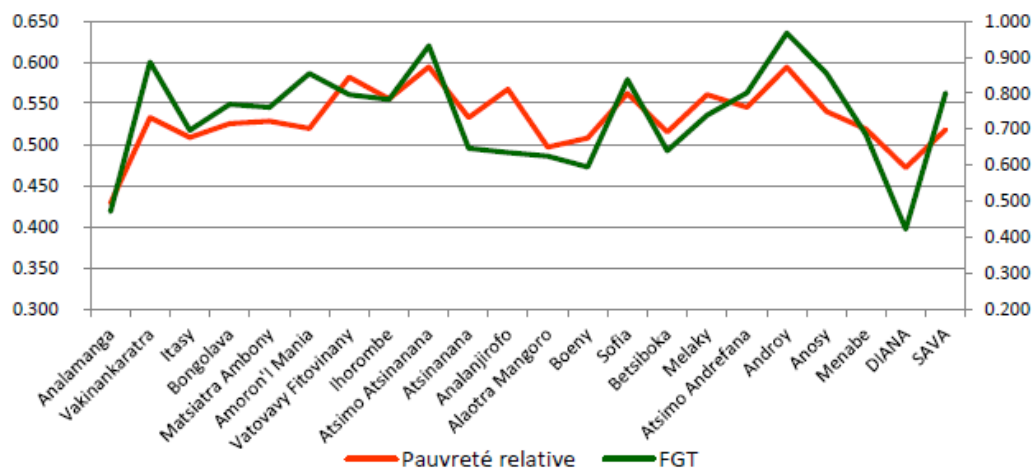
Table 11 : Proportion of poverty by region

	Expenses	Assets	Characteristics of accomodation	Three dimensional poverty	FGT
Analamanga	0,624	0,178	0,483	0,429	0,470
Vakinankaratra	0,841	0,176	0,581	0,533	0,886
Itasy	0,739	0,17	0,616	0,509	0,696
Bongolava	0,776	0,17	0,63	0,525	0,769
Matsiatra Ambony	0,778	0,187	0,62	0,528	0,761
Amoron'I Mania	0,803	0,176	0,579	0,519	0,855
Vatovavy Fitovinany	0,804	0,171	0,77	0,582	0,796
Ihorombe	0,785	0,161	0,72	0,555	0,782
Atsimo Atsinanana	0,885	0,134	0,763	0,594	0,931
Atsinanana	0,714	0,161	0,722	0,533	0,646
Analanjrofo	0,71	0,159	0,832	0,567	0,635
Alaotra Mangoro	0,682	0,164	0,645	0,497	0,625
Boeny	0,675	0,178	0,671	0,508	0,594
Sofia	0,82	0,154	0,713	0,562	0,838
Betsiboka	0,694	0,171	0,682	0,515	0,639
Melaky	0,771	0,167	0,746	0,561	0,739
Atsimo Andrefana	0,806	0,164	0,665	0,545	0,801
Androy	0,909	0,163	0,71	0,594	0,967
Anosy	0,84	0,144	0,637	0,540	0,854
Menabe	0,729	0,156	0,671	0,519	0,685
DIANA	0,622	0,167	0,627	0,472	0,422
SAVA	0,801	0,164	0,590	0,518	0,8
Total	0,708	0,163	0,651	0,507	0,715

Source : Author's calculation from INSTAT's ENSOMD 2012/2013 data

We have plotted simultaneously, in the graph below, the distribution of fuzzy poverty rates by region, according to the relative measure, as well as the incidence of poverty according to the FGT index in order to test graphically the consistency of the two results, while ensuring the reliability of the results obtained with the relative measure.

Figure 9 : Proportion of poverty by region



Source : Author

This graph shows that the two distributions follow the same rates as the region changes. This consistency is confirmed by the strong correlation between these two distributions, which have a correlation coefficient of 0.76. Therefore, we can argue that the results obtained with the relative measurement are practically reliable.

Conclusion of the second chapter

The adoption of a summary indicator would make it possible to summarize all the information contained in each of the variables and to assess their overall behavior. With this indicator, the measurement method we have proposed in this research will focus on multidimensional measurement in order to perfectly meet this concern of understanding all information and behaviors of all variables.

By considering the interrelationships between the different dimensions of poverty, and by posing some dimensions as cause variables and others as effect variables, it has been shown that it is possible to undermine the axiom of focus. In other words, it is possible to reduce the level of poverty for the same individual, even if some of his or her variables exceed the poverty line (normally, variables that exceed the line should be ignored by the poverty index).

If in theory this violation is possible, then the measure found and which meets this need has been called a "relative measure" since it makes it possible to calculate a poverty rate even above the poverty threshold without violating the other axioms, except the axiom of transfer.

In the light of the results on the allocation of income growth, which is considered to be a cause variable, it was much more asset oriented, at the expense of housing characteristics,

despite the fact that housing-related poverty is much higher than asset-related poverty. In any event, the decline in the level of relative poverty observed from these two effect variables was not as remarkable, so to speak, as the allocation would not focus solely on these two goods.

On the other hand, it was noted that the relative measure revealed an important aspect of poverty, according to which it is difficult for a poorer person to improve than a poorer one. As a result, such measures appear to be more realistic. Moreover, the functional form of the measure, which is almost concave, implies that a high concentration of the distribution at the bottom of the scale reflects the phenomenon of more resistant poverty, while the high concentration at the top of the scale makes it more sensitive to the decline. Moreover, this new measure appears to be reliable and robust since its structuring by region or by any other household characteristic is consistent with the structuring of other poverty measurement methods such as the one-dimensional measure of FGT.

Conclusion

In order to take account of the multidimensionality of poverty, the aggregation of a measure into a synthetic measure was used and facilitated by its axiomatic properties linked to its functional form. Despite the fact that the multidimensionality of poverty was unequivocal, the mere aggregation of a measure was not sufficient to reflect the importance of the interrelationships that would exist between the different dimensions. The consideration of these linkages or interrelations in the measurement of poverty was reflected in Bourguignon and Chakravarty (2003), through substitutability and/or complementarity between the variables or dimensions of poverty.

Our concern in the analysis of Bourguignon and Chakravarty (2003) was their use of a membership function taking the convex functional form, while the theory suggests a concave form. We have therefore turned to the search for a concave measure to conform to what the theory claims. Subsequently, our analysis went further by introducing the budgetary constraint variable in addition to the substitutable variables. Consumer theory introduced a budgetary constraint variable into the analysis of consumer behavior (poor household). To this end, we have taken up this theory by adding the budgetary constraint seen in the analysis by Bourguignon and Chakravarty (2003). Therefore, budgetary constraint was the cause variable for poverty and substitutable variables were the poverty effect variables, as suggested by Celiori and Zani (1989). The introduction of this budgetary constraint therefore caused the iso-poverty curve to move beyond the poverty lines. This displacement reflects a violation of the focus axiom. This tells us that it is possible for a poor household to move away from poverty if the variables on which it is not poor are increased by a certain amount.

Normally, according to this axiom, the measurement index ignores dimensions that exceed their thresholds. However, the measure that we have proposed allows us to capture this aspect and calculate a poverty rate for a variable that exceeds the poverty line. We have called this measure "relative measure." This measurement has the characteristic that allows the transfer principle to be modified as a function of the values of the parameters. However, its particularity lies in the fact that it reveals both a downward rigidity of poverty and an upward sensitivity.

Applications were made with urban household data prevalued between 2002 and 2012 in Madagascar. The results of these applications have shown that the change observed between these two years (2002 and 2012) has shown precisely the extent of the allocation of income (a variable that causes poverty) to the effect variables (durable goods holdings and housing characteristics).

On the one hand, there has been a considerable decline in poverty in relation to expenditure (income), which is accompanied by a slight decline in poverty linked to housing characteristics. On the other hand, this decline is quite significant for holdings. Indeed, in terms of the figures, the relative measure shows respective decreases of the order of 8.4% for expenditure, 5% for assets and 0.7% for housing characteristics. This implies that there is an allocation to other variables, despite a fairly significant improvement in living standards. In other words, an increase in income was not enough to move these two variables, and an increase in income above its threshold is always the right thing to do to reduce the level of poverty relative to each effect variable.

On the other hand, by applying the relative measure while leaving income freely above its threshold and keeping the other two dimensions below their thresholds, a difference of -3.7 percentage points between these two years was observed in the incidence of expenditure-related poverty, which is -1.2 percentage points overall. Indeed, the incidence of poverty in relation to expenditure was 66.7% if it had been kept below its threshold and it decreases to 63.1% when it was left freely above the threshold. That decline has dragged global poverty down from 45% to 43.8%.

The violation of the focus axiom was thus proved, and this implies that one should not ignore the variables to which the household is relatively wealthy, because the income making the household rich could obviously have an influence on the other variables to which the household is poor.

From the above, the relative measure may be of some use depending on the concerns of potential users. At the same time, it could also contribute to new research extensions. Two axioms have been strongly affected by this measure, and now the debate on the stability of axioms has special interests in the search for methods to eradicate the phenomenon of poverty.

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