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Review of Strategies in Games Theory and Its Application In Decision Making

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Abstract
The paper reviewed some strategies in game theory with emphasis on the economic applications of these strategies to champion decision making using both theoretic and numeral illustrations. These feats were achieved after articulating the concept of game theory, assumptions of game theory, classification of game or competitive environment, history and background of game theory, and limitations of game theory. Types of strategies in game theory, and the application thereof in decision making was illustrated with appropriate concepts and numerical illustrations. Lastly, equilibrium concepts in game theory were explained and demonstrated with some relevant illustrations. Furthermore, it was established that Nash Equilibrium is very relevant and consistent in game theory within the contexts of dominant strategy, mixed strategy and Prisoner's dilemma. For instance, in the case of prisoner's dilemma, the goal is to change the structure of the game so that the mutually advantageous outcome is a Nash Equilibrium. This entails that a firm, as an economic agent, needs to seek a competitive advantage by making its product less sensitive to the pricing of rival firms, differentiate its product with design improvements, service or advertising, seek cost advantage, and develop unique distribution channels.
KEYWORDS: Game theory, Nash Equilibrium, prisoner's dilemma, strategies

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I. Introduction: Concept of Game Theory

Game theory is the formal study of conflict and cooperation. It is a concept that encapsulates the statement of elements and conditions of a problem situation involving conflict of interest. The concept applies whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms or any combination of these. The concepts of game theory enable us to formulate, structure, analyse and understand strategic scenarios. Essentially, therefore, the theory of games is a theory of decision-making.

Game theory was created as a tool to mathematically model complex situations, or games. Typically, games model situations where one or more parties (players) have conflicting interests. Examples of players are political parties, governments, firms or businesses, prison inmates and professional sports franchises. The goal of a game theorist is to predict the decisions of and outcomes to the "players" of a "game". Game theory is not used to predict "winners" and "losers". The aim is not to tell us how to play the game, but to tell us certain properties involved with different ways of playing the game that might prove valuable".

Game theory is rooted in mathematics. As such, it can at times be complex and tedious in both its descriptions and applications. The real value of game theory, however, is not in the application of numbers to formulas, but in the critical thinking and deduction used in the analysis of a given game. A game makes explicit the rules governing players' interactions, the players feasible strategies, and their preferences over outcomes (Bicchieri and Sillari, 2005).

Game theory is the study of how rational agents behave in strategic situations or in games where each agent must first know decision of other agent before knowing the decision that is best for himself. Hence, game theory is an analytical tool for analysing how rational agents make their strategic decisions under collective mutual interdependence. The decisions are guided by alternative courses of actions or strategies, to each of which gain or loss is associated.

1.2. Problem Statement

The diversification and unpredicted nature of man has provoked researchers to embrace an infinite journey of unravelling the irrationality cum dynamism in human behaviour. Economics time immemorial has been preaching the rationality of individual-an axiom of utility theorist-so as to complement their models as well as embedded theories fit into the real-life concept, which have spur various branches of the discipline to evolve. However, the rationality of man has been perceived in his individualist as a lone decision maker amongst other alternative of his choice bidden, without recourse to when two or more of his fellows are engaged in attaining same feat in a competitive or win-win scenario. Questions of how do households, firms and government make decision has been considered primary focus of economics but lip staking inquiry emerge on how firms in an industry collide or otherwise to control price or quantity of the industry's commodity, how two individuals in a household rationalize their thought to have the highest share of a feat in order to be regarded and emerge superior over his opponent , how a nation's government will control the economies of others using their weakness in currency, resources and manpower to override them? These and more gave birth to the thought of rationalizing not to only for personification, but also to be better-off aftermath the choice become the dominant over the set of the dominated inferiors.

Following the stated above questions, the St. Petersburg paradox, serve as the eye opener for economist to think out of the box not only in solving individual rationality, but further to this to create means of rationality between individual, firms and government in making choices. The incident in Russian was later developed to game theory where different strategies/ approaches were fashioned out to tackle, likewise address dilemma met by various economic agents when faced in deciding the better option that will make them better-off both in reward and punishment as the case may dictate.

1.3 Assumptions of Game Theory

The following are some of the assumptions of Game Theory:

1. There are finite number of competitors (or players or agents) P= {1,2,3…., n}, {P1 ………,,Pn}.
2. The players are rational. That is, every player strives to maximise gains and minimise losses {U1,…,Un}.
3. Each player has finite number of possible courses of action (i.e., strategies) {S1 ,…,Sn}.
4. The choices are assumed to be made simultaneously, so that no player knows his opponent's choice until he has decided his own course of action.
5. There is a fixed and predetermined pay-off associated with each course of action.
6. Each agent/player knows the rule of the game.
7. A game can be characterised in a pay-off matrix or decision three.

1.4. Classification of Game or Competitive Environment

Game or competitive environment is one that is characterised by conflict of interest. Game environment has two major classifications, namely:

1. Two-Person Game

This is a competitive environment consisting of two players (opponents). The two-person-game can be zero-sum or non-zero sum. It is zero-sum if the sum of the returns to the players is zero. That is, one player's gain equals the other player's loss. It is non-zero sum if the sum of the returns to the players or participants is not zero. That is, one player's gain is not equal to the other player's loss. Winning in a two-person game is achieved either as a pure or strictly determined strategy or mixed strategy.

Examples of environment that typifies two-person game are:

(a) Duopolistic market structure; and

(b) Bargaining between the Academic Staff Union of Universities (ASUU) in Nigeria, on the one hand, and the Federal Government of Nigeria (FGN), on the other hand.

2. N-Person Game

This is a competitive environment consisting of three or more opponents or participants. N-person competitive environment is more realistic than two-person competitive environment, especially in practical real-life situations such as:

(a) Economics of incentives like the area of public finance;

(b) Multilateral trade relations among Nigeria and her trading partners;

(c) Business strategic interactions.

1.5. History and Background of Game Theory

The earliest example of a formal game-theoretic analysis is the study of a duopoly by Antoine Cournot in 1838. In 1921, the mathematician, Emile Borel suggested a formal theory of games. His suggestion was further advanced in 1928 by another mathematician, John von Neumann, based on what he called "*Theory of Parlour Games*".

Eventually, game theory became established and recognised in 1944 after the publication, *Theory of Games and Economic Behaviour*, by John von Neumann (a mathematician) and Oskar Morgenstern (an economist). The book provided much of the basic terminology and problem setup that are still in use today.

In 1950, John Nash demonstrated that finite games always have an equilibrium point, at which all players choose actions which are best for them given their opponents choices. This could be referred to as non-cooperative game, which since then has been a focal point of analysis. In 1950s and 1960s, game theory was broadened theoretically and applied to problems of war and politics. Since the 1970s, it has gained momentum in economic theory. It has also found applications in sociology and psychology, and has had established links with evolution and biology.

However, game theory received special attention in 1994 with the awarding of the Nobel Prize in economics to the game theorists John F. Nash, John C. Harsanyi and Reinhard Selten. At the end of the 1990s, a high-profile application of game theory has been the design of auctions. Prominent game theorists have been involved in the design of auctions for allocating rights to the use of bands of the electromagnetic spectrum to the mobile telecommunications industry. Most of these auctions were designed with the goal of allocating these resources more efficiently than traditional governmental practices, and additionally raised billions of dollars in the United States and Europe.

1.6. Limitations of Game Theory

Game following are some of the circumstances that constitute limitations to the application of Game Theory:

1. The circumstances under which strategic decisions are made is hardly appropriately depicted by a two-person environment. In most cases, the government or society, at least, is always a third or fourth party or relevant entity that must be considered in any strategic decision-making situation.
2. Strategic decision environment is hardly ever characterised by zero-sum. That is, many situations, each of the players or participants may gain directly or indirectly. For instance, if Nigeria hosts a global competitive sporting event like but does not win any medal; the country must have gained some other economic considerations such as product sales and increased revenue from hotel accommodations and allied services.
3. Players or participants do not necessarily have to simultaneously decide on courses of strategic actions. For example, in a duopolistic market structure there is a reaction function - price leader and price follower. Also, the ASUU may decide to extend its industrial strike action from warning strike to indefinite strike as a reaction to the strategy of the Federal Government of Nigeria in the negotiation process.
4. In real life strategic decision situation, it may be unrealistic to determine the precise pay-off or return associated to a game strategy in monetary terms.

II. Literature Review

2.1. Cooperative and Non-Cooperative Games

Cooperative game theory provides analytical tools to study the behaviour of rational players when they cooperate. The main focus of cooperative games describes the formation of cooperating groups of players that can strengthen the players' positions in a game. Lim (1999) views cooperative game theory concepts as sets of payoff combinations that satisfy both individual and group rationality. A typical example of how cooperative game theory can be applied naturally is situations arising in political science or international relations, where concepts like power are most important (Turocy & Von Stengel, 2001)

On the other hand, Non-cooperative game theory focuses on strategic choices resulting from interaction among competing players, each player selects its strategy independently for improving its own utility (Lim, 1999). Turocy & Von Stengel (2001) suggested that non-cooperative game theory specifically means, this branch of game theory explicitly represent the process in which players make choices out of their own interest. Turocy & Von Stengel (2001) further suggested that in the model of non-cooperative game theory the details of the ordering and timing of players' choices are crucial in determining the outcome of a game. Several concepts such as the Nash equilibrium exist for solving non-cooperative games. Lim (1999) suggested that the Nash equilibrium concept may be applied to games in both normal and strategic form, and provides a solution where each player maximises his payoff given the other players' strategies.

2.2. Dominance

Turocy & Von Stengel (2001) argues that since all players are assumed to be rational in game theory, they make choices which result in the outcome they prefer most, given what their opponents do. Both players are said to have dominant strategies. Kelly (2003.) explained dominant strategy as the best choice for a player for every possible choice by the other player. A dominant strategy has payoffs such that, regardless of the choices of other players, no other strategy would result in a higher payoff. Dominance is explained by the use of Prisoner's Dilemma that was first introduced by Tucker in 1950. Kelly (2003) further argued that this game captures the key tensions between individual and collective actions and the outcomes which are the consequence of these actions.

Davis (1997) explained dominance using an example called the Prisoner's Dilemma, where 'Two men are suspected of committing a crime together are arrested and placed in separate cells by the police. There is enough evidence to convict each of them of a minor offense, but not enough evidence to convict both of them of the major crime unless one of them acts as an informer against the other. Each suspect may either confess or remain silent, and each one knows the possible consequences of his action.' Davis (1997) further presented that they were both told that if they both confess; they both go to jail for five years. If they both do not confess then both go to jail for a year, but if only one of them confesses then the other goes to jail for twenty years while the confessor walks out free.

Avinash & Nalebuff (1991) also used the Prisoner's Dilemma to illustrate dominance of the game. Two suspected felons are caught by the police, and interrogated in separate rooms (Avinash & Nalebuff, 1991). They are each told the following:

- If you both confess, you will each go to jail for 10 years.
- If only one of you confesses, he gets only 1 year and the other gets 25 years.
- If neither of you confesses, you each get 3 years in jail.

All possible combinations can be presented on a table below, where all player's possible moves and outcomes are presented:

		First prisoner's decision	
Other prisoner's decision		Confess	Hold out
	Confess	10 years	25 years
	Hold out	1 year	3 years

Prisoner's Dilemma; Source: Avinash & Nalebuff (1991)

The game table above illustrates that the first prisoner will either get 10 years if he confesses or 25 if he does not. So, if the other prisoner confesses, the first would also prefer to confess. If the other prisoner holds out, the first prisoner will get 1 year if he confesses or 3 if he does not, so again he would prefer to confess. Both players are said to have dominant strategies (Avinash & Nalebuff, 1991). A dominant strategy has payoffs such that, irrespective of the choices of other players, no other strategy would result in a higher payoff. An ultimate observation here is that if both prisoners use their confess, they do not reach an optimal outcome.

2.3. Mixed strategies

Turocy and Von Stengel (2001) outlined that a game in strategic form does not always have a Nash equilibrium in which each player definitely chooses one of the strategies. But players base their random selection of strategies on certain probabilities. Mixed strategies are defined as a probability distribution over the set of actions. However, Rubinstein (1991) alternatively viewed mixed strategy as a belief held by all other players regarding a player's actions. Presented below is an illustration of mixed strategies equilibrium by an example of drunk driving; the police choose to set up checkpoints with probability 1/3. Assume if a player drinks Cola, he will get 0. If a player drinks Wine, he will get -2 with probability 1/3 and 1 with probability 2/3. Prisner (2014) also assumed that the value is the expected payoff;

$$\frac{1}{3} \times (-2) + \frac{2}{3} \times 1 = 0$$

The player is indifferent whether to drink Wine or Cola with any probability.

If a player drinks Wine with probability of 1/2 and gets to the police check points, he gets an expected payoff of -1 and if he does not;

$$\frac{1}{2} \times (-2) + \frac{1}{2} \times 0 = -1$$

It is also outlined that the police are also indifferent about setting up checkpoints and any mixed strategy. This results on mixed strategy equilibrium. Osborne (2002) argued that the concept of mixed strategy equilibrium in a strategic game does not motivate the player to introduce randomness in their behaviour. Players normally randomize deliberately to influence the other player's behaviour. Pindyck & Rubinfeld (2009) emphasized that there is no Nash equilibrium on game theory under mixed strategies. Pindyck& Rubinfeld (2009) further explained mixed strategies by use of matching pennies. In the game each player chooses either heads or tails and both players reveal their coin at the same time. If both are heads or tails, player one wins and if coins do match, player two wins. Nevertheless Rubinstein (1991) is of the view that mixed strategy equilibrium is then as common knowledge opportunities, this is because all the actions to which a strictly positive probability is assigned are ideal, given the beliefs.

2.4. Nash equilibrium

The Nash equilibrium is a game theoretic solution concept that is normally applied in economics. As previously outlined, Nash equilibrium was introduced by John Nash in 1950 and has emerged as one of the fundamental concepts of game theory (Kelly, 2003.). Nash equilibrium is a solution concept of a game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy (Osborne, 2002). However, Myerson (1999) viewed the concept of equilibrium as one of the most important and elegant ideas in game theory. Myerson (1999) also pointed out that a game can have many Nash equilibriums, and some of these equilibriums may be unreliable compared to what should be the outcome of a game. Some studies reflect that Nash equilibrium is concern about the actions that will be chosen by players in a strategic game (Osborne, 2002). Players have to know precisely what their opponents will choose (De Bruin, 2009). To do so, players should not base on the assumption that all players are rational. They rather focus on the basis of statistical information about previous game playing situations, if such information is available and reliable. Osborne (2002) also outlined an example where there is interaction between buyers and sellers. A buyer usually transacts only once with any given seller, or interacts repeatedly but anonymously. Each player chooses her action given her belief about the other players' actions. Osborne (2002) provided an example of Nash equilibrium on Prisoner’s Dilemma.

	Player 2		
		Quiet	Fink
Player 1	Quiet	2,2	0,3
	Fink	3,0	1,1

Prisoner's Dilemma; Source: Osborne (2002).

On the presented Prisoner’s Dilemma, Osborne (2002) argued that (Fink, Fink) is an exclusive Nash equilibrium. This action pair is said to be the equilibrium because given that player one chooses Fink, player two is also better off by choosing Fink than Quiet. Presented on the right column of the table, it is observed that Fink yields off player one a payoff of 1, while Quiet yields a payoff of 0. Also given that player one chooses Fink, player two is better off choosing Fink than Quiet, presented on the bottom row of the table it is observed that Fink yields player two a payoff of 1 whereas Quiet yields one a payoff of 0.

2.5. Extensive games with perfect information

Pindyck & Rubinfeld (2009) define extensive games as a representation of possible moves in a game in the form of decision tree. In strategic form games, players simultaneous choose their strategies without being aware of choices of other players. However, with extensive games, players can over time be informed about the actions of other players (Turocy & Von Stengel, 2001). This is also viewed as under perfect information since every player at some point becomes aware of the previous choices of other players. It is further highlighted that to avoid simultaneous movement on extensive game, only one player moves at a time. Osborne (2002) highlighted that this model allows the observation of the game in which each player can consider his plan of action not only at the beginning of the game but also at any point of time. However, strategic game restricts the observation of the game where each player chooses his plan of action once and for all. Extensive games can only consider unlimited possibilities, but the strategic game does not allow a player to reconsider his plan of action after some events in the game have unfolded. Extensive games with perfect information can be presented on a tree diagram, thus it’s also called a game tree with perfect information. Osborne (2002) outlined an example of extensive games with perfect information as follows:

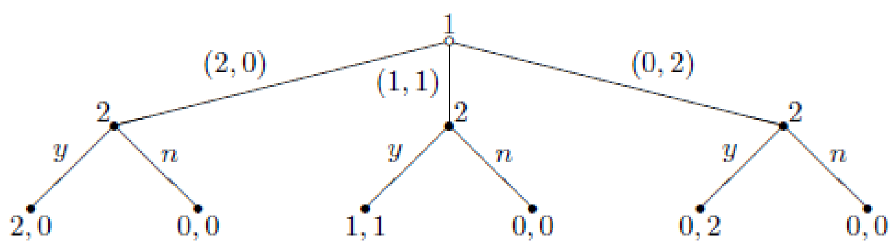

Figure 1 Prisoner's Dilemma; Source: Osborne (2002).

Figure 1 represents extensive games with perfect information on a tree diagram. The small circle at the top of the diagram represents the starting point of the game. The *1* above the small circle indicates that player 1 has to make the first move. The three branching points from the circle represents the possible actions of player 1 at the starting point of the game. The labels beside these branching points are the names of the actions to be taken. Each branching points leads to a small dot beside which is the label 2, indicating that player 2 takes an action after any history of length one. The labels beside the branching points that originate from these small dots and are the names of player 2's actions, y meaning *accept* and meaning *reject*. The numbers at the end of the branches are payoffs for player's preferences, the first number in each pair is payoffs for player 1 and the second number is payoff for player 2.

2.6 Extensive games with imperfect information

In environments with more than one player, each player's payoff is generally affected by the actions of the other players (Gipin & Sandholm, 2007). Thus, the ideal strategy of each player can depend on other players. Extensive games with imperfect information are one of the ways to deal with such strategies. Extensive games with imperfect information are defined as the games that are not fully observable. Osborne (2002) argued that when the player's information is imperfect in extensive games, a player need not to know what actions his rivals have taken before him. This means that when it is a player's turn to move, he does not have access to all of the information about the other player's decisions. Gilpin & Sandholm (2007) argued that such games, the decision of what to do at a point in time cannot generally be optimally made without considering decisions at all other points in time. This is because those other decisions affect the probabilities of being at different states at the current point in time.

For illustration, assume numbers of players are lined up, each player has two options either buy a new iPad (B) or do not buy (N). The quality of a new iPad is high (H) with probability of (p) 0,1 or low (L) with probability of 1-p. The quality is common to all players. Player 1 observes a private signal of new iPad (H,L), which is correct with probability of (p) 0,1 and a choice of preceding player (A1,……..,An). The net payoff from purchasing an iPad is 1 if the quality is good and -1 if the quality is bad.
Extensive games with imperfect information were further illustrated on a tree diagram by Osborne & Rubinstein (1994):

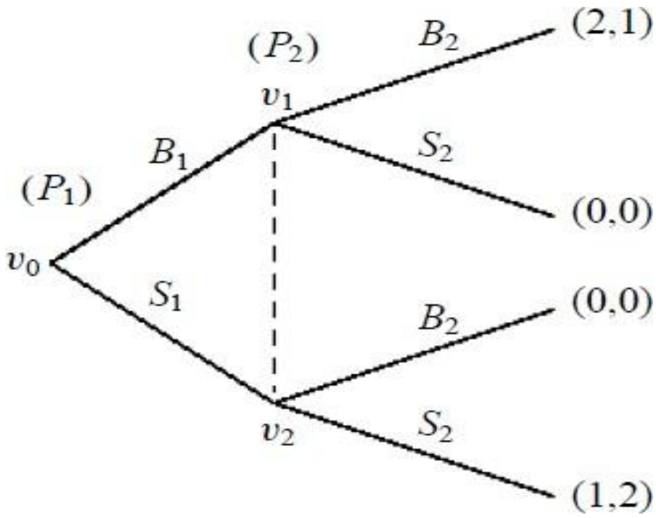

Figure 2 ***Prisoner's Dilemma; Source: Osborne & Rubinstein (1994)***

Figure 2 shows that the game starts at V_0, and P_2 must make a choice at branches V_1 and V_2 without knowing the choice of player P_1. They are then connected with a dotted line and label the edges coming out with common labels, B_2 and S_2. Player P_2 must make the choice of the edges with the same labels at both of these branches. These pair of branches (V_1 and V_2) is called the information sets.

2.7 Zero-sum games

Binmore (2007) defined zero-sum game as a mathematical representation of a situation in which a participant's gain or loss of utility is exactly balanced by the losses or gains of the utility of the other participant. If the total gains of the participants are added up, and the total losses are subtracted, they will sum to zero, thus it's called a zero-sum game. Turocy & Von Stengel (2001) outlined that the extreme case of players with fully opposed interests is demonstrated in the class of two player zero-sum games. The theory of Von Neumann and Morgenstern is mostly applied in games such as two-person zero-sum games, that is games with only two players in which one player wins what the other player loses. Mathematical description of the zero-sum games is when a two-person zero-sum game, the payoff function of Player II is the negative of the payoff of Player I (Turocy & Von Stengel, 2001).

In game theory, a game with only a few strategies can be easily represented by a matrix showing the payoff for each player along with the strategy they use (Brook, 2007). This can be represented in the form of zero-sum games where there are two players and every set of payoffs adds to zero. Zero sum games can also be viewed as a closed system, meaning everything that someone wins must be lost by someone else (Brook, 2007). Brook (2007) emphasized that zero-sum games are those in which the payoffs for each player sum to zero, and went on to outlined an example that shows the differences in representation of zero-sum and non-zero-sum games.

A			B		
	Stag	Hare		Stag	Hare
Stag	3,3	0,2	Stag	3	0
Hare	2,0	2,2	Hare	2	2

Prisoner's Dilemma; Source: Brook (2007)

The first game *"A"* represents the standard matrix game, while the second game "*B"* represents the zero-sum games. Regardless of the similarities in the both player's actions these two games are different. The player's action (Stag, Stag) returns the payoff 3 to both players in the first game, while in the second game (the zero-sum game) the same action player's returns a payoff of 3 for Player 1 but a payoff of –3 for Player 2, thus it results on 0 returns.

2.8. Auctions' bidding

Turocy & Von Stengel (2001) argued that the design and analysis of auctions is one of the achievements of game theory. Auction theory was established by the economist William Vickrey in 1961. It became more practical applied in the early 90's, when auctions generated billions of dollars through radio frequency spectrum for mobile telecommunication. Pindyck & Rubinfeld (2009) views bidding in auction through auction markets, which are defined as markets in which products are bought and sold through formal bidding processes. Klemperer (1999) views a game-theoretic auction model as a mathematical game represented by a set of players, a set of strategies available to each player, and a payoff vector corresponding to each combination of strategies.

Auctions have been used since olden days for the sale of a variety of objects (Klempere, 1999). In the present days, it is easy to find goods like fish, tobacco, flowers, horses, airplanes, art objects in auctions. Klempere (1999) explained how the auctions work in a practical sense as a situation where there is a valuable object; the bidders take an action that signals how much they are willing to pay for the object. There is a well-defined rule that assigns the object to one of the bidders according to all the bids, and all bidders know the rule. There is also a well-defined rule that dictates how much each bidder should pay, and all bidders know the rule. Studies further presented an example where there is queue for a scarce ticket, a prize will be a ticket, bidders will be the people in the queue, and the bids is the time spent waiting on line (Klempere, 1999).

According to Turocy & Von Stengel (2001), the most familiar type of auction is the familiar open ascending-bid auction, which is also called an English auction. In this type of auction, an object is put up for sale in the presents of the buyers. An auctioneer raises the price of the object as long as there are at least two interested bidders. The auction only stops when there is only one interested bidder left. The interested bidder gets the object at the price at which the last remaining opponent drops out.

More emphasis is also on an open descending price auction as another type of auction; it is also called Dutch auction. On this type of auction, the auctioneer begins by calling out a price high enough, where no bidder is even interested in buying the object at that certain price. When a certain bidder starts showing some interest, the price the price of an object gradually goes down to meet the bidder's indicates interest. The object is then sold to this bidder at the given price.

3.1 Types of Strategies in Game Theory, and Application in Decision Making

Strategies are courses of strategic action available to a player, from which the player can choose. It is alternative strategic options available to each player in a game situation. Average gain or loss to a player or participant in a game over an entire cycle of play as all players use optimum strategies depicts value or expectation of the game. A strategy can be pure, mixed or dominance.

3.1.1 Pure Strategy

A pure strategy is one in which the pay-off matrix contains a saddle point. That is, it is a strategic course of action which leaves the players with no other better alternatives, such that the payoff or return associated with it

is the optimum. As such, the saddle point is the value of the game; and the maximin (minimax) principle is used to determine the optimal strategies for the players. Therefore, it is the simplest type of game.
Examples of game situations under which the application of pure strategy is the most suitable in decision making are illustrated below.

1. Given that a competitive game situation involves two players, Ade and Nike, three (3) strategies (X, Y, and Z) are available for Ade while two strategies (X and Y) are available for Nike. The pay-off to Nike is 10000 when they both play Strategy X. When Nike plays Strategy X and Ade plays Strategy Y, the pay-off to Nike is 5000. Nike loses 20000 when she continues to use Strategy X while Ade switches to Strategy Z. And when Nike plays strategy Y while Ade uses strategy X, the loss to Nike reduces to 15000. When Ade adopts strategy Y while Nike continues to use Strategy Y, the return to Nike is 30000. The return to Nike reduces to 10000 as she continues with Strategy Y while Ade changes to Strategy Z. It can be shown that saddle point exists for Nike.
The game situation may be characterised in a pay-off matrix as follows:

(a) From Competitive Position of Ade

		Nike	
		Strategy X	**Strategy Y**
Ade	**Strategy X**	10000	-15000
	Strategy Y	5000	30000
	Strategy Z	-20000	10000

Sub-Game 1

$$
\begin{array}{ll}
 & \textbf{Ade} \qquad\qquad\qquad \text{Row Minimum:} \\
 & \begin{array}{cc} \text{X} & \text{Y} \end{array} \\
\textbf{Nike}\ \begin{array}{c} \text{X} \\ \text{Y} \end{array} & \begin{pmatrix} 10000 & -15000 \\ 5000 & 30000 \end{pmatrix} \begin{array}{c} -15000 \\ 5000 \end{array} \\
\text{Column Max:} & \begin{array}{cc} 10000 & 30000 \end{array}
\end{array}
$$

There is no saddle point in this sub-matrix of the game.

Sub-Game 2

$$
\begin{array}{ll}
 & \textbf{Nike} \qquad\qquad\qquad \text{Row Minimum:} \\
 & \begin{array}{cc} \text{X} & \text{Y} \end{array} \\
\textbf{Ade}\ \begin{array}{c} \text{X} \\ \text{Z} \end{array} & \begin{pmatrix} 10000 & -150000 \\ -20000 & 10000 \end{pmatrix} \begin{array}{c} -15000 \\ -20000 \end{array} \\
\text{Column Max:} & \begin{array}{cc} 10000 & 10000 \end{array}
\end{array}
$$

There is no saddle point in this sub-matrix of the game.

Sub-Game 3

$$
\begin{array}{cc|cc|c}
 & & \multicolumn{2}{c|}{\textbf{Nike}} & \text{Row Minimum:} \\
 & & X & Y & \\
\textbf{Ade} & Y & 5000 & 30000 & 5000 \\
 & Z & -20000 & 10000 & -20000 \\
\text{Column Max:} & & 5000 & 30000 &
\end{array}
$$

There is saddle point in this sub-matrix of the game.

Since saddle point exists in one of the sub-matrices of the game, it shows that pure strategy solution exists for Ade.

Advice: Strategy Y is the optimal strategic decision for Ade.

(b) From Competitive Position of Nike

		Ade		
		Strategy X	**Strategy Y**	**Strategy Z**
	Strategy X	10000	5000	-20000
Nike	**Strategy Y**	-15000	30000	10000

Sub-Game 1 ***Sub-Game 2*** ***Sub-Game 3***

$$
\begin{array}{cc}
 & \textbf{Ade} \\
 & \begin{array}{cc} X & Y \end{array} \\
\begin{array}{c} X \\ \textbf{Nike} \\ Y \end{array} & \begin{pmatrix} 10000 & 5000 \\ -15000 & 30000 \end{pmatrix}
\end{array};
\quad
\begin{array}{cc}
 & \textbf{Ade} \\
 & \begin{array}{cc} X & Z \end{array} \\
\begin{array}{c} X \\ \textbf{Nike} \\ Y \end{array} & \begin{pmatrix} 10000 & -20000 \\ -15000 & 10000 \end{pmatrix}
\end{array};
\quad
\begin{array}{cc}
 & \textbf{Ade} \\
 & \begin{array}{cc} Y & Z \end{array} \\
\begin{array}{c} X \\ \textbf{Nike} \\ Y \end{array} & \begin{pmatrix} 5000 & -20000 \\ 30000 & 10000 \end{pmatrix}
\end{array}
$$

Sub-Game 1

$$
\begin{array}{cc|cc|c}
 & & \multicolumn{2}{c|}{\textbf{Ade}} & \text{Row Minimum:} \\
 & & X & Y & \\
\textbf{Nike} & X & 10000 & 5000 & 5000 \\
 & Y & -15000 & 30000 & -15000 \\
\text{Column Max:} & & 10000 & 30000 &
\end{array}
$$

There is no saddle point in this sub-matrix of the game.

Sub-Game 2

$$
\begin{array}{cc|cc|c}
 & & \multicolumn{2}{c|}{\textbf{Ade}} & \text{Row Minimum:} \\
 & & X & Z & \\
\textbf{Nike} & X & 10000 & -20000 & -20000 \\
 & Y & -15000 & 10000 & -15000 \\
\text{Column Max:} & & 10000 & 10000 &
\end{array}
$$

There is no saddle point in this sub-matrix of the game.

Sub-Game 3

$$
\begin{array}{cc|cc|c}
 & & \multicolumn{2}{c|}{\textbf{Ade}} & \text{Row Minimum:} \\
 & & Y & Z & \\
\hline
\textbf{Nike} & X & 5000 & -20000 & -20000 \\
 & Y & 30000 & 10000 & 10000 \\
\hline
\text{Column Max:} & & 30000 & 10000 &
\end{array}
$$

There is saddle point in this sub-matrix of the game.

Since saddle point exists in one of the sub-matrices of the game, it shows that pure strategy solution exists for Nike.

Advice: The optimal strategic decision for Nike is also Strategy Y.

2. The following pay-off table represents the returns to two players, A and B, each of whom has four strategies, I, II, III, and IV. Given that the players are in the process of making strategic decisions, advise each of the players on the optimal strategy to adopt.

		Player B			
		Strategy I	**Strategy II**	**Strategy III**	**Strategy IV**
Player A	**Strategy I**	**-2**	**0**	**0**	**5**
	Strategy II	**4**	**2**	**1**	**3**
	Strategy III	**-4**	**-3**	**0**	**-2**
	Strategy IV	**5**	**3**	**-4**	**2**

Optimal Strategy for Each Player

		Player B				**Minimum**
		I	**II**	**III**	**IV**	
Player A	**I**	**-2**	**0**	**0**	**5**	**-2**
	II	**4**	**2**	**1**	**3**	**1**
	III	**-4**	**-3**	**0**	**-2**	**-4**
	IV	**5**	**3**	**-6**	**2**	**-6**
Maximum		**5**	**3**	**1**	**5**	

Advice: Since the maximin = the minimax = 1. Therefore, optimal strategies for both players are: Player A must use Strategy II; and Player B must use Strategy III. The value of the game is 1, which indicates that player A will gain 1 unit and player B will sacrifice 1 unit.

3. In a certain duopolistic market, one of the firms, 'Firm A', has three possible marketing strategies, Quantity Discount (QD), Price Discount (PD) and Media Advertisement (MA) (SP) while the other firm, 'Firm B', has two possible marketing strategies, Credit Sales (CS) and After Sales Service (ASS). Profits or that accrue to the firms depend on choices made from the possible marketing strategic alternatives. The strategic marketing alternatives with associated profits are as shown in the table below.

Marketing Strategies & Choices	**Profits for Choices from the Marketing Strategies**
Quantity Discount, Credit Sales	Profit of Firm B increases by ₦300000
Quantity Discount, After Sales Service	Profit of Firm A increases by ₦300000
Price Discount, Credit Sales	Profit of Firm B increases by ₦200000
Price Discount, After Sales Service	Profit of Firm A increases by ₦400000
Media Advertisement, Credit Sales	Profit of Firm A increases by ₦200000
Media Advertisement, After Sales Service	Profit of Firm A increases by ₦300000

The optimal marketing strategies and the expected average profit or loss can be determined for each of Firm A and Firm B in this competitive duopolistic market structure.

Optimal Marketing Strategies for Firm A and Firm B

To determine the optimal marketing strategies for Firm A and Firm B, the competitive environment should be expressed in a game pay-off matrix, and subsequently split into pay-off sub-matrices as follows:

(a) From Competitive Position of Firm A: Pay-off Matrix

		Firm B: Marketing Strategies	
		Credit Sales (CS)	After Sales Service (ASS)
Firm A: Marketing Strategies	Quantity Discount (QD)	-300000	300000
	Price Discount (PD)	-200000	400000
	Media Advertisement (MA)	200000	300000

Pay-off Sub-matrix 1

		Firm B		Row Minimum
	Strategy	**CS**	**ASS**	
Firm A	**QD**	-300000	300000	**-300000**
	PD	-200000	400000	**-200000**
Column Maximum		**-200000**	**400000**	

There is a saddle point. Therefore, pure strategy is appropriate.

Pay-off Sub-matrix 2

		Firm B		Row Minimum
	Strategy	**CS**	**ASS**	
Firm A	**QD**	-300000	300000	**-300000**
	MA	200000	300000	**200000**
Column Maximum		**200000**	**300000**	

There is a saddle point. Therefore, pure strategy is appropriate.

Pay-off Sub-matrix 3

		Firm B		Row Minimum
	Strategy	**CS**	**ASS**	
Firm A	**PD**	-200000	400000	**-200000**
	MA	200000	300000	**200000**
Column Maximum		**200000**	**400000**	

There is a saddle point. Therefore, pure strategy is appropriate.

The best competitive strategy for Firm A is Marketing Advertisement. Adopting the strategy would enable Firm A to sustain profit level of ₦200000, on the average.

(b) From Competitive Position of Firm B: Pay-off Matrix

		Firm A: Marketing Strategies		
	Strategies	**Quantity Discount (QD)**	**Price Discount (PD)**	**Media Advertisement (MA)**
Firm B: Marketing Strategies	**Credit Sales (CS)**	300000	200000	-200000
	After Sales Service (ASS)	-300000	-400000	-300000

Pay-off Sub-matrix 1

		Firm A		Row Minimum
	Strategy	**QD**	**PD**	
Firm B	**CS**	300000	200000	**200000**
	ASS	-300000	-400000	**-400000**
Column Maximum		**300000**	**200000**	

There is a saddle point. Therefore, pure strategy is appropriate.

Pay-off Sub-matrix 2

		Firm A		Row Minimum
	Strategy	**QD**	**MA**	
Firm B	**CS**	300000	-200000	**-200000**
	ASS	-300000	-300000	**-300000**

Column Maximum		**300000**	**-200000**	

There is a saddle point. Therefore, pure strategy is appropriate.

Pay-off Sub-matrix 3

		Firm A		Row Minimum
	Strategy	**PD**	**MA**	
Firm B	**CS**	200000	-200000	-200000
	ASS	-400000	-300000	**-400000**
Column Maximum		**200000**	**-200000**	

There is a saddle point. Therefore, pure strategy is appropriate.

The best competitive strategy for Firm B is Credit Sales. Adopting the strategy would enable Firm B to keep profit loss at amount not exceeding ₦200000, on the average.

3.1.2 Mixed Strategy

A mixed strategy is one which contains no saddle point. That is, a combination of strategic actions is required to optimise a player's return. In such game situation, the maximin (minimax) principle for solving a game problem breaks down. Therefore, the players require mixture of strategies for optimal solution, and value of the game.

Example of game situation under which the application of mixed strategy is appropriate in decision making is given below.

Two firms, MTN and GLO, provide homogeneous product, telecommunication services (M), to the Nigerian market during a specific time period. The gain of MTN in sales equals the loss of GLO. Each of the firms uses two strategies, namely:

Strategy 1: Door-to-Door Canvassing; and
Strategy 2: Electronic Media Advertising.

The sales manager of MTN observes the following pattern arising from past sales campaigns:

a. Sales revenue of MTN increases by ₦600,000 when both firms adopt Strategy 1.
b. MTN's sales revenue decreases by ₦700,000 when it adopts Strategy 1 but GLO uses Strategy 2.
c. Sales revenue of MTN decreases by ₦300,000 when it employs Strategy 2 but GLO adopts Strategy 1.
d. Sales revenue of MTN increases by ₦500,000 when both firms use Strategy 2.

Below is the game matrix representation, and subsequent evaluation of this competitive environment involving MTN and GLO from each of the firm's competitive position.

Game Matrix of the Firms' Competitive Environment

GLO

MTN

	Strategy 1	**Strategy 2**
Strategy 1	600000	-700000
Strategy 2	-300000	500000

Evaluation of the Game

(a) From Competitive Position of MTN

GLO

MTN

			Row Minimum
	Strategy 1	**Strategy 2**	
Strategy 1	600000	-700000	**-700000**
Strategy 2	-300000	500000	**-300000**
Column Maximum	**600000**	**500000**	

There is no saddle point. Therefore, MTN needs a mixed strategy.
Let the number of times MTN adopts Strategy 1 be X.
The number of times MTN uses Strategy 2 would be 1 – X
The mix strategy equation of the game for MTN is:
600000X – 300000(1 – X) = -700000X + 500000(1 – X)
600000X – 300000 + 300000X = -700000X + 500000 + 500000X
900000X – 300000 = -1200000X + 500000
2100000X = 800000

$$X = \frac{800000}{2100000} = \frac{8}{21}$$

The number of times MTN uses Strategy 2 would be

$$1 - X = 1 - \frac{8}{21} = \frac{13}{21}$$

Therefore, MTN should mix the strategies in the ratio 8: 13.
That is, MTN should use door-to-door canvassing 8 times and electronic media advertising 13 times during the period.
Value of the game for MTN (V_{MTN}) during the period would be:
V_{MTN} = 600000X – 300000(1 – X)
V_{MTN} = 600000X – 300000 + 300000X
V_{MTN} = 900000X – 300000

But $X = \frac{8}{21}$

$$=> V_{MTN} = 900000\left(\frac{8}{21}\right) - 300000$$

$$=> V_{MTN} = \left(\frac{7200000}{21}\right) - 300000$$

V_{MTN} = 342857.14 – 300000 = 42857.14
Thus, MTN should mix the strategies in the ratio 8:13 in order to gain N42857.14 on the average.

(b) From Competitive Position of GLO

MTN

GLO

			Row Minimum
	Strategy 1	**Strategy 2**	
Strategy 1	-600000	300000	**-600000**
Strategy 2	700000	-500000	**-500000**
Column Maximum	**700000**	**300000**	

There is no saddle point. Therefore, GLO needs a mixed strategy.
Let the number of times GLO adopts Strategy 1 be Y.
The number of times MTN uses Strategy 2 would be 1 – Y
The mix strategy equation of the game for GLO is:
-600000Y + 700000(1 – Y) = 300000Y - 500000(1 – Y)
-600000Y + 700000 – 700000Y = 300000Y - 500000 + 500000Y
-1300000Y – 800000Y = -1200000
-2100000Y = -1200000

$$Y = \frac{1200000}{2100000} = \frac{12}{21} = \frac{4}{7}$$

The number of times GLO chooses Strategy 2 would be

$$1 - Y = 1 - \frac{4}{7} = \frac{3}{7}$$

Therefore, GLO should mix the strategies in the ratio 4: 3.
That is, GLO should use door-to-door canvassing 4 times and electronic media advertising 3 times during the period.
Value of the game for GLO (V_{GLO}) during the period would be:
V_{GLO} = -600000Y + 700000(1 – Y)
V_{GLO} = -600000Y + 700000 – 700000Y
V_{GLO} = -1300000X + 700000

But $Y = \frac{4}{7}$

$$=> V_{GLO} = -1300000\left(\frac{4}{7}\right) + 700000$$

$$=> V_{GLO} = \left(\frac{-5200000}{7}\right) + 700000$$

V_{GLO} = -742857.14 + 700000 = -42857.14
Thus, GLO should mix the strategies in the ratio 4:3 in order to reduce loss to ₦42857.14 on the average.

3.1.3 Dominant Strategy and Dominated Strategy

A strategy is dominant if, in all respects, it is superior to other strategies. A strategy is dominated if, in all respects, it is inferior to other strategies. The principle of dominance states that if one strategy of a player dominates over the other strategies in all conditions then the dominated strategy can be ignored. A strategy dominates over the other only if it is superior to the other strategy in all conditions. As such, the dominated strategy should not be played. The concept of dominance is especially useful for evaluation of two-person zero-sum games where a saddle point does not exist. The rule of dominance strategy is that:

1. If all the elements of a column (e.g., i^{th} column) are greater than or equal to the corresponding elements of any other column (e.g., i^{th} column), then the ith column is dominated by the j^{th} column. As such, the j^{th} column can be deleted from the matrix.
2. If all the elements of a row (e.g., i^{th} row) are less than or equal to the corresponding elements of any other row (e.g., j^{th} column), then the ith row is dominated by the j^{th} row. As such, the i^{th} column can be deleted from the matrix.

The concept of dominant strategy and dominated strategy is illustrated below:
Given that the profit of an agricultural firm, Agrofarms Limited, can cultivate four types of crops A, B, C, and D, and that its profit from the sale of the crops when harvested depends on planting the right crops during the favourable periods. The periods are Wet Season and Dry Season. The crops and expected profits are given in the pay-off matrix below. Which of the crops should not be planted, no matter the season?

	PROFIT (₦'MILLION)	
	WET SEASON	**DRY SEASON**
Crop A	1000	1800
Crop B	2000	800
Crop C	1600	1000
Crop D	700	1600

Decision:

Profit from the sale of Crop A harvest is greater than profit for the sale of Crop D harvest in wet and dry seasons (i.e., Wet Season: 1000 > 700; and Dry Season: 1800 > 1600). This means that profit from Crop A dominates profit from crop D. Therefore, Crop D should not be planted, no matter the season.

IV. Equilibrium Concepts in Game Theory

Basically, there are two concepts of equilibrium solution techniques in game theory. These are:
(a) Dominant Strategy Equilibrium
(b) Nash Equilibrium

a. Dominant Strategy Equilibrium

This is equilibrium solution in which each player has a dominant strategy. A strategy is dominant when it is the best for the player notwithstanding other player's strategy.
The solution technique is usually expressed as:

$Ui\ (Si \quad S\tau);\ Ui\ (Si^{*} \quad S\tau)$

The equilibrium solution-yielding strategy is weakly dominated if $Ui\ (Si^{*} \quad S\tau) \geq Ui\ (Si \quad S\tau)$.
The strategy is strictly dominated if $Ui\ (Si^{*} \quad S\tau) > Ui\ (Si \quad S\tau)$.

An illustration is given below.

	X	Y	X
A	100000	100000	50000
B	100000	100000	15000
C	20000	40000	30000

A strictly dominates C; B strictly dominates C; B weakly dominates A.

Example

		Player 2	
		Strategy 1	**Strategy 1**
Player 1	**Strategy 1**	30 00	00 -40
	Strategy 2	20 40	-10 80

Considering Player 1
Strategy 1: 30 00
Strategy 2: 20 -10
Strategy 1 strictly dominates Strategy 2.
That is, if Ui (Si* Sτ) > Ui (Si Sτ). Therefore, Strategy 2 will not be used by Player 1.

Considering Player 2
Strategy 1: 00 -40
Strategy 2: 40 80
Strategy 2 strictly dominates Strategy 1.
That is, Ui (Si* Sτ) > Ui (Si Sτ). Therefore, Strategy 1 will not be used by Player 2.

b. Nash Equilibrium

Nash Equilibrium is formally defined or expressed as:

$$G = S_i \left\{ U_i \right\}_{i=1}^{n}$$

where G is Nash Equilibrium, *Si* is strategy of player *i*, and *Ui* is utility player *i* in the cycle of the entire game ranging from 1 to n.

S* = (Si* Si) € S

That is S* is a strategy associated with Nash Equilibrium; and it is contained in the strategies Si* and Si. S is a composite of the strategies.
Si* is Nash Equilibrium if and only if it augments Max Ui (Si Sτ).

Nash equilibrium is especially helpful in two-person games where players have more than two strategies. As such, formal analysis may become too long. Therefore, there is an easy numerical way to identify Nash Equilibrium on a pay-off matrix. However, the method is not appropriate when mixed or stochastic strategies are of interest.
The rule to identify Nash Equilibrium is as follows:
If the first pay-off value in the cell of a matrix is the maximum of the column cells, and the second pay-off value is also the maximum of the row cells, then the cell represents a Nash Equilibrium.
Therefore, to determine Nash Equilibrium cells in a pay-off matrix, we proceed as follows:
Find the maximum value of a column cell and check if the second value in the cell (pair of values) is the maximum of the row. If these conditions are met, then the cell represents a Nash Equilibrium. Check all columns similarly throughout the entire pay-off table to find all Nash Equilibrium cells. An N by N matrix may have between 0 and N by N (pure strategy) Nash Equilibria.
An illustration of multiple Nash Equilibria is shown in the 3 by 3 game matrix below.
Pay-Off Matrix
Pay-Off Matrix

	Strategy A	Strategy B	Strategy C
Strategy A	00 00	25 40	5 10
Strategy B	40 25	00 00	5 15
Strategy C	8 5	9 5	10 10

Using the rule, it is obvious that the Nash Equilibrium are (B, A), (A, B) and (C, C).
For cell (B, A) 40 is the maximum of the first column and 25 is the maximum of the maximum of the second row.
For cell (A, B) 25 is the second column maximum 40 is the first row maximum.
For cell (C, C) 10 is the third column maximum and 10 is the third row maximum.

For other cells, either one or both of the pairs in each cell are not the maximum of the corresponding columns and rows.

Stable and Unstable Nash Equilibrium

The concept of stability is useful in the analysis of many kinds of equilibrium, can also be applied to Nash Equilibrium.

Nash equilibrium for a mixed strategy game is stable if an infinitesimal or little change in probabilities for one player leads to a situation whereby two conditions hold:

i. The player who did not change has no better strategy in the new circumstance;

ii. The player who did change is now playing with a strictly worse strategy.

If these conditions are both met, then a player with the little change in his mixed-strategy will return to the Nash Equilibrium. Thus, the equilibrium is said to be stable.

If condition one does not hold, then the equilibrium is unstable. If only condition holds, then there is likely to be an infinite number of optimal strategies for the player who changed his strategy. John Nash showed that the latter situation could not arise in a range of well-defined games.

Stability is crucial in practical applications of Nash Equilibria, since the mixed-strategies of each player is not perfectly known, but has to be inferred from statistical distribution of his actions in the game. In this case, unstable equilibria are very unlikely to arise in practice since any minute change in the proportion of each strategy seen will lead to a change in strategy and break down of the equilibrium.

Occurrence of Nash Equilibrium

If a game has a unique Nash Equilibrium and is played among players under certain conditions, then the Nash Equilibrium strategy set will be adopted. Sufficient conditions to guarantee that Nash Equilibrium is played are:

i. The players will all do their utmost to maximise their expected pay-off as described by the game.

ii. The players are flawless in execution of their strategies.

iii. The players have sufficient intelligence to deduce the solution.

iv. There is common knowledge that all players meet these conditions, including this one. So not only must each player know that the other players meet the conditions, but also they must know that they all know that they meet them, and know that they know that they meet them, and so on.

Where the Conditions are Not Met

Examples of game theory in which these conditions are not met:

i. Incorrect Description of the Quantities that A Player Wishers to Maximise:

The first condition is not met if the game does not correctly describe the quantities a player wishes to maximise. As such, there is no particular reason for that player to adopt an equilibrium strategy. For instance, the prisoner's dilemma is not dilemma if either player is happy to be jailed indefinitely.

ii. Intentional or Accidental Imperfection in Extension:

For example, a computer capable of flawless logical play facing a second flawless computer will result in equilibrium. Introduction of imperfection will lead to its its disruptions either through loss to the player who makes the mistake, or through negation of the 4^{th} 'common knowledge' criterion leading to possible victory for the player. An example is a player suddenly putting a car into reverse in the game me of 'chicken-and-hawk', ensuring a no-loss no-win scenario.

iii. Complexity of the Game

In many cases, the third condition is not met because, even though the equilibrium must exist, it is unknown due to the complexity of the game. An example is the Chinese chess. Or if known, it may not be known to all players, as when playing tic-tac-toe with a small child who does desperately want to win (meeting the other criteria).

iv. Players Wrongly Distorting Each other's Rationality

The 4^{th} criterion of common knowledge may not be met. Even if all players do, in fact, players wrongly distorting each other's rationality may adopt counter-strategies to expected irrational play on their opponents' behalf. This is a major consideration in 'chicken' or an arms race game, for instance.

Where the Conditions are Met

Due to the limited conditions in which Nash Equilibrium can actually be observed, they are rarely treated as a guide to day-to-day behaviour, or observed in practice in human negotiations. However, as a theoretical concept, in economics, and evolutionary biology, the Nash Equilibrium has explanatory power-off in economics is money, and in evolutionary biology gene transmission, both are the fundamental bottom line of survival. Researchers who apply games theory in these fields claim that agents failing to maximise these for whatever reason will be competed out of market or environment, which are ascribed the ability to test all strategies.

This conclusion is drawn from the "stability" theory above. In these situations, the assumption that the strategy observed is actually a Nash Equilibrium has often been borne out by research.

V. Summary and Conclusion

In this term paper, strategies in game theory have been reviewed and applications to decision making have been espoused using both theoretic and numeral illustrations. These were done after articulating the concept of game theory, assumptions of game theory, classification of game or competitive environment, history and background of game theory, and limitations of game theory. Types of strategies in game theory, and the application thereof in decision making have been illustrated with appropriate concepts and numerical illustrations. Lastly, equilibrium concepts in game theory were explained and demonstrated with some relevant illustrations.

The paper concludes that Nash Equilibrium is very relevant and consistent in game theory within the contexts of dominant strategy, mixed strategy and Prisoner's dilemma. For instance, in the case of prisoner's dilemma, the goal is to change the structure of the game so that the mutually advantageous outcome is a Nash Equilibrium. This entails that a firm, as an economic agent, needs to seek a competitive advantage by making its product less sensitive to the pricing of rival firms, differentiate its product with design improvements, service or advertising, seek cost advantage, and develop unique distribution channels.

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