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Determination of Premiums for Different Discrete Analogues of Continuous Loss Distributions

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Abstract*: The premium value (P_{max}) that an insured willing to pay is determined using risk aversion function for different utility functions. Therefore in the present paper we made an attempt to determine P_{max} for the risk aversion function assuming the loss random variable to follow different forms of discrete analogues continuous loss distributions. We give numerical illustrations using different parameter values of discrete analogues continuous loss distributions.*
Key Words: *Insurance premium, risk aversion, utility function, discretization and loss distribution.*

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I. Introduction

For any insurance contract to be mutually advantageous to the insurer and the insured, premium setting is an important task for an actuary. Therefore the most important thing for insurance companies is the setting of premiums for different types of policies or different levels of risk. In Actuarial Science, many authors have contributed towards the calculations of premium utility theory (Goovaerts et al., 1984: Wang, 1996: Wang and Young, 1997). There exits an economic theory that describes the reasons why insured are willing to pay a premium more than the mathematical expectation of their loss, that is, the net premium (Kaas et al., 2004). These approaches to pricing insurance contracts treat insurance losses as positive random variables and produce premium that are higher than the expected value of the insurance loss. Kapoor and Jain (2011) determined maximum premium by considering different forms of the utility function assuming the loss random variable to follow different forms of continuous statistical distributions.

Sometimes in real life it is difficult or inconvenient to get samples from a continuous distribution. Almost always the observed values are actually discrete because they are measured to only a finite number of decimal places and cannot really constitute all points in a continuum. Even if the measurements are taken on a continuous scale the observations may be recorded in a way making discrete model more appropriate. From the above discussion it can be inferred that many a times in real world the original variables may be continuous in nature but discrete by observation and hence it is reasonable and convenient to model the situation by an appropriate discrete distribution generated from the underlying continuous models preserving one or more important traits of the continuous distribution. Deriving discrete analogues (discretization) of continuous distribution has drawn attention of researchers. In recent decades a large number of research papers dealing with discrete distribution derived by discretizing a continuous one have appeared in a scattered manner in existing statistical literatures (Holland, 1975).

Mallappa and Talawar (2019) determined the maximum premium (P_{max}) for the insured to pay by considering different forms of utility functions. Assuming the loss random variable to follow different forms of discrete analogues continuous distributions considered comparisons of $P_{max}$for linear, quadratic, exponential and fractional power utility functions are given. Numerical illustrations are considered using different parameter values of discrete analogues continuous distributions. In the present paper we attempt to determine $P_{max}$for different forms of the risk aversion functions assuming the loss random variable to follow different forms of discrete analogues continuous distributions. We give numerical illustrations using different parameter values of discrete analogues continuous distributions.

1.1 Risk Aversion

Wang (1996), Wang and Young (1998) distinguish between two types of risk aversion. One type is based on an individual's attitude towards wealth under expected utility theory while the other is based on varying probabilities under dual theory. The authors believe that insurance entities reflect different levelsof risk aversion based on their sizes. The utility function $u(w)$ is defined on a set of prospects and represents preferences over these prospects. The utility function satisfies the principle of non-satiation, that is$u'(w) > 0$. This means that $u(w)$is an increasing function of wealth w and people prefer more wealth to less. In insurance and finance sector, the investor preferences are assumed to be influenced by their attitude towards risk, which can be expressed in terms of properties of utility functions. Investors can be risk-averse, risk-neutral or risk seeking (Dickson 2005). A risk-averse (risk seeking) investor values an incremental increase (decrease) in wealth less highly than the incremental decrease (increase). For a risk averse (risk seeking) investor, the investor, the utility function $u(w)$ is strictly concave (convex), that is,$u''(w) < (>)0$. A risk neutral investor is indifferent towards risk and for him$u'(w) > 0$ and$u''(w) = 0$. The form of this utility function can be chosen to model individual's preferences according to whether or not, he likes, dislikes, or is indifferent to risk. The higher the curvature of$u(w)$, the higher will be risk aversion. However, since expected utility functions are not uniquely defined, a measure that stays constant is the Arrow-Pratt measure of absolute risk-aversion (ARA) (Pratt, 1964). This is defined as $A(w) = -\frac{u''(w)}{u'(w)}$.Increasing/Decreasing absolute risk aversion (IFRA/DFRA) implies that the utility functions positively/negatively skewed, that is, $u''(w) < (>)0$ (Haim, 2006). The risk-averse investor prefers to use either exponential or quadratic or fractional power utility function.

1.2 Risk Aversion Coefficient

Given the utility function $u(w)$, how can we approximate the maximum premium P_{max} for a risk? Let μ and σ^2 denote the mean and variance of X. Using the first terms in the expansions of $u(.)$in $w - \mu$, we obtain

$u(w - P_{max}) \approx u(w - \mu) + (\mu - P_{max})u'(w - \mu)$;

$u(w - X) \approx u(w - \mu) + (\mu - X)u'(w - \mu) + \frac{1}{2}(\mu - X)^2u''(w - \mu)$(1)

Taking expectations on the both sides of the latter approximation yields

$E[u(w - X)] \approx u(w - \mu) + \frac{1}{2}\sigma^2u''(w - \mu)$(2)

From equations (1) and (2), we get

$\frac{1}{2}\sigma^2u''(w - \mu) \approx (\mu - P_{max})u'(w - \mu)$ Where $E[u(w - X)] = u(w - P_{max})$ (3)

Therefore, the maximum premium P_{max} for a risk is approximately

$P_{max} \approx \mu - \frac{1}{2}\frac{u''(w-\mu)}{u'(w-\mu)}$(4)

This suggests the following definition; the (absolute) risk aversion coefficient $r(w)$ of the utility function $u(.)$ at a wealth w is given by

$r(w) = -\frac{u''(w)}{u'(w)}$ (5)

Then the maximum premium P_{max} to be paid for a risk X is approximately

$P_{max} \approx \mu + \frac{1}{2}r(w - \mu)\sigma^2$ (6)

Note that $r(w)$ does not change when $u(w)$ is replaced by $au(w) + b$. Then from equation (6) we see that the risk aversion coefficient indeed reflects the degree of risk aversion, the more is the risk aversion one is, the larger the premium one is prepared to pay.

II. Computation of maximum premium values using different Risk aversion functions

a) Exponential Utility Function

$u(w) = -ae^{-aw}, \quad a > 0$

$$r(w) = -\frac{u''(w)}{u'(w)} = a \text{ and } r(w - \mu) = a \text{ where } a = 105, w = 100 \text{ and } w \leq a$$

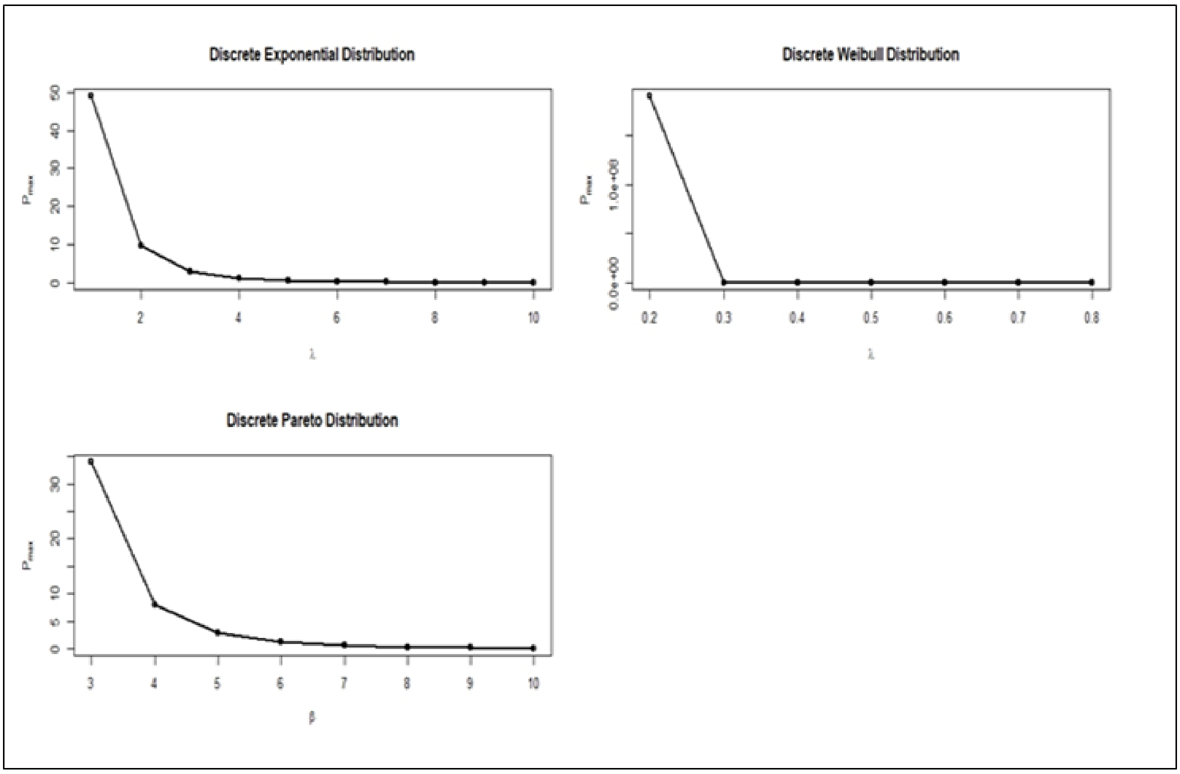
Then maximum premium value is$P_{max} = \mu + \frac{a}{2}\sigma^2$

Table 1: Maximum premium values for different discretized loss distributions under the exponential risk aversion function.

Discretized Continuous Distributions	Maximum Premium Value (P_{max})
Discrete Exponential(λ)	$\frac{e^{-\lambda}}{(1-e^{-\lambda})} + \frac{a}{2}\left[\left(\frac{e^{-\lambda}(1+e^{-\lambda})}{(1-e^{-\lambda})^2}\right) - \left(\frac{e^{-\lambda}}{(1-e^{-\lambda})}\right)^2\right]$
Discrete Gamma(m, λ)	$\frac{me^{-\lambda}}{(1-e^{-\lambda})} + \frac{a}{2}\left[\left(\frac{me^{-\lambda}(1+me^{-\lambda})}{(1-e^{-\lambda})^2}\right) - \left(\frac{me^{-\lambda}}{(1-e^{-\lambda})}\right)^2\right]$

Discrete Weibull (λ)	$$\sum_{k=0}^{\infty} e^{-k^{\lambda}} + \frac{a}{2}\left[\left(2\sum_{k=1}^{\infty} e^{-(k+1)^{\lambda}} + \sum_{k=1}^{\infty} e^{-k^{\lambda}}\right) - \left(\sum_{k=0}^{\infty} e^{-k^{\lambda}}\right)^{2}\right]$$
Discrete Burr(α,β)	$$\sum_{k=1}^{\infty} \frac{1}{(1+k^{\alpha})^{\beta}} + \frac{a}{2}\left[\left(\sum_{k=1}^{\infty} \frac{(2k-1)}{(1+k^{\alpha})^{\beta}}\right) - \left(\sum_{k=1}^{\infty} \frac{1}{(1+k^{\alpha})^{\beta}}\right)^{2}\right]$$
Discrete Pareto(β)	$$\sum_{k=1}^{\infty} \frac{1}{(1+k)^{\beta}} + \frac{a}{2}\left[\left(\sum_{k=1}^{\infty} \frac{(2k-1)}{(1+k)^{\beta}}\right) - \left(\sum_{k=1}^{\infty} \frac{1}{(1+k)^{\beta}}\right)^{2}\right]$$

The following figures illustrate the tendency of maximum premium P_{max} for different discrete analogues version of continuous loss distributions with different parametric values.



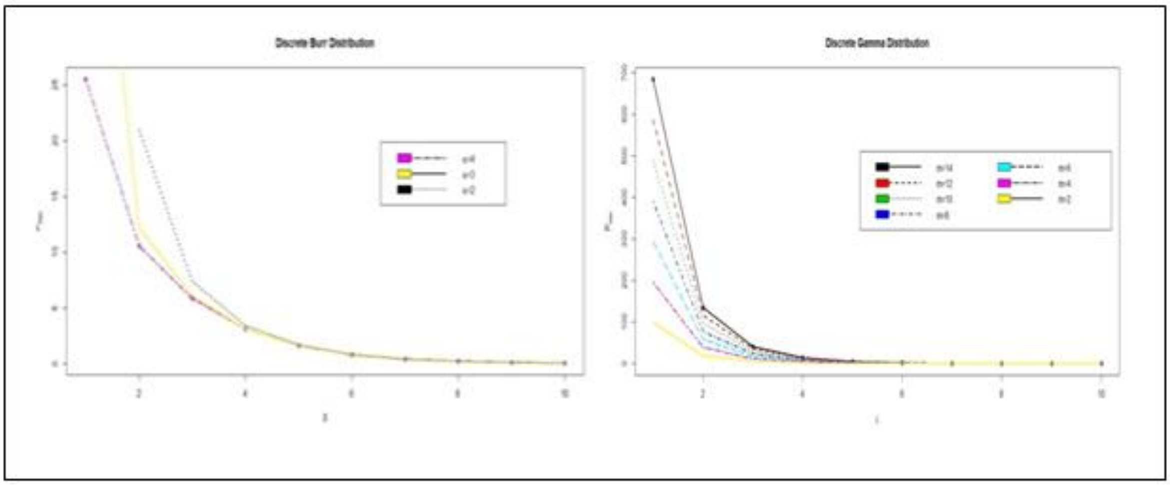


Fig.1(a)-(e) : Maximum premium values at initial wealth $w = 100$ and $a = 105$ i.e.,$a \geq w$

If the insured people assume a risk aversion function of exponential utility function, from the above graphs we can conclude the following.

1. If the insured loss assumes discrete Exponential, Pareto, Burr distributions, the maximum premium value declines as parameter value enlarge. This situation is advantageous to the insured he will ready to pay very less premiums. Also here premium values are more when if we use utility function while calculating premiums.
2. For discrete Weibullif $\lambda < 0.3$, then premium values are very high and later stabilizes as parameter enlarges, in this case also it will advantageous to the insured peoples.
3. If we assume loss assumes discrete Gamma and if parameter value $0 < \lambda < 4$, then in this situation premium values are very high and stabilizes as the parameter enlarges. It is also good for insured peoples.

(b). Quadratic Utility Function

Quadratic Utility function is given as follows:

$u(w) = -(a-w)^2, a \geq w$

$u(w-\mu) = -(a-w+\mu)^2$

$u'(w-\mu) = 2(a-w+\mu)$

$u''(w-\mu) = -2$

Risk aversion function of quadratic utility function is given by

$$r(w-\mu) = -\frac{u''(w-\mu)}{u'(w-\mu)} = \frac{1}{(a-(w-\mu))}$$

Then maximum premium value is

$$P_{max} = \mu + \left(\frac{1}{2(a-(w-\mu))}\right)\sigma^2$$

Table 2: Maximum premium values for different discretized loss distributions under the quadratic risk aversion function.

Discretized Continuous Distributions	**Maximum Premium Value (P_{max})**
Discrete Exponential(λ)	$\frac{e^{-\lambda}}{(1-e^{-\lambda})} + \frac{1}{2\left(a-\left(w-\frac{e^{-\lambda}}{(1-e^{-\lambda})}\right)\right)}\left[\left(\frac{e^{-\lambda}(1+e^{-\lambda})}{(1-e^{-\lambda})^2}\right)-\left(\frac{e^{-\lambda}}{(1-e^{-\lambda})}\right)^2\right]$
Discrete Gamma(m,λ)	$\frac{me^{-\lambda}}{(1-e^{-\lambda})} + \frac{1}{2\left(a-\left(w-\frac{me^{-\lambda}}{(1-e^{-\lambda})}\right)\right)}\left[\left(\frac{me^{-\lambda}(1+me^{-\lambda})}{(1-e^{-\lambda})^2}\right)-\left(\frac{me^{-\lambda}}{(1-e^{-\lambda})}\right)^2\right]$
DiscreteWeibull (λ)	$\sum_{k=0}^{\infty} e^{-k^\lambda} + \frac{1}{2\left(a-(w-\sum_{k=0}^{\infty} e^{-k^\lambda})\right)}\left[\left(2\sum_{k=1}^{\infty} e^{-(k+1)^\lambda}+\sum_{k=1}^{\infty} e^{-k^\lambda}\right)-\left(\sum_{k=0}^{\infty} e^{-k^\lambda}\right)^2\right]$
Discrete Burr(α,β)	$\sum_{k=1}^{\infty}\frac{1}{(1+k^\alpha)^\beta} + \frac{1}{2\left(a-\left(w-\sum_{k=1}^{\infty}\frac{1}{(1+k^\alpha)^\beta}\right)\right)}\left[\left(\sum_{k=1}^{\infty}\frac{(2k-1)}{(1+k^\alpha)^\beta}\right)-\left(\sum_{k=1}^{\infty}\frac{1}{(1+k^\alpha)^\beta}\right)^2\right]$
Discrete Pareto(β)	$\sum_{k=1}^{\infty}\frac{1}{(1+k)^\beta} + \frac{1}{2\left(a-\left(w-\sum_{k=1}^{\infty}\frac{1}{(1+k)^\beta}\right)\right)}\left[\left(\sum_{k=1}^{\infty}\frac{(2k-1)}{(1+k)^\beta}\right)-\left(\sum_{k=1}^{\infty}\frac{1}{(1+k)^\beta}\right)^2\right]$

The following figures illustrate the tendency of maximum premium P_{max} for different discrete analogues version of continuous loss distributions with different parametric values.

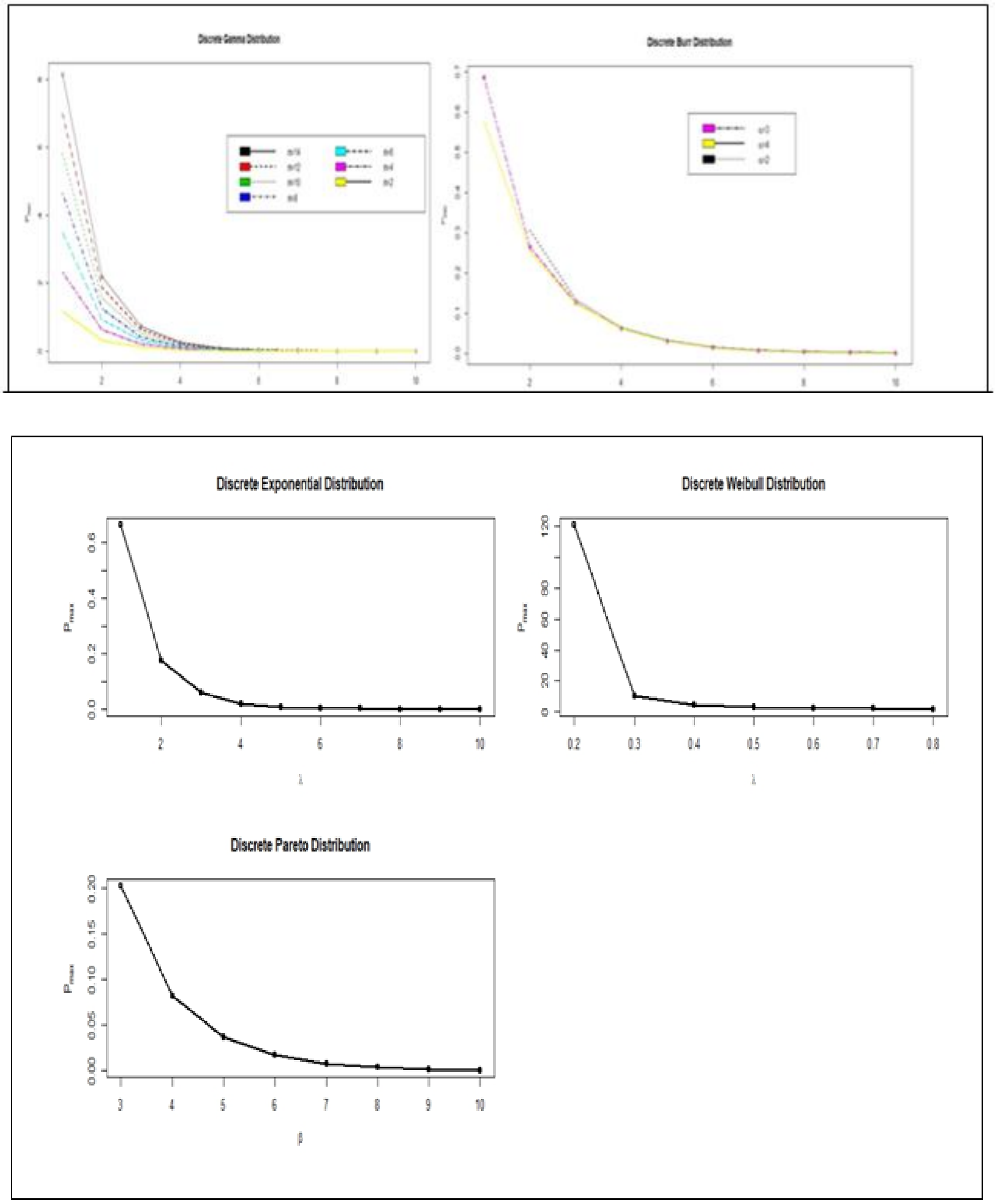


Fig.2(a)-(e) : Maximum premium values for initial wealth $w = 100$ and $a = 105$ i.e., $a \geq w$

If the insured people assume a risk aversion function of quadratic utility function, from the above graphs we conclude the following

1. If the insured loss assumes discrete Exponential, Pareto, Gamma distributions, the maximum premium value declines as parametric value enlarges. This situation is advantageous for the insured as he will ready to pay fewer premiums.
2. For discrete Weibull distribution, there is a presence of high premium values at the smaller parameter values λ and the later premium stabilizes as the parameter value increases. It is also advantageous to the insured people.
3. With the discrete Burr loss distribution for the parameter values $\alpha = 2,3,4$ the maximum premium values coincide as the parameter value enlarges.

(c) Risk Aversion Function for Fractional Power Utility Function

Fractional power utility function is given as follows:

$u(w) = w^c, \qquad 0 < c < 1$

$u(w-\mu) = (w-\mu)^c$

$u'(w-\mu) = c(w-\mu)^{c-1}$

$u''(w-\mu) = c(c-1)(w-\mu)^{c-2}$

Risk aversion function of quadratic utility function is given by

$$r(w-\mu) = -\frac{u''(w-\mu)}{u'(w-\mu)} = -\frac{(c-1)}{(w-\mu)}, \qquad 0 < c < 1$$

Then maximum premium value is

$$P_{max} = \mu - \frac{1}{2}\left(\frac{c-1}{(w-\mu)}\right)\sigma^2$$

Table 3: Maximum premium values for different discretized loss distributions under the fractional power utility risk aversion function.

Discretized Continuous Distributions	Maximum Premium Value (P_{max})
Discrete Exponential(λ)	$\frac{e^{-\lambda}}{(1-e^{-\lambda})} + \frac{1}{2}\left(\frac{c-1}{(w-\left(\frac{e^{-\lambda}}{(1-e^{-\lambda})}\right))}\right)\left[\left(\frac{e^{-\lambda}(1+e^{-\lambda})}{(1-e^{-\lambda})^2}\right) - \left(\frac{e^{-\lambda}}{(1-e^{-\lambda})}\right)^2\right]$
Discrete Gamma(m,λ)	$\frac{me^{-\lambda}}{(1-e^{-\lambda})} + \frac{1}{2}\left(\frac{c-1}{(w-\left(\frac{me^{-\lambda}}{(1-e^{-\lambda})}\right))}\right)\left[\left(\frac{me^{-\lambda}(1+me^{-\lambda})}{(1-e^{-\lambda})^2}\right) - \left(\frac{me^{-\lambda}}{(1-e^{-\lambda})}\right)^2\right]$
Discrete Weibull (λ)	$\sum_{k=0}^{\infty} e^{-k^{\lambda}} + \frac{1}{2}\left(\frac{c-1}{(w-(\sum_{k=0}^{\infty} e^{-k^{\lambda}}))}\right)\left[\left(2\sum_{k=1}^{\infty} e^{-(k+1)^{\lambda}} + \sum_{k=1}^{\infty} e^{-k^{\lambda}}\right) - \left(\sum_{k=0}^{\infty} e^{-k^{\lambda}}\right)^2\right]$
Discrete Burr(α,β)	$\sum_{k=1}^{\infty}\frac{1}{(1+k^{\alpha})^{\beta}} + \frac{1}{2}\left(\frac{c-1}{(w-\left(\sum_{k=1}^{\infty}\frac{1}{(1+k^{\alpha})^{\beta}}\right))}\right)\left[\left(\sum_{k=1}^{\infty}\frac{(2k-1)}{(1+k^{\alpha})^{\beta}}\right) - \left(\sum_{k=1}^{\infty}\frac{1}{(1+k^{\alpha})^{\beta}}\right)^2\right]$
Discrete Pareto(β)	$\sum_{k=1}^{\infty}\frac{1}{(1+k)^{\beta}} + \frac{1}{2}\left(\frac{c-1}{(w-\left(\sum_{k=1}^{\infty}\frac{1}{(1+k)^{\beta}}\right))}\right)\left[\left(\sum_{k=1}^{\infty}\frac{(2k-1)}{(1+k)^{\beta}}\right) - \left(\sum_{k=1}^{\infty}\frac{1}{(1+k)^{\beta}}\right)^2\right]$

The following figures illustrate the tendency of maximum premium valuesP_{max} for different discrete version of continuous loss distributions with different parametric values.

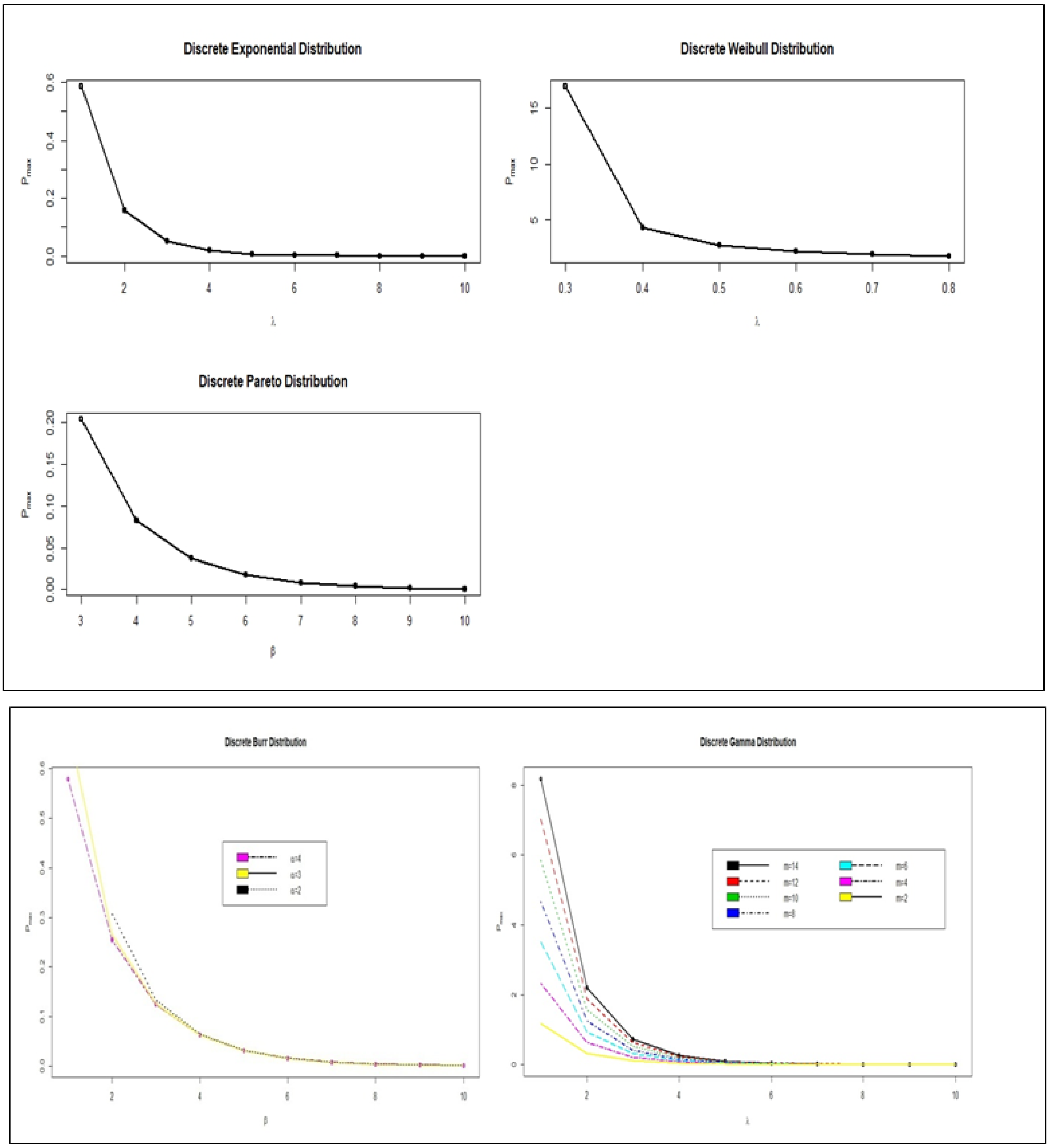


Fig.3(a)-(e) : Maximum premium values for initial wealth$w = 100$ and $0 < c < 1$

If the insured people assume a risk aversion function of factional power utility function, from the above graphs we conclude the following

1. If the insured's loss follows discrete Exponential, Pareto, Gamma and Burr distributions. The maximum premium value declines as the parameter enlarges and later stabilizes as the parameter value enlarged more. This is good from insured point of view.
2. For the discrete Weibull distribution, there is a presence of high premium values (more than the premium values if we use utility function) at the smaller values of λ and later it stabilizes as the parameter value enlarges. It is also advantageous to the insured peoples.

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