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Modellingvar[1] By The Three And Four Ordering Moments Ofyield Distribution

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Abstract
There are severaldifferentapproaches of assessingrisk in the financialliterature. Regulation has renderedita function of Value-at-Risk. As a reminder of the inadequacy of existingapproaches, the recentcrisis has prompted us to developtoolsthatprovide more detailon losses and gains. In recentyears, modern statisticshave developed a series of probabilistictoolsthat are nowbeingusedexplicitlyin finance. This isin particularthe case of the use of the third- and fourth-ordermoments in the calculation of Valut-at-Risk in studyingleptokurticity or non-gaussianity, and the asymmetry of the distribution of returns. The purpose of thisworkis to extendthismethodology to forms of financial assets like CAC40 companies.
The analysis of regulations and theirpost-crisismodification helps to identify the issues at stake and the difficulty of riskassessment. The presentation of the concept of third- and fourth-order moments exhibitstheirproperties and shows thattheyprovide more information about the distribution tailsthan the classic moments (mean and variance). The distribution of returns for thesesecurities shows thatthisis a case of financial assets whosebehaviouris far from a normal distribution and thereforerequiresspecial techniques. Finally, empiricalanalysis of financial stocks derivedfrom the CAC40 over a long period shows the benefit of the third- and fourth-ordermoments in calibrating the laws and constructing more robustestimators of quantiles thanthoseconstructedusing normal distribution or historical distribution.
***Key words** : Value-at-Risk, financialmodelling, autocorrelationfunctions, probabilities, distributions, moments,*
***JEL classification** : G32, G35, M31*

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I. Introduction

Financial theoryinvolves the study of the prices of financialgoods and, more importantly, their temporal evolution. The financial model isgenerallybased on a representation of the prices of financial assets (or interest rate levels) (Mahamat, 2017, 2018).

Suchrepresentationmaybepursued in order to betterunderstandfinancialmarkets or to improvetools for improvingfinancialmanagement:risk management, asset allocation, development of new financialproducts.

Financial modeling isoften a balancebetweenbeing in line withfinancialmarketexperience and beingeasy to use.A model thataims to generate all the statisticalcharacteristics of the observations usually leads to a complicated model thatisoftendifficult to use, as theoreticalcalculations are typicallydifficult to carry out.Conversely, oversimplifyingmodelsdefinitelyallows us to completeseveralcalculations, in our case wewillsimulate the Value-at-Risk based on the Mont Carlo simulation, thencalculate the Gaussian Value-at-Risk withdifferentquantile levels, and thenfinallywe move on to the Value-at-Risk basedon approaches (Cornish Fisher, Gram Charlier, and Jonson) thattakeintoaccount the third and fourthorder moments of the distribution. Weanalyze the main statisticalproperties of the financialseries and propose a consistent modeling of most of theseproperties.

First used in insurance (ruin concept) and then in trading rooms (JP Morgan - RiskMetrics), Value-at-Risk (VaR) has played a very important role in the analysis of risk and financialreserves. The complexity of market instruments and their incorporation into more complicated portfolios (arbitrage, hedges, multiple asset classes) stimulatedresearch to improvethismethod (RoseléChim and Radjou, 2020).Havingbecomeessential for institutions withcomplexactivities, the regulator has made itthe core of lossassessmentmodels and profiles.

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VaRwascriticized for itsshortcomings (variety of resultsdepending on the lawsused, verydifferentrisk information depending on the confidence levelused, non-additivity). Moreover, the regulationssoonimposedbacktesting and acknowledgedtheirlack of robustness as a penalizing factor.

The crisishighlighted new or insufficientlyformalizedrisks (liquidity, model risk, endogeneity, systemicrisk). It called for a betterunderstanding of the behaviour of assets in the extremes (maximum loss or gain).

Techniques developedfromorderstatistics in order to buildestimatorsthatbettertakeintoaccount the extremes of distributions gave rise to the concept of third- and fourth-ordermoments. Their usage has led to more effective outcomes. In particular, this usage provesuseful for the estimation of the parametricVaR, whichwouldprovide more detail on the realization of gains and losses.

In thispaperwe use moments from distributions thatweapply to the return of CAC 40: the first part examines the stylizedfactsthatallow us to identify the problem of non-normality of autocorrelation, or volatility. The second part isdevoted to the presentation of VaR, and finally the last part is the subject of a study of the modelling of the twoseries of CAC 40 prices for a periodfrom 02/January/2008 to 28/December/2013 to whichweassociateseveralmodelling and estimates of VaR, and itsempiricalapplication; and thenwe end with the Backtest.

1 Value-at-Risk

By definition, VaRis the maximum lossthat a portfolio manager can incurwith a givenprobability over a certain period of time. Assumingthatthisprobabilityis 95%, the errormargin for this maximum lossisonly 5%. If the distribution of cash flows in a portfolio obeys a normal distribution. Let us also assume that the random variable X represents the value of the portfolio, with X~N (μ, σ2). The random variable X can thusberewritten in terms of the standard normal variable ε, ε ~ N(0,1) :

$$\Pr(r_h < \text{VaR}) = p \qquad (1)$$

Withr_h = Ln(V_{t+h} /V_t) the return of the asset at horizon h, and V_t the value of the index at time t. By construction, this VaR is a generally negative number.

∅(P) is the quantile function of the reducedcentred normal distribution ; with the known values of p and h, the VaR can alsobewrittenas:

$$\text{VaR} = \propto_h + \sigma_h \times \emptyset^{-1}(P) \qquad (2)$$

Where Où $\propto_h$ $and\sigma_h$ are the level of quantile and the standard deviation of returns at horizon h, respectively, and $\emptyset^{-1}$(P) is the quantile function of the reducedcentred normal distribution.

The value of VaRreflects the amount of lossthat the investorcould not exceed a certain probability over a well-defined time horizon. This approximation does not takeintoaccountextremeeventsthatcouldoccur and thatcouldresult in more severelosses. This leads the investor to makebiaseddecisionsbased on VaR by underestimatinglosses. Nevertheless, ES avoidsthisVaRshortcoming. Indeed, ittakesintoaccountextremeeventsthatcouldoccur. It isdefined as the average of portfolio lossesthat are above the VaRlevel.

As mentionedabove, financial portfolios can have returnswithasymmetric and leptokurtic distributions. In most cases, the distribution isalsounknown and varies over a different horizon. In thisstudy, weexamined the performance of the Gram-Charlier, Cornish-Fisher, and Johnson, as approachesthat use all of the first four moments of returns, and provideapproximatequantitiesrelated to the unknown distribution of a portfolio's return. For the VaR formula (1), thisapproximatequantitywillbe for $\emptyset^{-1}$(P), the quantile function of the distribution of returns. This partial expectation can becalculatedusing Gram-Charlier, Cornish-Fisher and Johnson approximations of the truedensityfunction. The followingsubsectionsdiscusses how to obtaintheseapproximate values for each of the three cases considered.

1.1 Gram-Charlier

A first approach to VaR and ES calculationwithequations (1) and (2) is the use of the approximate Gram-Charlier densitygiven in the Appendix.

The VaR formula in equation (1) requires the quantile function for the approximate Gram-Charlier density. This quantity can beobtainedwith the distribution function for the approximate Gram-Charlier density. This distribution function can thenbeinvertednumerically to calculate the VaR. The expression of the distribution functionisdetailed in the appendices, and the Gram-Charlier approximation isgivenby:

$$\emptyset_{GC}(\text{ P};k_3,k_4) = \emptyset_N - \frac{k_3}{6}[f_N \times (k^2-1)] - \frac{(k_4-3)}{24}[f_N \times k(k^2-3)] \qquad (3)$$

$$\text{Where}\emptyset_{GC}(\text{ z };k_3,k_4) = \left[1+\frac{k_3}{6}(z^3-3z)+\frac{(k_4-3)}{24}(z^4-6z^2+3)\right] \times f_N(z)$$

Where$\emptyset_N$ and f_N are, respectively, the standard normal density and distribution function evaluated at k, the$k_3 = E[z^3]$ is the skewness coefficient and $k_4 = E[z^4]$ is the kurtosis coefficient.

The Gram-Charlier VaRisthencalculated as follows:

$VaR_{GC} = \propto_h + \sigma_h \emptyset_{GC}^{-1}(z;\ k_3, k_4)$ (4)

2.1 Cornish Fisher

A second approach to VaR and ES usingequations (1) and (2) isproduced by the Cornish-Fisher approach (Zangari)[1996]). This approachprovides a form of approximation based on the known quantile functionneeded to calculateVaR, with the values of the knownskewness and flattening coefficients. The Cornish-Fisher approximation isthenwritten as follows:

$$w_{\propto} = Z_{\propto} + \frac{Z_{\propto}^2 - 1}{6} \times k_3 + \frac{Z_{\propto}^3 - 3Z_{\propto}}{24} \times (k_4 - 3) - \frac{2Z_{\propto}^3 - 5Z_{\propto}}{36} \times {k_3}^2$$

In thisexpression, $w_{\propto}$is the percentile corrected for the threshold distribution $\propto$, $Z_{\propto} = \Phi_N^{-1}(\propto)$, where $\propto$is the quantile level, $\Phi_N^{-1}(\propto)$is the quantile function of the reducedcentered normal distribution, $k_4$is the kurtosis coefficient and $k_3$is the skewness coefficient. Usingthis quantile function, the Cornish-Fisher VaRisthenwrittenas:

$$VaR_{CF,\propto} = \mu + \left(Z_{\propto} + \frac{Z_{\propto}^2 - 1}{6} k_3 + \frac{Z_{\propto}^3 - 3Z_{\propto}}{24} (k_4 - 3) - \frac{2Z_{\propto}^3 - 5Z_{\propto}}{36} {k_3}^2 \right) \sigma$$

Where μ and σ are the mean and standard deviation, respectively.

Or it can bewrittenas:$VaR_{CF,\propto} = \mu - w_{\propto} \times \sigma$

For distributions of skewnessbelowzero or negative and kurtosis greaterthan 3, the VaRobtainedisshiftedwith respect to GaussianVaR and thereforeallows for deviationsfrom "normality" to betakenintoaccount. Nevertheless, by construction, thisVaRcorrectlyrepresents the risk if $k_3$isaroundzero, and $k_4$is close to 3. If thesetwo conditions are not satisfied, the Cornish-Fisher approximation is no longer appropriate (Lhabitant) [2004], and othermethodsshouldbeused.

The main goal of thisstudyis to find a methodthatis compatible with the problem of leptocurtity and/or asymmetricyield distribution.

However, itis possible to numericallycalculatethisquantity. This approachrequiresnumericallyreversing the Cornish-Fisher approximation to a dichotomyprocedure in order to obtain a probability close to the quantile value.

As with the approximate Gram-Charlier density, the approximate quantile functionsgenerated by the Cornish-Fisher approach are not alwaysdesirableproperties. The functiongeneratedis not always a monotonicfunction for all pairs of asymmetry and flattening. Outside of this set, Cornish-Fisher expansion provides non-monotonic quantiles either in the tail of the distributions.

3.1 Johnson'sapproach

A thirdapproach to VaR and ES calculatedwithequations (1) and (2) is the Johnson density system. This methodologypresented by Simonato (2010) allows the first four moments to beused as the main input in a VaR model. The requiredsteps for VaRcalculationsispresented .

Consider a continuousrandom variable z with an unknown distribution thatneeds to beapproximated. Johnson (1949) proposes a set of "normalized" translations. Theseallow the transformation of the continuous variable z into a standard normal variable y and has the followinggeneralform:

$$y = a + b \times g\left(\frac{z - c}{d}\right)$$

Where a and b are shapeparameters, c is a location parameter, d is a scaleparameter, and g(.) is a functionwhoseshapedefines the four families of Johnson's system distributions.

$$g(\mu) = \begin{cases} Ln(\mu) \\ Ln\left(\mu + \sqrt{\mu^2 + 1}\right) \\ Ln\left(\frac{\mu}{1-\mu}\right) \\ \mu \end{cases}$$

They correspond to the log-normal family, the unboundedfamily, the boundedfamily and the normal family.

Thus, the process of using the Johnson system thuscomes down to defining the values of a, b, c and d that are associatedwith the moments of the distribution.

Hill et al (1976) proposed an algorithmthatallows us to choose the appropriatefamily (the form of the function g(.)) and the values of the parametersrequired to approximatethisunknown distribution, when the first four moments of the function are known.

When the parameters are determined in the mannerpresentedabove, the Johnson random variable can beexpressed as the inverse of the normalized translation presentedabove:

$z = c + d.\, g^{-1}\left(\frac{y-a}{b}\right),$

$$\text{Where : } g^{-1}(\mu) = \begin{cases} e^{\mu} \\ (e^{\mu} - e^{-\mu})/2 \\ 1/(1 + e^{-\mu}) \\ \mu \end{cases}$$

Which in order correspond to the log-normal family, the unboundedfamily, the boundedfamily and the normal family.

The quantitiesrequired for the VaR and ES formulas are obtainedfrom the skewness coefficient and the flattening coefficient of the standardizedyield distribution. Using a mean of zero, a standard deviation of one, and the desiredskewness and flattening coefficients within the Hill algorithm, wefind the values of parameters a, b, c, and d.

Once this first stepiscompleted, itisthen possible to measure the risk of this distribution.

The distribution quantitiesrelated to the calculation of VaR are obtained by calculating the quantile par function:

$$\Phi_J^{-1}(p; a, b, c, d) = c + d.g^{-1}\left(\frac{\Phi_N^{-1}(p) - a}{b}\right)$$

Where$\Phi_N^{-1}(p)$is the inverse standard normal quantile function of a random variable valued at p. The VaR is then defined as:

$$VaR_J = \mu_h + \sigma_h \times \Phi_J^{-1}(p; a, b, c, d)$$

2 Empirical Application and Monte Carlo Simulation

Weillustrate the usefulness of takingintoaccountthe third- and fourthordermoments in calculatingVaRfrom the series of Total Et Bouygues assets over a dailyperiodfrom 02/01/2008 to 28/12/2013, thatis, 1563 observations. The indices include the stylizedfacts as defined by Cont(2000) withpricevolatility and non-stationarity. Analyses on the stationarypriceseriesrevealothercharacteristics of financialseries: no auto-correlation of returns but auto-correlation of returnssquared, asymmetry and leptokurticity of the distribution of returns, and volatility clusters. In addition to highlighting ARCH effects, GARCH's application of the BDS test rejects the hypothesis of linear structures.

Descriptive analyses and preliminary tests

Charts 1 and 2 describe total and Bouygues price trends. They show a non-stationarityconfirmed by unit root tests. To station ourseries, we use the first differences in the log prices, which are approximations of the financialreturns of ourselectedsecurities.

2.1 Share performance during the studyperiod

Assets under management experienced a sharpdeclinefollowing the 2008 crisis. This decline, the largesteverobserved, is the result of a combination of negative performance, unitholderdisinvestments and unit liquidations.

Figure 1 - Evolution of TOTAL and BOUYGUES prices

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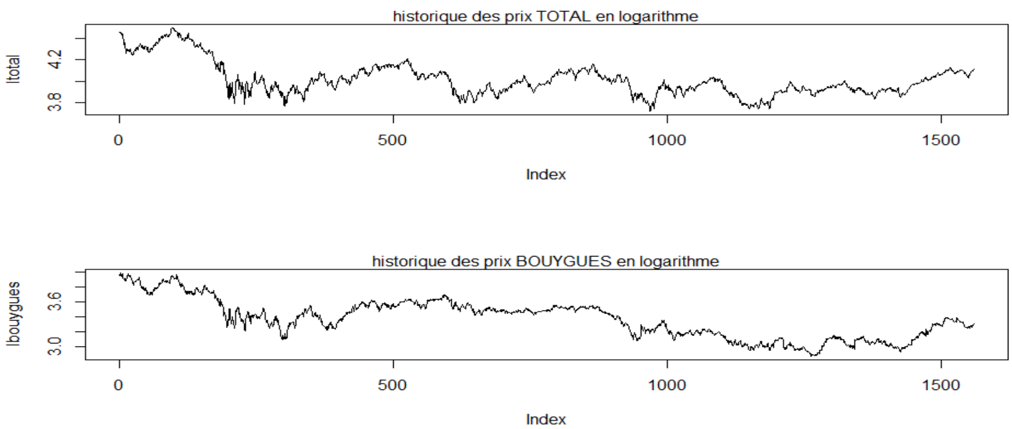


Convincedthat fluctuations in asset pricesconsist of a global trend and an asset-specific factor itself, ithedgesits portfolios by acquiringundervalued assets and sellingovervalued assets.The descriptive statistics in Tables 1 and 2 indicatethatreturns are volatile, leptokurtic and asymmetric: the distributions of returns are not Gaussian distributions. The shortcomings of linearmodels lead us to consider a non-linearapproach to the process of generating return series. To explainthis option, we use tests thatallow us to determinewhether a seriesisi.i.d.Theresults of the tests are given in the appendices for different epsilon values and for different dimensions.

TOTAL: descriptive daily performance statistics

Average	0.03527903
Standard deviation	0.9996982
Min	-1
Max	1
Skewness	-0.070602
Kurtosis	1.004985
Autocorrélation	3.11%
Autocorrélation squared	21.12%
Jarque–Bera (p-value)	2.2e-16%
Ljung–Box (p-value)	0.3486 %
Ljung–Box squared returns (p-value)	0.0%

Table 1*-Statistics Security description TOTAL*

BOUYGUES: descriptive statistics of daily performance

Average	-0.000420461
Standard deviation	0.02478897
Min	-0.1287917
Max	0.1566574
Skewness	0.4575874
Kurtosis	8.315181
Autocorrélation	3.11%
Autocorrelationsquared	21.2%
Jarque–Bera (p-value)	2.2e-16%
Ljung–Box (p-value)	0.1559 %
Ljung–Box squared returns (p-value)	2.2e-16%

Table 2*-Statistics Title description BOUYGUES*

Comparing the two indices, wefindthat for the total index, consideration of the function'sthird and fourth moments isgreater.Theseresultssuggest the use of the Normal Act in calculatingVaR and the offering of lower performance in the riskcalculation of this index.

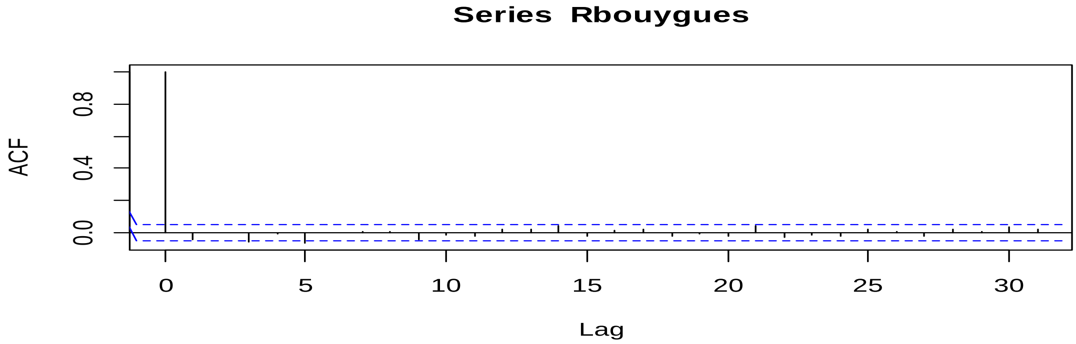
2.2 Autocorrelation test

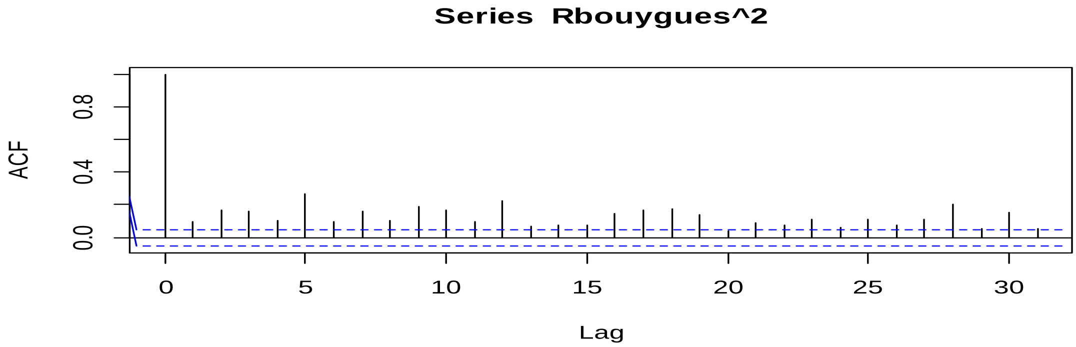
In general, autocorrelationisused to characterizelineardependencies in residualseries (that's to saytrend- and seasonally-adjusted time series). This isbecause trend and season are deterministic components and itmakeslittlesense to estimatestatisticalproperties of deterministicquantities. Moreover, if the seriesunderstudy has itscharacteristicschanging over time, it can bedifficult to estimateitsstatisticalpropertiesbecausethereisusuallyonly one realization of the process, whichis not sufficient to make an estimate. But it'sveryhelpful to considerwhat a rawseries' empiricalautocorrelationwith trend and/or seasonalitywould look like.

Autocorrelationfunctions for Bouygues yieldseries :

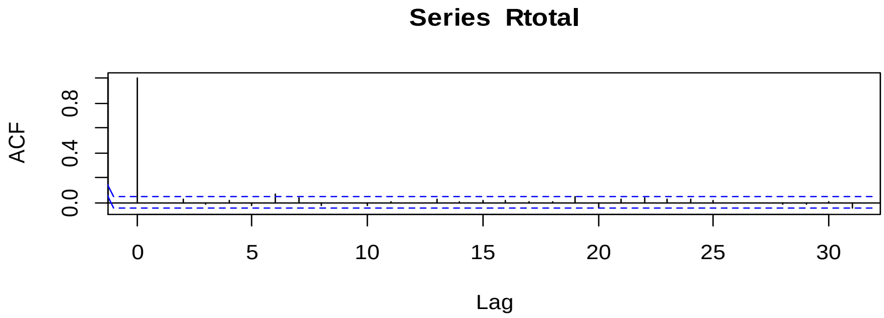
It provides information on the variability of the series and on the time links passing through the intermediate variables y_{t-1}, y_{t-1}.

Autocorrelationfunctions for the yieldseriesBOUYGUES:





Autocorrelationfunctions for the yieldseries TOTAL



The GARCH tests highlight a phenomenon of intermittency in the twoyieldseries and the autocorrelation tests confirm the presence of volatility clusters.

2.3 Non-Normality Tests

The Shapiro-Wilk test and the QQ plot below are used to determinewhether the Total and Bouygues sharepricereturnswillsatisfy the assumption of normality. The QQ plot'sdistinctly non-linear pattern helps us to ignore the presumption of normality at first glance.Figure 3 - Empirical Quantile of Total and Theoretical Share Price

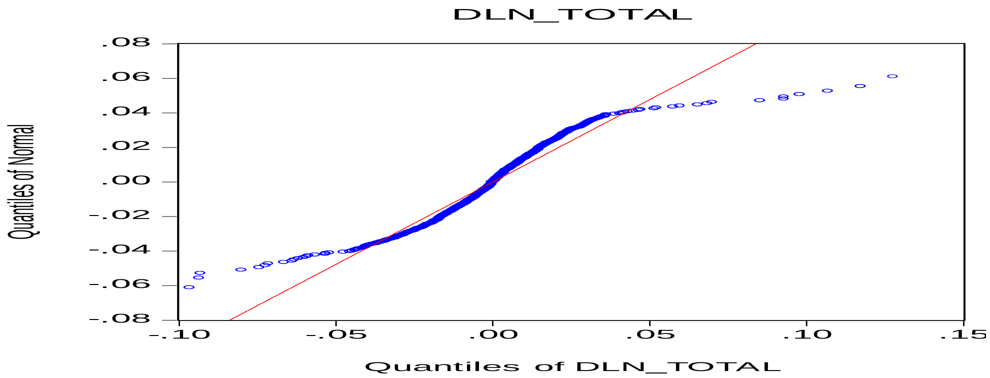
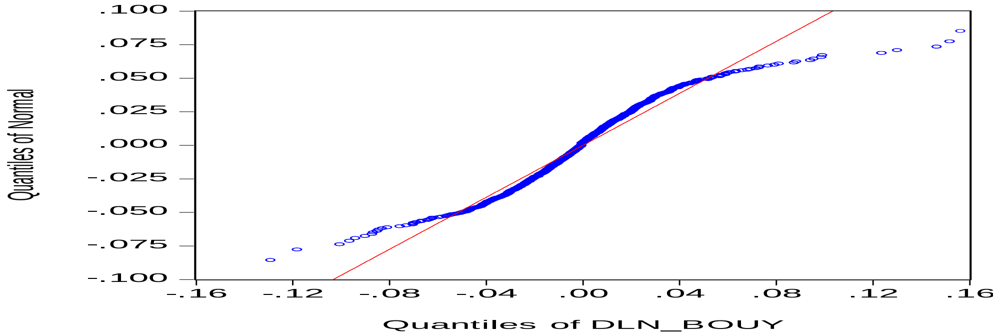


Figure 4 - Empirical quantile of the BOUYGUES and Theoretical course



Empirical quantiles of logarithmicreturnsnormalized by their standard deviation. The plots clearly show the convergence of the distribution towards the Gaussian, presented by the solid line, as the scaleincreases.

2.3 Monte Carlo Simulation

The basic idea of simulation is to build an experimental or simulator device, whichwill "act like" or simulate the system of interest in some important aspects in a quick and inexpensiveway. In the context of quantitative analysis, simulation ispresented as a means of experimentationbased on a mathematical model. Althoughboth simulation and optimization use quantitative models, they are based on verydifferent concepts.

Monte Carlo simulation VaRconsists of generating possible scenarios on the portfolio by consideringmarketfactors. Linsmeier et al. (1996) explained the followingprocedure: First, we must first determine or hypothetically pose a specific distribution thatmostadequatelyrepresents the possible changes in ourmarketfactors. In thisway, our distribution parameters can beestimated. Once thesesteps are completed, the use of a randomgenerator (Excel or Matlab for example) isrequired to obtain a number of hypothetical values N of changes in the values of marketfactors. This N numberof values is at least 1000 (10,000 in our case) in order to obtainaccurateresults and isoftengreaterthan 10,000. These N hypotheticalresults are used to obtain N values in our portfolio fromwhichwe can deductdaily gains and losses. Finally, thesedaily gains or losses, which are oftenexpressed as returns, are ordered in the sameway as the VaRand represents the maximum lossfound at confidence level 1-α.

In our case, wewilltherefore assume thatsecuritieslossesoccurrandomly. A quantification of the loss by the Loss Distribution Approachwillbeusedbased on the Normal distribution law. The frequencyisalsomeasured by the Normal distribution. The probability of lossisobtained as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\delta}\right)^2}$$

It seems to us,in the histogrambelow,that the Skewnessis not far fromzero, whichmeansthat the distribution ispracticallysymmetrical; and the Kurtosis is not far fromthat of a normal law. We can thereforeapproach the distribution of this simulation by a reducedcentered normal distribution.

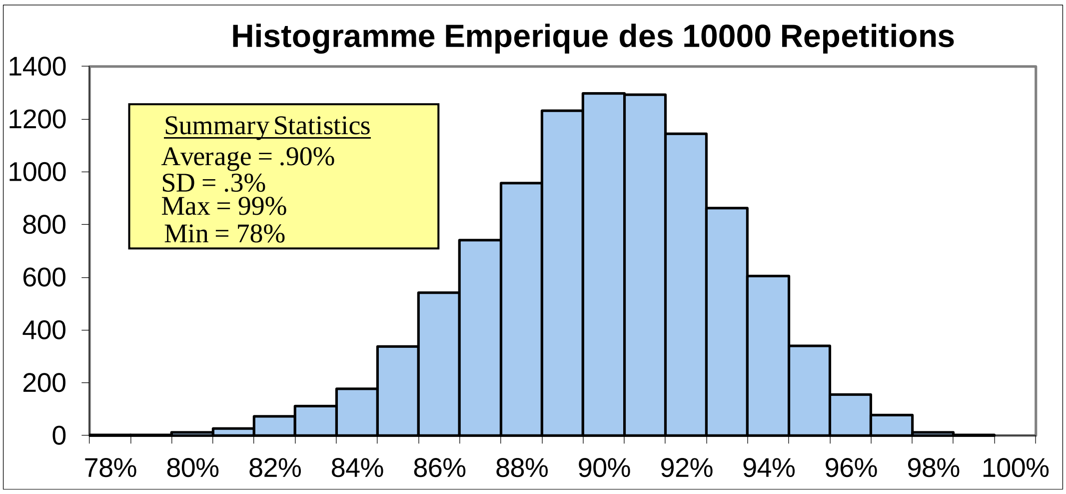


Figure 3 - Histogram of the simulated Monte Carlo series

2.4 Descriptive Statistics of Monte Carlo Simulation

Focusing on the works of (RoseléChim 2017), afterlooking at the first two moments of the simulation, the asymmetry coefficient is the thirdelementof focus in this basic analysis.

REPETITIONS	10000
AVERAGE	-0,011993309
STANDARD DEVIATION	1,007595757
SKEWNESS	0,011836949
KURTOSIS	3.013229

Table 1-Descriptive Statistics of Monte Carlo Simulation

Assuminga normal distribution, this coefficient isequal to zero. In general, the skewness coefficient of a distribution is positive if the right tail, in our simulation case itisequal to 0.012, isalmostsymmetrical.

The fourth and final moment is the flattening coefficient. This coefficient thusmakesit possible to describe how returns are concentratedaround the mean. A high value of the flattening coefficient meansthat more of the variance in the data comesfromextremedeviations. It isequal to 3 in this simulation, sowe can saythatour simulation isalmostGaussian.

GaussianVaR of the Monte Carlo simulation fordifferent confidence levels	
VaR (1%)	-2,349615464
VaR (5%)	-1,674526307
VaR (10%)	-1,301715877
VaR (90%)	1,27772926
VaR (95%)	1,65053969
VaR (99%)	2,325628847

***Table 2-** VaR of the Monte Carlo simulation*

These Values at Risk have a veryspecificmeaning. This meansthat at a confidence level of for example, 1%, 5%, and 10%, the lossamountsshould not exceed the values shown in Table 2.

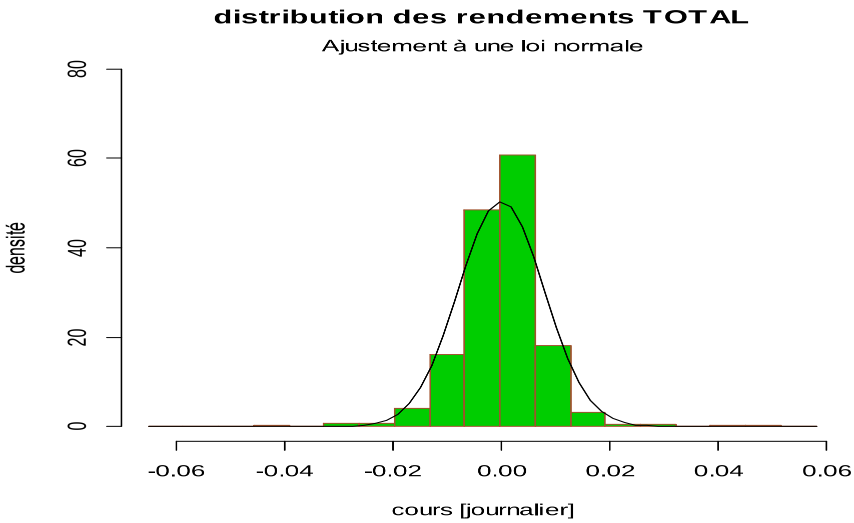
The confidence levels of 90%, 95% and 99% correspond,respectively, to the gains thatshould not beexceeded in normal situations.

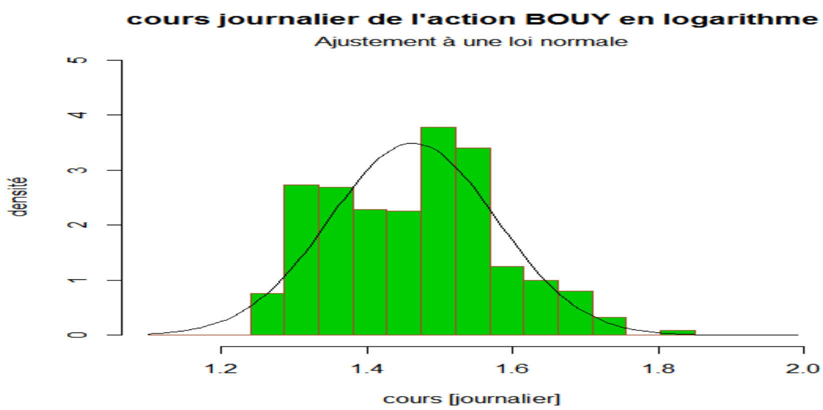
2.5 Empirical Modeling

In order to takeintoaccount the previouscharacteristics of ouryieldseries, we propose a battery of models by differentmethodsthat use the third and fourthordermoments of the yield distribution, namely the Gram Charlier method, the Cornish Fisher extension, and the Jonson approach.
In this section, all results are presentedin tabularform, similar to the previouscase. The tables thusintegrate the robustness of the resultsthrough the samemethodology for a given index and a precisewindow.

Quality of fit

The graphs below show the empirical distribution of returns for each asset, with the normal distribution adjusted for the samemeans and standard deviation.





On the adjustment of the normal distribution to the distribution of the returns of the two stocks, the leptokurticityproblemappearsveryclear ; the shape of the tails of the distributions willbestudiedwithdifferentmodels in order to compare the results.
The results are used to computethe VaR, based on the parametricapproach, and are presented by quantile, for the GaussianVaR.Each asset can beconsidered as an example of how the VaRcalculations are performed.
The 1%, 5% and 10% quantiles are maximum losses at 99%, 95%, and 90%,probabilities, respectively. The minus signmeans a loss (leftside of the distribution). On the other hand, the 90%, 95% and 99% quantiles are maximum gains at 10%, 0.5%, and 0.1%, probabilities,respectively.Here, the positive value means a gain (right part of the distribution).

VaR Gaussienne	R.Total	R.Bouygues
Moyenne	0,000235	0,000102
Ecart-Type	0,01823869	0,01577882
Skewness	-0,00358	-7,04535

Kurtosis	9,3448668	21,881975
VaR(1%)	-0,04219	-0,03660
VaR(5%)	-0,02976	-0,02585
VaR(10%)	-0,0231	-0,02011
VaR(90%)	0,023609	0,020324
VaR(95%)	0,030236	0,0260568
VaR(99%)	0,042665	0,036809

Table3:GaussianVaR to 1%, 5%,10%, 90% and to 99%

At the 1%, 5% and 10% thresholds, it can beseenthat the smaller the margin of error, the greater the absoluteincrease in VaR (the lossbecomessubstantial).

At the 90%, 95%, and 99% thresholds, wesee the exact opposite of the previous section, i.e. the higher the margin of error, the smaller the gain.

Nowweattempt to answer the followingquestion: how to have settings richer in information andlesssimplifying, using more moments?

Interestingadvancesappearwith the use of the first four moments. These last points willbedeveloped in the rest of this document.

Approaches to VaRcalculationbased on developments by Cornish-Fisher, Gram Charlier, and Jonson aim to modify the multiple associatedwith the normal distribution in order to integrate the third and fourth moments of the distribution of returns. Theseapproachesprovide an approximateanalytical expression of the quantile of a distribution as a function of its moments.

By limiting the threeapproachesmentionedabove to its first terms, an analytical expression of VaRisobtainedusing the expectation μ, the standard deviation σ, the Skewness and the Kurtosis of returns.

The tables below show the degrees of asymmetry, the levels of kurtosis, and the $w_\propto$statisticcalculatedfrom the aboveequations (approximation by Gram Charlier, Cornish-Fisher, and Jonson) for the 1%, 5%, 10%, 90%, 95%, and 99% thresholds.

	R.Total			**R.Bouygues**		
	Gram Charlier	**Cornish Fisher**	**Jonson**	**Gram Charlier**	**Cornish Fisher**	**Jonson**
W (1%)	11,6834	3,77863	0,248642	-225,35	38,3539	0,035762
W (5%)	-7,2532	1,5279	0,17695	-153,55	-6,323	0,0332
W (10%)	-1,2191	0,81887	0,137351	-104,06	-15,228	0,03179
W (90%)	-1,24539	-0,8196	0,1366	-104,701	13,7289	0,02201
W (95%)	-7,260138	-1,52996	0,17619	-102,511	2,27876	0,020602
W (99%)	11,7668	-3,7838	0,24789	100,137	-48,646	0,01804

Table 5:Statistics at the thresholds of 1%, 5%, 10%, 90%, 95% and 99%.

As can beseen in the tables, excess kurtosis slightlydominates the asymmetry in the calculation of Cornish-Fisher expansion. By using 2.32 (1% threshold) as a multiple in the GaussianVaRequation, the risk of these assets isthereforegreatlyunderestimated.

Wealso note that the statistics (quantile $w\_\propto$) of all the thresholds in the Jonson approach are verylowcompared to the twopreviousones, explained by the dynamism of thisapproach.

Results and comparisons

The results are presented by quantiles, for each model. Each asset can beseen as an example of the comparison of VaRcalculationsfrom one model to another.

Quantiles of 1%, 5% and 10%

VaR	1%			5%			10%		
	VaR G.C	VaRC.F	VaR N	VaR G.C	VaRC.F	VaR N	VaR G.C	VaRC.F	VaR N
Total	-0,212	-0,06	-0,04	-0,13252	-0,02763	-0,029	0,022471	-0,0147	-0,0
Bouygues	-0,555	-0,61	-0,04	24,67103	0,099887	-0,025	16,49001	0,240384	-0,0

Table 7.1- VaR by modelswith 1%, 5% and 10% quantiles

1%, 5%, and 10% quantiles are maximum losses at 99%, 95%, and 90% Probabilities. The minus signmeans a loss (leftside of the distribution).

The threemodels have been grouped(Gaussian, Gram Charlier and Cornish Fisher) underdifferentlossthresholds in the same table for comparison.
The estimate of losseswith Gram Charlier VaRisgenerally close to Cornish Fisher VaR.
GaussianVaRgiveshigherlossesthanCornish Fisher VaR and Gram Charlier VaR, and thisistrue for bothsecurities.

Cornish-Fisher VaR leads to findings far awayfromGaussianVaR and overestimateslosses in all situations.The differencesbetween the results of the models at the differentthresholds are reduced for securitieswith more or lesssymmetrical distributions.

90%, 95% and 99% quantiles

For the samereasonas the previous table, but this time we are talking about gains instead of losses.
The 90%, 95% and 99% quantiles are maximum gains at 10%, 5% and 1% Probabilities. The plus signmeans a gain (right part of the distribution).

VaR	90%			95%			99%		
	VaR G.C	VaRC.F	VaR N	VaR G.C	VaRC.F	VaR N	VaR G.C	VaRC.F	VaR N
Total	0,02295	0,0151	0,026	0,13265	0,0281	0,03023	-0,214	0,0692	0,0
Bouygues	18,1094	-0,216	0,020	25,0964	-0,035	0,02605	-1,579	0,7676	0,0

Table 7.2- VaRwith 90%, 95% and 99% quantiles

Again, the Gram Carlier VaR and the Cornich Fisher VaR are close.
GaussianVaRgiveslower gains thanthoseobtainedwith Gram Carlier VaR, and Cornich Fisher VaR in most cases, especially in highlyskewed distributions.
Cornish Fisher VaRgivesevengreater gains thanthoseestimatedwithGaussianVaR.
The differences are greater for calculationsperformed at the 99%, 95% and 90% quantile. Same observation on the left and right tails of the distribution: the gaps becomewider as they move awayfrom the centre.

Jonson VaR: 1%, 5% and 10% Quantiles

The 1%, 5% and 10% quantiles are maximum losses at 90%, 95% and 99% Probabilities. The minus signmeans a loss (leftside of the distribution).
The Jonson VaRiscalculatedaccording to the families of the followinglaws: Normal family, Log-Normal family, Borne family, and non-Borne family.

VaR Jonson	1%				5%				10%			
Loi	Normale	Log-Normale	Borné	Non Borné	Normale	Log-Normale	Borné	Non Borné	Normale	Log-Normale	Borné	Non Borné
Total	-13,009	-6,7180481	-23,9	3,142025	-16,188	-5,399211	-12,77	5,390891	-18,555	-4,7107621	-9,49	6,918636
Bouygues	-879,34	-85,667707	-119	389,7925	-1035,6	-73,723885	-96,5	473,8897	-1048,8	-72,881349	-95	480,8924

Table 7.3- Jonson VaRwith 1%, 5% and 10% quantiles

The Jonson VaRcalculatedaccording to the log-normal family and the Cornish Fisher VaR are close.
The Jonson VaRwith the Borne familyresults in higher gains than the Jonson VaRwith the Non-Borne family, in most cases. Generally, the results are closer in the cases of the Jonson VaRwith the Borne family and the Jonson VaRwith the Normal family (distributions withshapeparameters close to zero).

Jonson VaR: 90%, 95% and 99% Quantiles

The 90%, 95% and 99% quantile are maximum gains at 10%, 5% and 1% probabilities. The plus signmeans a gain (right part of the distribution).

VaR Jonson	1%				5%				10%			
Loi	Normale	Log-Normale	Borné	Non Borné	Normale	Log-Normale	Borné	Non Borné	Normale	Log-Normale	Borné	Non Borné
Total	-13,009	-6,7180481	-23,9	3,142025	-16,188	-5,399211	-12,77	5,390891	-18,555	-4,7107621	-9,49	6,918636
Bouygues	-879,34	-85,667707	-119	389,7925	-1035,6	-73,723885	-96,5	473,8897	-1048,8	-72,881349	-95	480,8924

Table 7.4- Jonson VaRwith 90%, 95% and 99% quantiles

In the VaRcalculationaccording to Jonson'smodels, the deviations are larger for calculations at the differentlevels of quantiles.
Same observation on the left and right tails of the distribution: the deviationsincrease as one moves awayfrom the centre.

3. VaRwith3rd and 4th moments set at the 5% threshold

Whilekeepingour respective series as such, we have assigneddifferent values to the skewness and kurtosis parameters, in order to see the impacts of this change on the VaRcalculation. Total and Bouygues' VaRcalculations are performed at the 5% threshold.
In the tables (8.1 and 8.2) below, which show the results for the differentmodels, itisclearthat the furtherawayfrom the position of normality, i.e. skewness =0 and kurtosis =3, the more uncontrollable the risksbecome.

Skewness	-1			0			1		
Kurtosis	VaR GC	VaR CF	VaR J.log N	VaR GC	VaR CF	VaR J.log N	VaR GC	VaR CF	VaR J.log N
0	-0,4247	-0,0253	0,0261	-0,406	-0,030	0,41879	-0,389	-0,0358	1,002396
1,5	-0,3381	-0,0248	1,63577	-0,320	-0,030	15,7381	-0,3024	-0,0352	-0,79373
3	-0,2515	-0,0242	-25,183	**-0,233**	**-0,029**	**-5,0181**	-0,2158	-0,0347	-0,96156
4,5	-0,1649	-0,0237	-9,4983	-0,147	-0,029	-4,4852	-0,1292	-0,0342	-1,32575
5,5	-0,1072	-0,0234	-8,5323	-0,089	-0,029	-4,5682	-0,0715	-0,0339	-1,57547

Table 8.1- Total VaR: for fixed moments 3 and 4

Skewness	-1			0			1		
Kurtosis	VaR GC	VaR CF	VaR J.log	VaR GC	VaR CF	VaR J.log	VaR GC	VaR CF	VaR J.log
0	-0,7187	-0,022	-0,9993	-0,688	-0,026	0,00074	-0,6583	-0,031	1,000887
1,5	-0,5722	-0,022	1,25142	-0,542	-0,026	12,6476	-0,5118	-0,031	-0,56546
3	-0,4258	-0,021	-23,229	**-0,395**	**-0,025**	**-4,3949**	-0,3654	-0,03	-0,70513
4,5	-0,2793	-0,021	-8,4548	-0,249	-0,025	-3,9087	-0,2189	-0,03	-1,01969
5,5	-0,1816	-0,02	-7,5822	-0,151	-0,025	-3,9762	-0,1213	-0,029	-1,23575

Table 8.2- Bouygues VaR: for fixed moments 3 and 4

The coloured values are the results of the differentmodels in accordance with the normalitycriteria.
GaussianVaRgives the lowestlossestimates, especially for the mostextreme quantiles, except in cases where the distributions are symmetrical (shapeparameter close to zero).
The development of Cornish Fisher is not satisfactory. Indeed, Cornish Fisher VaRsystematicallygiveshigherestimates of losses and gains than the Gram Charlier VaR and the VaRcalculatedusingJonson'smodels. The Jonson VaRmodels are generally close to eachother. In summary the approachdevelopedfrom the Jonson extension istherefore close to reality.

3.1 Backtesting

The classicapproachadopted by manyauthorsis to provideVaRforecaststakingintoaccountonly the long position, i.e. for negativereturns. However, the forecastingcapacity of the modelsthat are proposed must beassessed in both long and short positions. Actors participating in the financialmarkets are not onlycurious about the maximum lossthat a fall in the price of the asset theyholdcouldgenerate, but theymay, in a short position, beconcerned about the maximum increase in the price of an asset theyintend to acquire like RoseléChim and Radjou, (2020) has demonstrated. Wepresent the results of backtesting in short and long positions, for conditional and unconditionalhedges, in-sample and out-of-sample. In addition, we use the GARCH (1 1) model forecast.

3.2Methodology

Instead of contrasting the predictions of the model to the realizations, wetake the decision to continue with the simulation by creating as many scenarios as possible.Weimplicitly assume thatJonson'sapproachcalculatedaccording to the log-normal family corresponds to the real model becauseit uses the most information on the distribution tails. Wethenmeasure the errorsproduced by thatothermethods of VaRcalculation. If these computation methodsgenerate large errors, thiswillvalidate the relative contribution of the third and fourthordermoments.

In thisbacktest, wewill test the validity of the VaRlevelscalculatedabove by simulating data in the law of eachsecurityestimatedthanks to the third and fourthorder moments and by calculating the number of times, on average, that the VaRs are exceeded. The standard deviation of thesenumbers of exceedancesisalsocalculated.
Wegenerate N=1000 data for each asset and calculate the mean values of the exceedances and the corresponding standard deviations.
These analyses are performed for 1%, 5%, 10%, 90%, 95% and 99% quantiles,respectively.

3.3 Results

The tables belowpresent, by quantile, the mean and standard deviation of the number of exceedances of the VaRspecifiedwitheach model.

5% quantile exceedance

Action	VaR Gaussienne		VaR G. Charlier		VaR C. Fisher		VaR Jonson	
	Average	**Deviation**	**Average**	**Deviation**	**Average**	**Deviation**	**Average**	**Deviation**
Total	0,505	0,49997	0,562	0,4961	0,505	0,4999	0	0
Bouygues	0,505	0,49997	1	0	0,552	0,4972	0	0

Table-9.1: Comparative table of VaR at the 5% threshold

GaussianVaRunderestimates or overestimateslosses as the case maybe. Losses are generallyoverestimated in the case of Bouygues shares, whose distribution ishighlyskewed.
Cornish Fisher generallyoverestimateslosses (oftenexceeding 0.5).
NB: By constructingthe simulation itself, the number of overshootings for the Jonson VaRiszero. This isonly a numericalconsequence of the simulation, sowe notice thatthis model has not recordedanyovershoot.

Exceedence for 95% quantile

Action	VaR Normale		VaR G. Charlier		VaR C. Fisher		VaR Jonson	
	Average	**Deviation**	**Average**	**Deviation**	**Average**	**Deviation**	**Average**	**Deviation**
Total	0,528	0,526	0,562	0,49614	0,527	0,49927	0	0
Bouygues	0,49921	0,499	1	0	0,5	0,5	0	0

Table-9.1: Comparative table of VaR at the 5% threshold

GaussianVaRmostoftenoverestimates the gains (oftenlessthan 0.5). The results are more in line with the true model for the mostsymmetrical distribution (Total case in particular).
The Cornish Fisher VaRgivesunsatisfactoryresults, withovershootssometimessignificantlyabove or below the expectedlevel, whichalternatelyunderestimates or overestimatesearnings, except in the case of Total, which has a more or lesssymmetrical distribution.
The tables of overruns for the remaining quantiles are in the Appendices.
The GaussianVaR model errorissignificant, leadingmostoften to underestimatinglosses and overestimating gains. However, theseresults are more nuancedwhen the distributions are symmetrical. Backtestingalso leads to rejectingVaRestimates made from the Cornish Fisher developmentbecausethis model tends to overestimatelosses.

ClosingRemarks

This researchwork has focused on many of the elementsinvolved in designing and assessing a measure of risk in finance. The crisis has reminded us of a simple lessonthat the habit of prosperity has succeeded in hiding:thereis no suchthing as fast and risk-free wealth. Growth in the value of goodsislimited by time and risk.

Beyond calculation, risk must betested and thentranslatedintoacts of prudent management, simplicity must befavoured ("Whatis simple iswrong. Whatiscomplicatedisunusable" P. Valery). It isthenclearwhy Value-at-Risk (VaR) isstilluseddespiteitsshortcomings. Value-at-Risk (VaR) iscommonlyused by regulators and practitioners to manageexposures to marketrisks.

In the various sections, we have examined the performance of differentmethodologiesused to measureVaR. As a result, wefoundthatamongourdifferentmethodologies, the normal distribution approachis the least accurate and thisis not surprising. Appliedresearch has developed formulas to compensate for the inability of GaussianVaR to adapt to the asymmetry in the distributions of returns on financial assets. In particular, Gram Charlier'smethod, the development of Cornish Fisher and the extension of Jonson are corrections to the GaussianVaR formula consisting of introducing kurtosis and skewnessintoits expression.

Judgingitnecessary to identify a model thatcorrectlyfits the shape of the tails of distributions, we have exploited the properties of toolsdeveloped by modern statistics, the third and fourthordermoments. Theirpropertiesmakeit possible to better capture information on extreme values. Model estimation allows us to build a more robustindicator of VaR. This is a parametric Jonson VaRbased on the log-normal family of four parameters.

The use of Johnson'smethodologywiththird and fourth-order moments provides a bettermeasure of riskthan the normal distribution in general. Theseresults are consistent with the literature and demonstrate the relevance of usingthemratherthan the normal distribution.

Furthermore, severalbacktests show that Jonson VaRis a more accurate model thanGaussianVaR and givesresultsthat are both more convincing and more stable thanCornish Fisher VaR and Gram Charlier VaR.

The Jonson VaRistherefore a greatstepforward. The VaRcalculationisthenverytheoretical and the actuallosses are higherthanexpected. Takingintoaccount the disappearance of the market in VaRmodelswouldthereforebe a significantimprovement.

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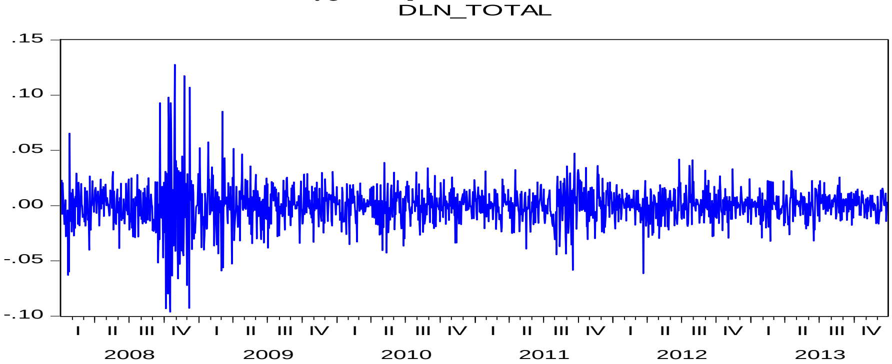
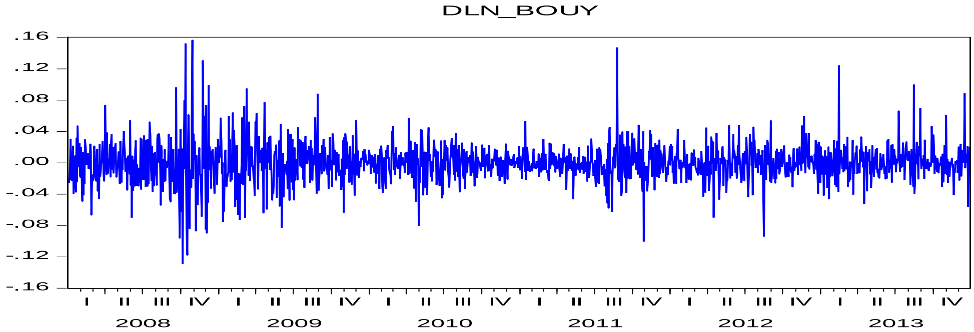
Appendixes

Historicallogarithmicreturns: Total and Bouygues Equities





Yieldnormality test

The normal distribution occursmostoften in statistics and mostestimates are made assumingthat the distribution of the empirical data is normal.

However, itis rare for an empirical data distribution to beperfectly normal. It isthereforenecessary to make approximations and establishcriteria to "approximate" any distribution to the normal distribution.

A normal distribution ischaracterizedmainly by twoparameters: the mean and the variance (second order moment). Twoother important parametersused to characterize a normal distribution are the Skewness and Kurtosis coefficients.

Skewness or skewness coefficient

The Skewness coefficient S_kmeasures the asymmetry of the distribution;itisassociatedwith the moment of order 3. It isgiven by :$S_k = \frac{1}{N}\sum\left(\frac{R_j-\bar{R}}{\delta_P}\right)^3$

The empiricalskewness S iscompared to that of a reduced and centred normal distribution which is 0 :

- If S_k= 0 ,then the distribution issymmetrical and has a good chance of being close to a normal distribution (but not a sufficient condition).
- If S_k >0, then the distribution spreads to the right and issaid to bepositivelyskewed.
- If Si S_k< 0, then the distribution spreads to the left and issaid to benegativelyskewed.

The graphs belowillustratethisbehavior.

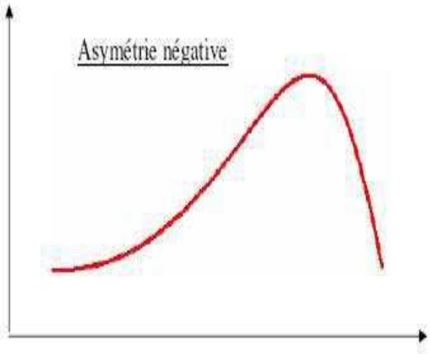


FIGURE .1 :S_k<0

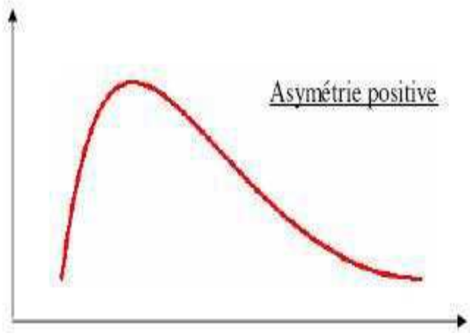


FIGURE.2 :S_k>0

Kurtosis or flattening coefficient

The Kurtosis coefficient K measures the flattening of a distribution.
It is a measure of the degree of concentration of observations at the "tail" of the distribution. It isassociatedwith the moment of order 4. In our case, itisgiven by : $K = \frac{1}{N}\sum\left(\frac{R_j-\bar{R}}{\delta_P}\right)^4$

Kurtosis iscompared to a reducedcentered normal distribution of 3 :

- If K = 3, then the distribution issaid to bemesocurtic, its "tail" is close to that of a normal distribution.
- If K > 3, then the distribution issaid to beleptocurtic, it has a thicker "tail".
- If K < 3, then the distribution issaid to beplatocurtic, with a thinner "tail".

Autocorrelations of series 'Rtotal', by lag

Autocorrelations of series ‘Rtotal’, by lag

1.000	-0.005	0.022	-0.015	0.012	-0.021	0.067	0.037	-0.020	-0.005	-0.023
0.011	-0.002	0.024	0.007	0.017	0.019	0.005	0.006	0.050	-0.042	0.026
0.035	0.024	0.027	0.018	-0.009	0.000	-0.014	-0.017	0.003	-0.048	

This resultshouldberead by looking at the values 2 by 2 vertically: the upper value represents the offset and the lower value the coefficient.

The graphs below have been createdwith the R software;theyrepresent the superimposedhistograms of a distribution of the normal distribution of returns of ourvarioussecurities.

On thesehistograms, we have superimposed the density graph of a normal distribution with the samecharacteristics (Variance, Mean) as the returns, to better observe the behaviour of the returns.

Model [3] with constant and trend

Null Hypothesis: D(LN_TOTAL) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic - based on SIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.512793	0.0000
Test critical values:	1% level	-3.963941	
	5% level	-3.412695	
	10% level	-3.128318	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LN_TOTAL,2)
Method: Least Squares
Date: 30/12/14 Time: 00:40
Sample (adjusted): 8/01/2008 27/12/2013
Included observations: 1559 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LN_TOTAL(-1))	-12.57737	2.281487	-5.512793	0.0000
D(LN_TOTAL(-1),2)	10.77824	1.804862	5.971781	0.0000
D(LN_TOTAL(-2),2)	10.01643	1.235633	8.106317	0.0000
C	-0.056026	0.045094	-1.242411	0.2143
@TREND(2/01/2008)	9.69E-05	4.99E-05	1.940401	0.0525

R-squared	0.042877	Mean dependent var	0.022897
Adjusted R-squared	0.040413	S.D. dependent var	0.904994
S.E. of regression	0.886519	Akaike info criterion	2.600173
Sum squared resid	1221.313	Schwarz criterion	2.617338
Log likelihood	-2021.835	Hannan-Quinn criter.	2.606555
F-statistic	17.40392	Durbin-Watson stat	1.042017
Prob(F-statistic)	0.000000		

The t of the trend coefficient iscomparedwith the value given by the Dickey-Fuller table. Weseethat t = 1.94 < 2.78, the hypothesis H0 isaccepted: the trend is not significantlydifferentfromzero. Wethen move on to model [2] without constant and without trend.

Model [2] with constant and without trend

Null Hypothesis: D(LN_TOTAL) has a unit root
Exogenous: Constant
Lag Length: 2 (Automatic - based on SIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.447491	0.0000
Test critical values:	1% level	-3.434338	
	5% level	-2.863189	
	10% level	-2.567696	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LN_TOTAL,2)
Method: Least Squares
Date: 30/12/14 Time: 00:29
Sample (adjusted): 8/01/2008 27/12/2013
Included observations: 1559 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LN_TOTAL(-1))	-12.43279	2.282297	-5.447491	0.0000
D(LN_TOTAL(-1),2)	10.68236	1.805789	5.915617	0.0000
D(LN_TOTAL(-2),2)	9.970029	1.236499	8.063111	0.0000
C	0.019850	0.022480	0.883019	0.3774

R-squared	0.040558	Mean dependent var	0.022897
Adjusted R-squared	0.038707	S.D. dependent var	0.904994
S.E. of regression	0.887307	Akaike info criterion	2.601310
Sum squared resid	1224.272	Schwarz criterion	2.615042
Log likelihood	-2023.722	Hannan-Quinn criter.	2.606416
F-statistic	21.91121	Durbin-Watson stat	1.039150
Prob(F-statistic)	0.000000		

We can seethat the coefficient of the constant est is not significantlydifferentfromzerobecausewe have t = 0.88 < 2.52. Wethen move on to model [1] without constant and without trend.

Model [1] without constant and without trend

Null Hypothesis: D(LN_TOTAL) has a unit root
Exogenous: None
Lag Length: 2 (Automatic - based on SIC, maxlag=23)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.472171	0.0000
Test critical values:	1% level	-2.566446	
	5% level	-1.941027	
	10% level	-1.616563	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LN_TOTAL,2)
Method: Least Squares
Date: 30/12/14 Time: 00:45
Sample (adjusted): 8/01/2008 27/12/2013
Included observations: 1559 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LN_TOTAL(-1))	-12.48417	2.281393	-5.472171	0.0000
D(LN_TOTAL(-1),2)	10.71649	1.805248	5.936300	0.0000
D(LN_TOTAL(-2),2)	9.986708	1.236267	8.078115	0.0000

R-squared	0.040077	Mean dependent var	0.022897
Adjusted R-squared	0.038843	S.D. dependent var	0.904994
S.E. of regression	0.887244	Akaike info criterion	2.600529
Sum squared resid	1224.886	Schwarz criterion	2.610827
Log likelihood	-2024.112	Hannan-Quinn criter.	2.604358
Durbin-Watson stat	1.038756		

We have 2.60 > -1.94 (for a risk of 5%), the hypothesis H0 isverified:

Non-stationaryseries.
The GARCH tests

Dependent Variable: DLN_TOTAL
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 30/12/14 Time: 15:20
Sample (adjusted): 4/01/2008 27/12/2013
Included observations: 1561 after adjustments
Convergence achieved after 30 iterations
MA Backcast: 3/01/2008
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-2)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.011549	0.024497	0.471431	0.6373
AR(1)	0.017395	4.383303	0.003968	0.9968
MA(1)	0.053678	4.196323	0.012792	0.9898
Variance Equation				
C	0.573576	0.025415	22.56800	0.0000
RESID(-1)^2	-24.00163	6.711900	-3.575981	0.0003
RESID(-2)^2	-15.44432	13.55731	-1.139187	0.2546

R-squared	-0.000179	Mean dependent var	0.022624
Adjusted R-squared	-0.001463	S.D. dependent var	0.904214
S.E. of regression	0.904875	Akaike info criterion	2.679419
Sum squared resid	1275.690	Schwarz criterion	2.699995
Log likelihood	-2085.287	Hannan-Quinn criter.	2.687069
Durbin-Watson stat	1.001002		
Inverted AR Roots	.02		
Inverted MA Roots	-.05		

Table 3- GARCH Test

Dependent Variable: DLN_TOTAL
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 30/12/14 Time: 15:20
Sample (adjusted): 4/01/2008 27/12/2013
Included observations: 1561 after adjustments
Convergence achieved after 30 iterations
MA Backcast: 3/01/2008
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-2)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.011549	0.024497	0.471431	0.6373
AR(1)	0.017395	4.383303	0.003968	0.9968
MA(1)	0.053678	4.196323	0.012792	0.9898
Variance Equation				
C	0.573576	0.025415	22.56800	0.0000
RESID(-1)^2	-24.00163	6.711900	-3.575981	0.0003
RESID(-2)^2	-15.44432	13.55731	-1.139187	0.2546

R-squared	-0.000179	Mean dependent var	0.022624
Adjusted R-squared	-0.001463	S.D. dependent var	0.904214
S.E. of regression	0.904875	Akaike info criterion	2.679419
Sum squared resid	1275.690	Schwarz criterion	2.699995
Log likelihood	-2085.287	Hannan-Quinn criter.	2.687069
Durbin-Watson stat	1.001002		
Inverted AR Roots	.02		
Inverted MA Roots	-.05		

Table 3-BOUYGUESGARCH Test

Dependent Variable: DLN_BOUY
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 30/12/14 Time: 15:45
Sample (adjusted): 7/01/2008 27/12/2013
Included observations: 1560 after adjustments
Convergence achieved after 269 iterations
MA Backcast: 4/01/2008
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2 + C(8)*GARCH(-1)
+ C(9)*GARCH(-2)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.002744	0.000408	-6.720858	0.0000
AR(1)	-1.032389	0.031388	-32.89075	0.0000
AR(2)	-0.037912	0.029831	-1.270902	0.2038
MA(1)	0.958018	0.012233	78.31202	0.0000
Variance Equation				
C	5.68E-05	8.18E-06	6.942426	0.0000
RESID(-1)^2	1.563557	0.053350	29.30773	0.0000
RESID(-2)^2	-0.890710	0.073347	-12.14374	0.0000
GARCH(-1)	0.643791	0.049793	12.92939	0.0000
GARCH(-2)	0.022997	0.019406	1.185035	0.2360

R-squared	-0.025321	Mean dependent var	-0.000184
Adjusted R-squared	-0.027298	S.D. dependent var	0.036369
S.E. of regression	0.036862	Akaike info criterion	-4.225470
Sum squared resid	2.114279	Schwarz criterion	-4.194591
Log likelihood	3304.867	Hannan-Quinn criter.	-4.213989
Durbin-Watson stat	1.913965		
Inverted AR Roots	-.04	-.99	
Inverted MA Roots	-.96		

Staticbacktest

Overruns for 1% quantile

Equity	VaR Normale		VaRG.Charlier		VaRC.Fisher		VaR Jonson	
	Mean	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**
Total	0,5	0,5	0,428	0,494789	0,486	0,499804	0	0
Bouygues	0,5	0,5	0,999	0,031607	0,275	0,446514	0	0

Overruns for 10% quantile

Equity	VaR Normale		VaRG.Charlier		VaRC.Fisher		VaR Jonson	
	Mean	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**
Total	0,505	0,49997	0,525	0,49937	0,506	0,499964	0	0
Bouygues	0,505	0,49997	1	0	0,602	0,489485	0	0

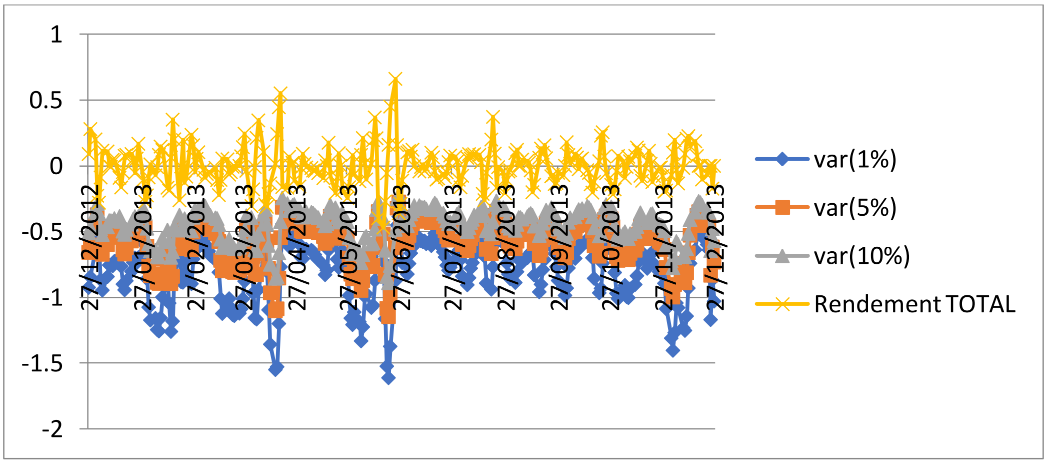
Overruns for 90% quantile

Equity	VaR Normale		VaRG.Charlier		VaRC.Fisher		VaR Jonson	
	Mean	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**
Total	0,525	0,49934	0,525	0,499375	0,519	0,499639	0	0
Bouygues	0,525	0,49934	1	0	0,427	0,494642	0	0

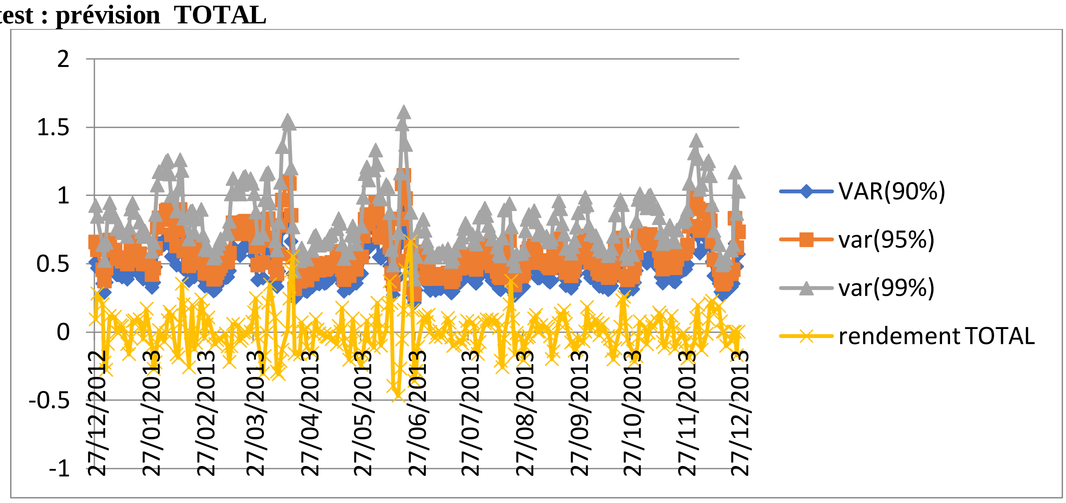
Overruns for 99% quantile

Action	VaR Normale		VaRG.Charlier		VaRC.Fisher		VaR Jonson	
	Mean	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**
Total	0,536	0,498702	0,428	0,494789	0,541	0,498316	0	0
Bouygues	0,531	0,499038	0,068	0,251746	0,79	0,407308	0	0

Backtest : Prévision TOTAL



Backtest : prévision TOTAL



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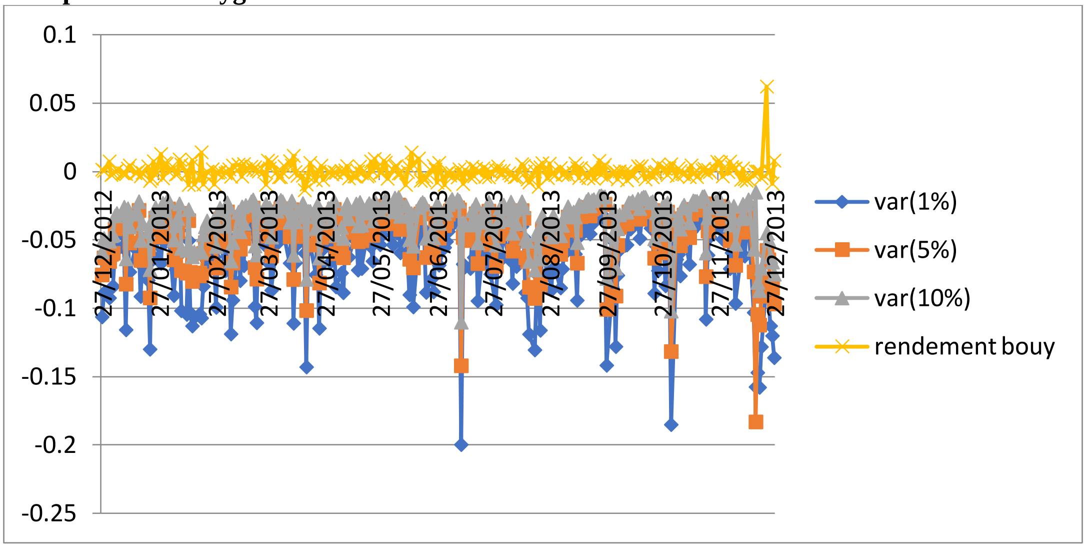

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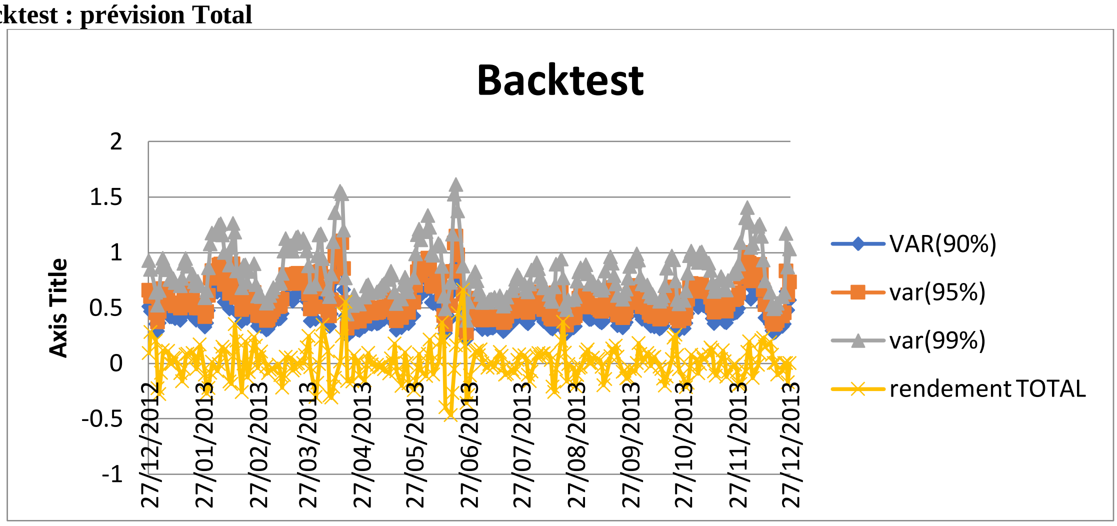

Paul RoseléChim, et. al. " Modellingvar By The Three And Four Ordering Moments Ofyield Distribution." *IOSR Journal of Economics and Finance (IOSR-JEF)*, 12(1), 2021, pp. 08-27.