

## Calculation and visualization of inductances and magnetic force of a coil iron nucleus

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**Abstract:** The paper describes the investigation and development of a structure and performance characteristics of a coil iron nucleus cylindrical (C.I.N.C).

The coil iron nucleus cylindrical is a nonlinear electro radio in which the moving of the nucleus in a sense or in other causes change in inductance and can reach extreme values at the superposition of nucleus and coil centers. The variation of the inductance and the degree of freedom of movement of the nucleus can lead to a device with electromechanical conversion. The aim of this paper is the determination and visualization of self inductance and mutual of the (C.I.N.C) based on geometric dimensions and the displacement of the nucleus.

**Keywords:** Coil, iron, nucleus cylindrical, linear motor, inductance.

### I. INTRODUCTION

The cylindrical coil with iron nucleus can be explored as a tubular linear motor oscillation (C.I.N.C) [3]. This type of engine is based on the periodic variation of the inductance, which causes the change in the behaviour of the electrical circuit with the existence of a series capacitance.

The C.I.N.C with mobile induct exists in two disposition, vertical and horizontal [4],[5].

The stationary part consists of a solenoid which forms part of the resonant RLC circuit. The ferromagnetic cylinder inserted in the solenoid constitutes the moving part.

Parametric electromechanical elements, comprehend a class of single machine which present certain common characteristics [6], [7]. The process of energy conversion is similar to some type of asynchronous motor [1]. Power conversion is due to a regular change of the parameters, that is why is called parametric machines. The functioning of this type of machine is based on the application of electromechanical oscillations. [7] and [8].

### II. Inductances as function of core displacement

The ferromagnetic nucleus core is integral part which reflects the position or displacement. This nucleus is pushed to a variable length  $l_f$  in a coil of  $N$  turns uniformly distributed over a length 'l' as shown by fig II.1

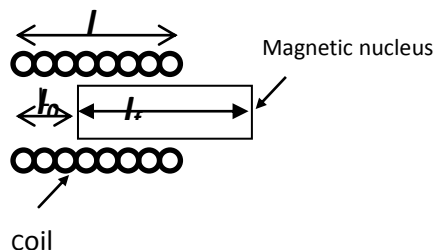


Fig II.1 Diagram of a coil with plunger

The self-induction coil coefficient  $L$  depends on the penetration of the nucleus. The calculation of  $L$  is done by considering the inductor as the association of an inductance of the air length  $l_0$ , with a self-induction coefficient  $L_0$  in series with an inductance iron nucleus of length  $l_f$ , with self-induction coefficient of  $L_f$  and a mutual  $M$ .

This inductance is expressed as:

$$L = L_0 + L_f + 2M \quad (1)$$

With:

$$M = K \sqrt{L_0 \cdot L_f} \quad (2)$$

$k$  is the coupling coefficient which is assumed constant ( $0 \leq k \leq 1$ ) Depending on various characteristics of the circuit we can write:

$$L_0 = \mu_0 \cdot \frac{N^2}{l^2} S_0 \cdot l_0 = \mu_0 \frac{N^2}{L^2} S(l-l_f) \quad (3)$$

$$L_f = \mu_0 \cdot \frac{N^2}{l^2} [S_0 + (\mu_f - 1)S_f] l_f \quad (4)$$

The expression of L is then:

$$L = \mu_0 \cdot \frac{N^2}{l^2} (S_0 l + (\mu_0 - 1) \cdot S_0 l + 2k \sqrt{S_0 [S_0 + (\mu_p - 1)]} \cdot \sqrt{(l-l_f) \cdot l_f})$$

The displacement  $\Delta l_f$  of the nucleus, leads to a variation  $\Delta L$  of the inductance, which depends on  $l_f$  which is a nonlinear function of  $\Delta l_f$  as in the case of air gap variable inductance of  $L_f$ . [2].

### III. Inductances equations development

To determine the expression (1) we need to calculate the inductances  $L_0$ ,  $L_f$  and  $M$ :

#### III.1. Inductance without nucleus

In this case the expression of the inductance is given by:

$$L_0 = \mu_0 \frac{N^2}{l} S_0 \cdot \frac{l_0}{l} = \mu_0 \frac{N^2}{l} \cdot \frac{S_0(l-l_f)}{l} \quad (6)$$

Where:  $l = l_0 + l_f$

$S_0$  is the vacuum section

$$l_0 = \mu_0 \frac{N^2}{l} S_0 - \frac{\mu_0 N^2 S_0 l_f}{l} \quad (7)$$

We have:

$$L_\infty = \mu_0 \cdot \pi \cdot r^2 \cdot n^2 \cdot l \quad (8)$$

With:  $n = \frac{N}{l}$

$$\Rightarrow L_\infty = \frac{\mu_0 \cdot N^2 S_0}{l^2} \cdot l = \frac{\mu_0 \cdot N^2 \cdot S_0}{l} \quad (9)$$

So:

$$L_0 = L_\infty - L_\infty \cdot \frac{l_f}{l} = L_\infty \cdot (1 - \frac{l_f}{l}) \quad (10)$$

$l_f$  is a function of displacement  $x$ , to be determined later.

We set:  $a = L_\infty$

So:  $b = \frac{L_\infty}{l}$

$$L_0 = a - b \cdot l_f \quad (11)$$

Inductance of the nucleus

$$L_f = \frac{\mu_0 \cdot N^2}{l} [S_0 + (\mu_f - 1) \cdot S_f] \cdot \frac{l_f}{l} \quad (12)$$

With:

$$L_f = \frac{L_\infty}{S_0} [S_0 + (\mu_f - 1) \cdot S_f] \cdot \frac{l_f}{l} \quad (13)$$

$$L_f = \frac{L_\infty}{S_0} l_f + L_\infty \cdot \frac{S_f}{S_0 \cdot l} (\mu_f - 1) \cdot l_f \quad (14)$$

We set: 
$$c = b \cdot \frac{S_f}{S_0} \cdot (\mu_f - 1)$$

And we replace by the constant (b) we have:

$$L_f = (b + c) \cdot I_f \tag{15}$$

**III.2. Mutual inductance**

In equation (2) we replace the expressions (10) and (14) we will have:

$$M = \frac{L_\infty}{S_0} \cdot \frac{k}{l} \sqrt{S_0[S_0 + (\mu_f - 1) \cdot S_f] \cdot (l - l_f) \cdot I_f} \tag{16}$$

We set: 
$$d = S_0[S_0 + (\mu_f - 1) \cdot S_f]$$

And we replace the constant (b) we have:

$$M = \frac{k}{S_0} \cdot b \cdot \sqrt{d \cdot (l - l_f) \cdot I_f} \tag{17}$$

**III.3. Total inductance (coil over nucleus)**

The total inductance of the coil is determined by taking the expressions of the different inductances obtained before

$$L = L_\infty + \frac{L_\infty S_f}{S_0 l} (\mu_f - 1) I_f + \frac{L_\infty 2k}{S_0 l} \sqrt{S_0 \{S_0 + (\mu_f - 1) S_f\} (l - l_f) I_f} \tag{18}$$

The inductance expressed as a function of the constants (a, b, c and d) is:

$$L = a + c \cdot I_f + \frac{2k}{S_0} \cdot b \cdot \sqrt{d \cdot (l - l_f) \cdot I_f} \tag{19}$$

**IV. COIL PARAMETERS**

**IV.1. Coil without iron nucleus**

Under variable tension of 50 Hz frequency, the parameters indicated in table IV.1 are measured by considering the circuit indicated in fig IV.1

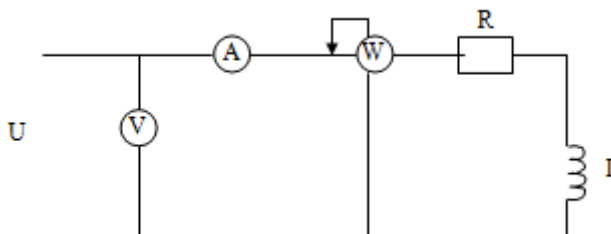


Fig IV.1 Practical measuring circuit

Table IV.1

Measured parameters

$U_L$ (v)	$I$ (A)	$Z$
9.5	0.45	21.1
13.0	0.6	22
15.6	0.7	22.28
18.3	0.8	22.87

The inductive reactance:  $X_0 = \sqrt{Z^2 - R^2} = L \cdot \omega$ ;

Inductance:  $L_0 = X_0 / \omega$ ;

The impedance:  $Z_{0 \text{ average}} = 22 \Omega$  ;

The internal resistance of the coil:  $R = 10.55 \Omega$ ;

The inductance of the coil without its nucleus  $L_0 = 0.06$  H.

The internal resistance of the coil without nucleus can be determined by using the same circuit feed by direct current.

**IV.2. Coil with iron's nucleus**

Before performing the test, we must provide the core inside the coil and make sure it is centered and gradually feeding the circuit with an alternative tension of 50Hz frequency. The indications of measuring devices are shown in table IV.2

**Table IV.2**  
The indications of measuring

$U_L$ (v)	I(A)	P(w)	$I_r$	$U_r$	$U_L$	Z
16,7	0.05	0.83	0.0025	332	-	334
35	0.1	0.95	0.01	95	-	350
53	0.15	1.05	0.0225	46.67	25.11	353.33
96	0.3	2.06	0.09	22.89	93.23	320
150	0.48	4.32	0.2304	18.75	148.82	312.5
225	0.7	9.45	0.49	19.3	224.17	321.43

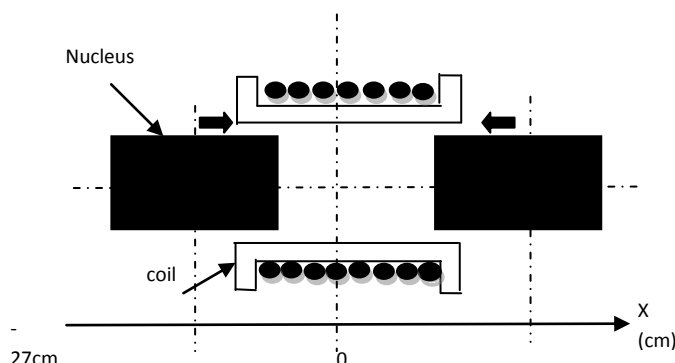
with:  $Z_{average}=332\Omega$  ,  $R=10,55\Omega$  ,  $L=1.116H$

**IV.3. Inductance Variation in function of nucleus position**

The coil with iron's nucleus is characterized by the variation of its inductance in function of the movement of the nucleus inside the coil. Taking reference as the superposition of the coil's and the nucleus's centre as indicated in figure IV.2.

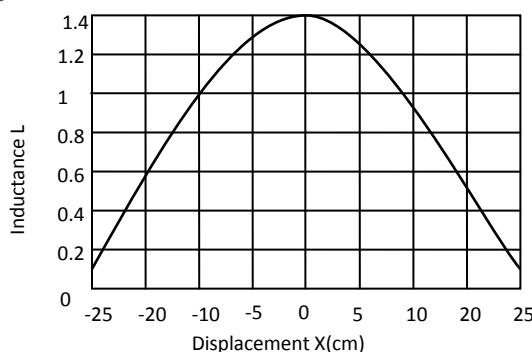
The manually movement of the nucleus along the axis X passes through 03 zones. For the left and right areas, the coil is without nucleus and the inductance takes a minimum value.

However for the intermediate area the nucleus occupied gradually the total of the coil to pursue the air that it contains and in this case the inductance becomes max.



**Fig IV.2** Representative patterns of the core displacement.

The experiment results taken under a constant tension of  $U_{eff} = 50V$  with the assumption that the resistance stay constant, are shown by fig IV.3



**Fig IV.3** Variation of the inductance L in function of displacement X

**IV.4. Determination of the length of the iron as function of the displacement**

The expressions [(11), (15), (17), (18)], of this article are function of length  $l_0$  and  $l_f$  inside the coil which are also function of x

The superposition of the two curves  $l_f = f(x)$  and  $l_0 = g(x)$  presented in fig IV.4 shows the evolution of the gradual penetration of the nucleus into the coil by chasing the air in it.

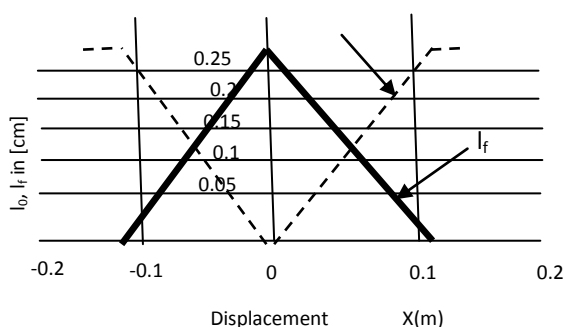


Fig IV.4 Superposition of curves  $l_0$   $l_f$

The mathematics expressions of these two functions are determined by interpolation giving:

$$l_f = 0.2e^{(-200x^2)} \quad (20)$$

And

$$l_0 = l - (0.2e^{(-200x^2)}) \quad (21)$$

From equations governing the behavior of the coil with and without a nucleus, a MATLAB program was developed for the calculation of inductances  $L_0$ ,  $L_f$  and  $M$ , according to the dimensions of the coil and core taking into account the varying displacement of the core in the coil.

### V. Inductance of the coil with iron's nucleus (nucleus center)

An inductor with iron nucleus consists of a coil with 3000 spires and a length  $l$  with negligible resistance.

The core is composed of sheets of very good quality with a coefficient equal to 0.9 and with a length  $l_n$  as indicated in table V.1

The coil is feed with a tension of 115V and a frequency  $f = 50\text{Hz}$ . It is assumed that the reluctance of the iron  $\mathcal{R}$  is negligible compared to that of air, the latter is equivalent to a reluctance of a gap with Length  $e$ .

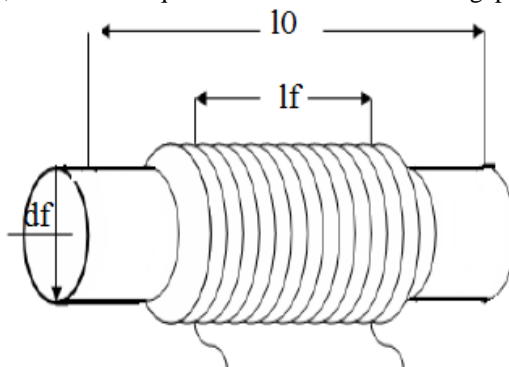


Fig V.1 coil iron nucleus.

The section of the nucleus is given by:

$$S_f = \frac{\pi \cdot d_f^2}{4} \quad (22)$$

The section of the useful iron is:

$$S_{uti} = S_f \cdot k_{uti} \quad (23)$$

The induction  $B$  is determined by the formula of KAPP:

$$B = \frac{U_{ef}}{4.4N \cdot f \cdot S_{uti}} \quad (24)$$

It can also be calculated by the formula of Hopkinson.

The length of the gap is fictional:

$$e = 0.28 \cdot \sqrt{\frac{S_{uti} \cdot I}{l_n}} \quad (25)$$

The volume of air to magnetize:

$$V = e \cdot S_{uti} \quad (26)$$

The reactive power needed for the gap:

$$Q = \frac{B^2 \cdot \omega \cdot V}{2.5} \cdot 10^6 \quad (27)$$

The active power is low because f and B are weak.

The fictional absorbed current is:

$$I_r = \frac{Q}{U_{ef}} \quad (28)$$

This current can be considered as total current, which can also be determined by applying the formula of Hopkinson.

$$NI_{ef} = \mathfrak{R} \cdot \phi \quad (29)$$

So:

$$I_{ef} = \frac{\mathfrak{R} \cdot \phi_m}{N \cdot \sqrt{2}} = \frac{1}{S} \left( \frac{l_n}{\mu_0 \mu_r} + \frac{e}{\mu_0} \right) \cdot \frac{\phi_m}{N \cdot \sqrt{2}} \quad (30)$$

With:

$$\frac{l}{\mu_0 \cdot \mu_r} \approx 0 \quad (31)$$

The impedance of the circuit is:

$$Z = L \cdot \omega = \frac{U_{ef}}{I_{ef}} \quad (32)$$

When the inductance of the coil is:

$$L = \frac{Z}{\omega} \quad (33)$$

A MATLAB program has been designed to perform calculations of inductance for a coil with centered nucleus.

## VI. The coil characteristics

The characteristics of the coil considered are shown in the table VI.1, which summarizes all the electrical and geometrical dimensions.

**Table VI.1**  
Electrical and geometrical dimensions of C.I.N.C

Elements	Dimensional	Symbols	Values
Coil	Length	l	22.0[cm]
	Outside diameter	d	6.4[cm]
	Inside diameter	d <sub>0</sub>	3.0[cm]
	Conductor diameter	d	1 [mm]
	Inductance Without core	L <sub>0</sub>	0.04 [H]
	nucleus inductor	L	1.1[H]
	internal resistance	R	10 [Ω]
Nucleus	Length	l <sub>r</sub>	26[cm]
	Diameter	d <sub>r</sub>	2.9 [cm]
	Mass	m	0.65[mm]
Air gap	Thickness	e <sub>t</sub>	0.1 [mm]

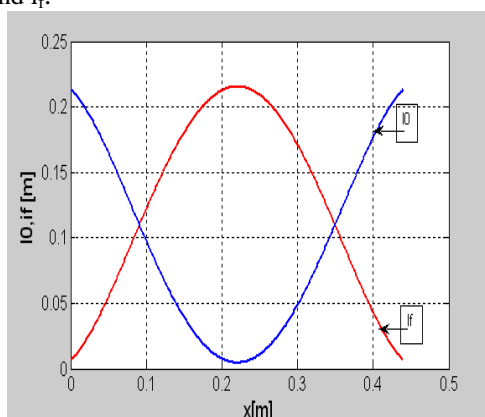
## VII. Calculation and visualization of inductors according to the position of the nucleus

The results of simulation of the inductances L<sub>0</sub>, L<sub>r</sub>, M and L in function of displacement (x) of the nucleus affected under MATLAB program, are giving by the following figures:

FigVII.1 represents the variation of the nucleus position inside the coil. We note that during the penetration of iron in the coil and chasing air, the length of it increases gradually to its maximum value which

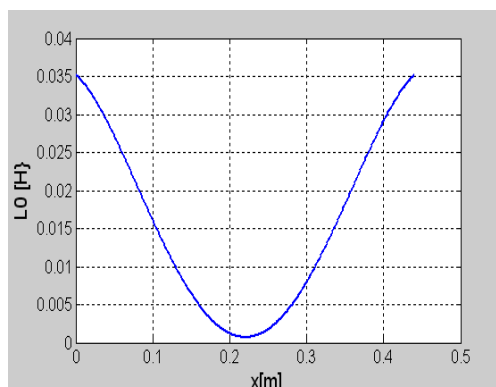
corresponds to the total occupation, then a regression of the curve that corresponds to t progressive removal core, knowing that the length of the coil  $l = l_0 + l_f$ .

The curve of evolution of the air inside the coil is represented by Fig VII.1 and on which are superimposed the two curves  $l_0$  and  $l_f$ .



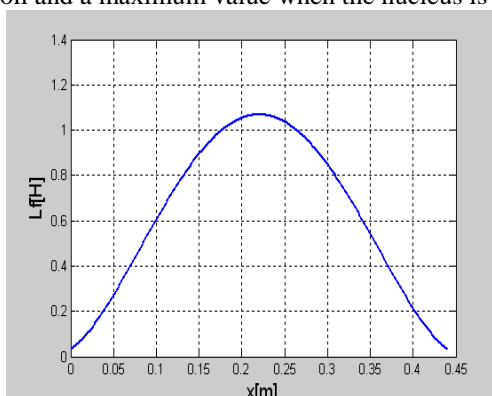
FigVII.1 Superposition of  $l_0 = f(x)$  and  $l_f = g(x)$

Fig VII.2 represents the inductance of the coil with movement of the nucleus in function of the nucleus position inside the coil. When the nucleus is in center,  $L_0$  pass by its minimum and it takes again its proper value when it is completely clear.



FigVII.2 Inductance of the air in function of the nucleus position

The fig VII.3 represents the inductance of iron in function of the position of the nucleus. It takes minimum values outside of the coil and a maximum value when the nucleus is in central.



FigVII.3 Inductance of iron in function of the position of the nucleus

FigVII.4 shows the variation of mutual inductance in function of its position inside the coil and takes the most important values almost to this of the total inductance of the coil.

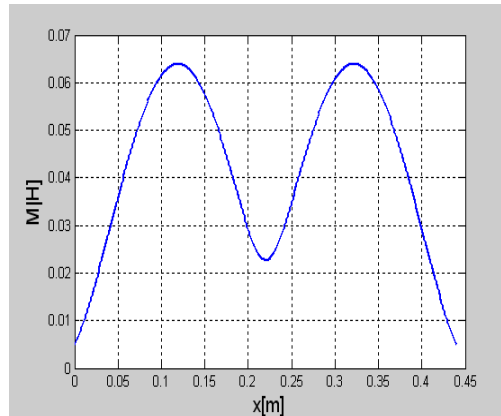


Fig VII.4 Variation of the mutual-inductance in function of (x)

The total inductance of the coil is represented by curve VII.5, we notice that it is affected by the presence of the nucleus and it takes a maximum value equal to 1.1Henry.

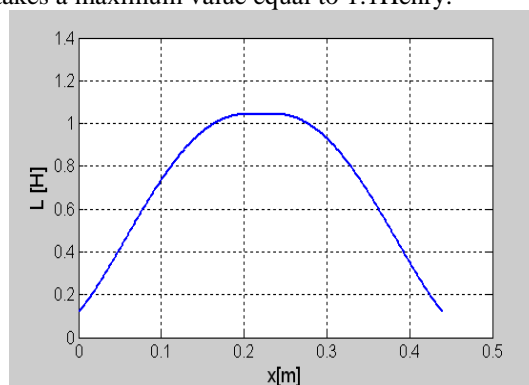
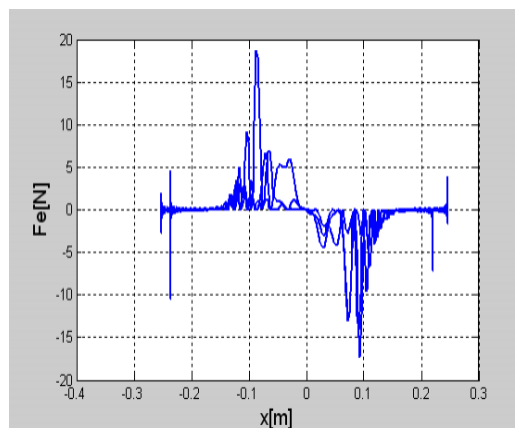


Fig VII.5 Variation of the total inductance in function of (x)

The curve FigVII.6 of the magnetic force given by the mathematical software (sigma plot9) used in our research shows that it depends on the displacement x following the shape of the derivative of the inductance L, which is a very delicate operation, so it is simply derived inductance:  $F_e = 0.5 \cdot i^2 \cdot (dL / dx)$ .



FigVII.6 Variation of the force in function of (x)

### VIII. CONCLUSION

The elaborate work have give a satisfactory results, the inductances determined can be exploit for any eventual simulation of a tubular linear motor all we exploit the equations of KIRCHOFF in level of the inductor (coil) and induces (nucleus) to model the (C.I.N.C) .

The object of our study is to determine the inductance of the motor linear oscillations (C.I.N.C) which led eventually to easily find the magnetic force that governs the operation of the engine, We note also that here the nucleus comes in the middle of the coil force changes direction and the ring returns to its original position causing the oscillation.



We can treat the problem of ferro-resonance which is a very interesting subject to study this type of motor. The results of the simulation, presented in the paper, illustrate the performance of the (C.I.N.C).

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