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Design Of Compensator Using Moment Matching Algorithm

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Abstract
A novel method of model reduction was created for stable linear dynamic systems on a vast scale. The authors have tried to preserve time moments and prevailing modes. Through this reduction, the overall significant properties contained in the large-scale complete order model are translated into the lower order system, enabling the generalized pole clustering approach to be used to compute the approximate denominator. The factor division procedure is used to produce the approximate numerator. This leads to the creation of a lower order system. Comparison research is carried out to show its efficacy, to emphasize some of its key characteristics, and to achieve its correctness. The approximate model identified by the suggested method is compared with the reduced order models computed from the recently proposed methods and well-known model reduction schemes in two standard numerical instances. The design of the compensator employing the moment matching algorithm, and the simplified model is highlighted as well in the study. A classic numerical example from the literature is used to validate and illustrate the compensator design.

Keywords: *Moment Matching Algorithm, Controller, dominating modes, large-scale systems, model order reduction (MOR), and pole clustering.*

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I. Introduction

These days, a variety of intricate, multidimensional systems are employed in societal, technological, and environmental activities. When a system's dimensions are so great that traditional research, modeling, control, design, and computation techniques are unable to produce acceptable results with practical computational efforts, the system is referred to as huge scale. Lower dimensions systems that are comparable to the large dimensional system are needed to solve such an issue. For engineers, then, lower order systems are highly preferred in the analysis, synthesis, and simulation of large-scale systems due to their ability to reduce costs, design time, and simplify implementation. The technique of distinguishing a lower dimensional system from a higher dimensional system is known as model order reduction, or MOR. The key components of the prior system are still present in the lower dimensional system in MOR. In several scientific and technical disciplines, the MOR is often used to simplify higher dimensional real time systems [7, 10, 23, 28, 38]. The model reduction was first proposed in the frequency domain large-scale system simulation to lower the order of the transfer function of linear dynamic systems [6]. In this frequency range, the stability equation [4, 5], the Padé approximation [42], the Routh approximation [19, 20], and the Routh stability [60]. These methods have certain drawbacks, such as the possibility that the Padé approximation will result in an unstable lower order system even while the underlying system is stable [32]. These methods have certain drawbacks, such as the possibility that the Padé approximation will result in an unstable lower order system even while the underlying system is stable [32]. According to [20], the Routh approximation method is unable to minimize the order of the non-strictly transfer function. According to [44], failure of the Routh-Hurwitz method of reduction the Routh stability approach. The Routh approximation approach is not able to minimize the order of the non-strictly transfer function, according to [20]. It is found that [40], the Routh stability technique does not preserve the dominant poles in non-minimum phase systems. For non-minimum phase systems, large-scale systems cannot be made simpler using the stability equation technique [40]. [17], [32-36], [1]. To simplify high dimensional big scale systems in the temporal domain, several model diminution strategies are put forth [2, 3, 15, 40, 55]. Of them, balanced realization is the most often applied technique for linear time-invariant system simplification on a large scale. It was first suggested for linear time invariant model reduction. The observability and controllability Grampian's are utilized to determine less observable and controllable states. This strategy truncates the least controllable and observable states to determine the reduced model. This approach occasionally results in a smaller model whose steady state response does not match the response of the whole model. Several MOR techniques are suggested to address this issue [13, 11, 35]. One of the most popular MOR techniques for the simplification of large-scale dimensions linear systems among these approaches is the pole clustering technique [50]. This method clusters the complicated original system's poles (zeros), then uses the centers of those clusters to compute. By preserving the dominant poles of the original system in the reduced model, this strategy guarantees the stability of the reduced model if the original system is stable. Nevertheless, there are a few drawbacks to this approach as well. For example, additional simulation time and mathematical calculations are required to determine the tuning factor and gain adjustment factor, which are necessary to match the temporal responses of the higher order original system and lower order systems appropriately. Several model reduction techniques based on the pole clustering technique have been developed to overcome these restrictions [46, 48, 53]. This work proposes a new, straightforward, and programmable model reduction technique that avoids the pole clustering method's drawbacks. This approach uses the factor division method to derive the numerator polynomial and the generalized pole clustering method to compute the denominator of the simplified model [33, 47]. In the reduced order model, this technique guarantees the retention of the original system's stability and initial time moments.

II. Problem Statement

An nth-order transfer function of higher order SISO linear time invariant (LTI) system is considered as

$$G(s) = \frac{N(s)}{D(s)} = \frac{d_0+d_1 s+\cdots+d_{n-1}s^{n-1}}{e_0+e_1 s+e_2 s^2+\cdots+e_n s^n} \quad (1)$$

The objective of the paper is to compute the unknown parameters of rth-order ($r < n$) reduced model and its nature and performance are approximately the same as the original system and it is defined as the following transfer function

$$R_r(s) = \frac{Q_r(s)}{P_r(s)} = \frac{q_0+q_1 s+q_2 s^2+\cdots+q_{r-1}s^{r-1}}{p_0+p_1 s+p_2 s^2+\cdots+p_{r-1}s^{r-1}+p_r s^r} \quad (2)$$

The transfer matrix of multi-input multi output (MIMO) system linear time-invariant (LTI) system is defined as below

$$[G(s)] = \frac{1}{D(s)}\begin{bmatrix} a_{11}(s) & a_{12}(s) & \cdots & a_{1u}(s) \\ a_{21}(s) & a_{22}(s) & \cdots & a_{2u}(s) \\ \vdots & \vdots & \vdots & \vdots \\ a_{v1}(s) & a_{v2}(s) & \cdots & a_{vu}(s) \end{bmatrix} \quad (3)$$

$$= [g_{ij}(s)]_{v\times u} \quad (4)$$

where $i = 1, 2, 3, \ldots, v;\ j = 1, 2, 3, \ldots, u,$ and u and v are the inputs and outputs of original system respectively. The $g_{ij}(s)$ can be written as

$$g_{ij}(s) = \frac{a_{ij}(s)}{D(s)} \quad (5)$$

This contribution's goal is to compute the original system's estimated model while maintaining all the system's key characteristics in the comparable model. The approximated model's transfer matrix is specified as

$$[R_r(s)] = \frac{1}{P_r(s)}\begin{bmatrix} b_{11}(s) & b_{12}(s) & \cdots & b_{1u}(s) \\ b_{21}(s) & b_{22}(s) & \cdots & b_{2u}(s) \\ \vdots & \vdots & \vdots & \vdots \\ b_{v1}(s) & b_{v2}(s) & \cdots & b_{vu}(s) \end{bmatrix} \quad (6)$$

$$= [r_{ij}(s)]_{v\times u} \quad (7)$$

where $i = 1, 2, 3, \ldots, v;\ j = 1, 2, 3, \ldots, u$. Hence $r_{ij}(s)$ can be defined as

$$r_{ij}(s) = \frac{b_{ij}(s)}{P_r(s)} \quad (8)$$

The elements that make up the reduced transfer function matrix r_{ij} (s) and the real transfer function matrix $g_{ij}(s)$) are different. For the same kind of input, the predicted reduced model responds about the same as the original system.

Procedures for Proposed Method

The two primary processes for determining the lower order system in the suggested technique are as follows. By identifying the centers of these clusters and the clusters of poles of the higher order original system, the denominator polynomial of the approximated system is obtained. The steps taken to create the clusters and cluster centers are listed below.

Determination of denominator polynomial

Any dynamic system's temporal and frequency responses are contingent upon the quantity, location, and density (the number of poles inside a given region) of poles. Most MOR approaches directly ignore the poles of higher order systems that are distant from the imaginary axis [36, 37, 39, 59]. In the suggested method, the dominant poles are calculated by adding the poles that are farthest from the s-plane origin to determine the

lower order system. The poles of the higher order system are arranged into 'r' numbers of groups in the r^{th} order reduced system. A group's pole shows its influence on the cluster center of that group. Based on their dominance - that is, their relative significant distance from the s-plane origin, the original systems' poles are arranged in clusters. The principles listed below are used to cluster the poles.
1) The clustering of real and complex poles is different.
2) Separate clusters ought to be set up for the poles located in the left and right halves of the s-plane.
3) The lower order model preserves the poles located at the origin of the s-plane and along the vertical axis.

Clustering of real poles
Examine the following transfer function for the original system of equation (1) in its pole-zero form.

$$G_p(s) = \frac{N(s)}{D(s)}$$

Where,

$$N(s) = (s + z_1)(s + z_2) \ldots (s + z_{r-1})(s + z_r)(s + z_{r+1}) \ldots (s + z_{2r-1})(s + z_{2r})(s + z_{2r+1})\ldots(s + z_{n-1}) \quad (9)$$

and

$$D(s) = (s + p_1)(s + p_2) \ldots (s + p_{r-1})(s + p_r)(s + p_{r+1}) \ldots (s + p_{2r-1})(s + p_{2r})(s + p_{2r+1})\ldots(s + p_n) \quad (10)$$

Considering that the reduced system has an order of r (r<n), there are r number of clusters of the poles in the r^{th} order reduced system. arranging the poles in the left half of the s-plane in increasing magnitude order as
Pole : $-p_1, -p_2, \ldots, -p_i, \ldots, -p_n \quad \forall \quad |p_i| < |p_{i+1}|$ (11)

The first cluster contains the first pole, and the second cluster contains the second pole. The technique is repeated up to the last pole. In a similar manner, the r^{th} pole is placed in the r^{th} cluster, the $(r+1)^{th}$ pole is placed in the first cluster, the $(r+2)^{th}$ pole is placed in the second cluster, and the $(2r)^{th}$ pole is placed in the r^{th} cluster. Upon identifying every pole within each group, the cluster centers are acquired using the subsequent formula.

$$c_1 = -\left\{\left(\left[\frac{1}{|p_1|^X}\right] + \left[\frac{1}{|p_{r+1}|^X}\right] + \left[\frac{1}{|p_{2r+1}|^X}\right] + \cdots \left[\frac{1}{|p_k|^X}\right]\right)/k\right\}^{-(1/X)} \quad (12)$$

$$c_2 = -\left\{\left(\left[\frac{1}{|p_2|^X}\right] + \left[\frac{1}{|p_{r+2}|^X}\right] + \left[\frac{1}{|p_{2r+2}|^X}\right] + \cdots \left[\frac{1}{|p_l|^X}\right]\right)/l\right\}^{-(1/X)} \quad (13)$$

$$\cdots \quad \cdots \quad \cdots \cdots \quad \cdots$$

$$c_r = -\left\{\left(\left[\frac{1}{|p_r|^X}\right] + \left[\frac{1}{|p_{2r}|^X}\right] + \left[\frac{1}{|p_{3r}|^X}\right] + \cdots \left[\frac{1}{|p_m|^X}\right]\right)/m\right\}^{-(1/X)} \quad (14)$$

where X is the order of the cluster roots and c_1, c_2, ….., c_r are the cluster centers of the poles. Depending on the precision of the lower order system, the value of X can be any natural number. Additionally, k, l, and m represent the number of poles located in clusters 1, 2, and r, respectively. Depending on the poles of the higher order systems, the values of these variables may be the same or differ. The denominator of the lower order system is determined as follows the cluster centers have been established.

$$P_r(s) = (s - c_{1p})(s - c_{2p}) \ldots (s - c_{rp}) \quad (15)$$

The cluster centers in equations (12–14) rely on the poles that are closest to the s-plane origin for a given value of X. As the value of X climbed from more than one, the cluster center moved closer to the dominant pole of the cluster. Therefore, it can be said that the cluster center is dependent on the cluster's dominant pole and that its magnitude is roughly closer to the dominant pole. It is also evident from equations (12–14) that the suggested clustering approach becomes the pole clustering method as defined by [50] when X equals to 1.

Clustering of complex poles
The nth order large scale system with only complex poles has the following denominator polynomial.

$$D(s) = (s + a_{1p} \pm jb_{1p})(s + a_{2p} \pm jb_{2p})\ldots(s + a_{rp/2} \pm jb_{rp/2})\ldots(s + a_{np/2} \pm jb_{np/2}) \quad (16)$$

A pair of complicated pole groups (r/2) is constructed for the r^{th} order reduced system. Like how real poles are arranged in clusters, the complex poles located in the left half of the s-plane are arranged in ascending order and placed in (r/2) pairs of clusters. These clusters' cluster centers are

$$A_{1p} \pm jB_{1p} = -\left\{\left(\left[\frac{1}{|a_{1p}|^X}\right] + \left[\frac{1}{|a_{(r+1)p}|^X}\right] + \left[\frac{1}{|a_{(2r+1p)}|^X}\right] + \cdots + \left[\frac{1}{|a_{kp}|^X}\right]\right)/k\right\}^{-(1/X)} \pm$$

$$\left\{\left(\left[\frac{1}{|b_{1p}|^X}\right]+\left[\frac{1}{|b_{(r+1)p}|^X}\right]+\left[\frac{1}{|b_{(2r+1)p}|^X}\right]+\cdots+\left[\frac{1}{|b_{kp}|^X}\right]\right)/k\right\}^{-(1/X)} \quad (17)$$

$$A_{2p} \pm jB_{2p} = -\left\{\left(\left[\frac{1}{|a_{2p}|^X}\right]+\left[\frac{1}{|a_{(r+2)p}|^X}\right]+\left[\frac{1}{|a_{(2r+2)p}|^X}\right]+\cdots+\left[\frac{1}{|a_{lp}|^X}\right]\right)/l\right\}^{-(1/X)} \pm$$

$$\left\{\left(\left[\frac{1}{|b_{2p}|^X}\right]+\left[\frac{1}{|b_{(r+2)p}|^X}\right]+\left[\frac{1}{|b_{(2r+2)p}|^X}\right]+\cdots+\left[\frac{1}{|b_{lp}|^X}\right]\right)/l\right\}^{-(1/X)} \quad (18)$$

…. …. …

$$A_{rp/2} \pm jB_{rp/2} = -\left\{\left(\left[\frac{1}{|a_{rp/2}|^X}\right]+\left[\frac{1}{|a_{rp}|^X}\right]+\left[\frac{1}{|a_{(3r/2)p}|^X}\right]+\cdots+\left[\frac{1}{|a_{mp}|^X}\right]\right)/m\right\}^{-(1/X)} \pm$$

$$\left\{\left(\left[\frac{1}{|b_{rp/2}|^X}\right]+\left[\frac{1}{|b_{rp}|^X}\right]+\left[\frac{1}{|b_{(3r/2)p}|^X}\right]+\cdots+\left[\frac{1}{|b_{mp}|^X}\right]\right)/m\right\}^{-(1/X)} \quad (19)$$

where k, l, and m are the numbers of complex poles placed in cluster-1, cluster-2, and cluster-r, respectively; X is the order of the clusters' root; and ($A_{rp/2}$+j$B_{rp/2}$) are the cluster centers of the complex poles. Preferably, the value of X is more than one, and it also relies on how accurate the lower order system is supposed to be. Following the cluster centers' computation, the lower order system's denominator is

$$P_r(s) = \left(s - A_{1p} \pm jB_{1p}\right)\left(s - A_{2p} \pm jB_{2p}\right) \ldots \left(s - A_{rp/2} \pm jB_{rp/2}\right) \quad (20)$$

Clustering of real and complex poles

The method described in the preceding technique is used to compute the cluster centers for the real and complex poles independently. The order of the lower order system determines the number of clusters for both real and complex poles. Assuming α cluster centers for real poles and β cluster centers for complex poles in the rth order lower order model, the reduced order system's denominator polynomial is

$$P_r(s) = \prod_{i=1,j=x+1}^{x,y/2}\left(s - c_{ip}\right)\left(s - A_{jp} \pm jB_{jp}\right) \quad | \quad \alpha + \beta = r \quad (21)$$

Obtaining the lower order system's numerator polynomial

The factor division approach is used to find the numerator polynomial of the reduced system [21, 35, 47]. Using this method, first compare the higher order actual system of equation (1)'s transfer function with that of the lower order system of equation (2), as shown below.

$$Q_r(s) = \frac{N(s)}{D(s)} \times P_r(s)$$

$$= \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{r-1}s^{r-1} + \cdots + a_{n+r-1}s^{n+r-1}}{e_0 + e_1 s + e_2 s^2 + \cdots + e_{n-1}s^{n-1} + e_n s^n} \quad (22)$$

The numerator polynomial $Q_r(s)$ is a power series of $\left(\frac{N(s)}{D(s)} \times P_r(s)\right)$ about $s = 0$ and it can be easily calculated by using the moment matching procedure discussed by [19]. This algorithm ensures the retention of initial "r" time moments of the higher order system in the lower order system, and it is defined as [19, 26, 27, 47].

$$q_0 = \frac{a_0}{e_0} \quad \begin{cases} a_0 & a_1 & a_2 & \ldots & a_{r-1} \\ e_0 & e_1 & e_2 & \ldots & e_{r-1} \end{cases}$$

$$q_1 = \frac{k_0}{e_0} \quad \begin{cases} k_0 & k_1 & k_2 & \ldots & k_{r-2} \\ e_0 & e_1 & e_2 & \ldots & e_{r-2} \end{cases}$$

$$q_2 = \frac{l_0}{e_0} \quad \begin{cases} l_0 & l_1 & l_2 & \ldots & l_{r-3} \\ e_0 & e_1 & e_2 & \ldots & e_{r-3} \end{cases}$$

$$\vdots \quad (23)$$

$$q_{r-2} = \frac{u_0}{e_0} \quad \begin{cases} u_0 & u_1 \\ e_0 & e_1 \end{cases}$$

$$q_{r-1} = \frac{v_0}{e_0} \quad \begin{cases} v_0 \\ e_0 \end{cases}$$

where

$$k_i = a_{i+1} - q_0 e_{i+1}, \ i = 0, 1, \ldots, r-2,$$
$$l_i = k_{i+1} - q_1 e_{i+1}, \ i = 0, 1, \ldots, r-3$$
$$\vdots$$
$$v_0 = u_1 - q_{r-2} e_1.$$

The numerator coefficients q_i (i=0, 1, 2, r-1) are calculated by using equation (23). Therefore, it guarantees the preservation of initial "r" time moments of the large-scale model in the rth order reduced model. The numerator polynomial of the reduced order system is commutated by using the factor division method discussed in [19, 21, 27, 33, 47].

III. Method For Design Of Compensator

The controllers design and simulation of the higher order systems is a complicated task. As the system order increases, the cost and complexity of the controller design increased. This difficulty can be circumvented if a “good” lower order model is obtainable for the higher order model and controller design is carried out by using the lower order model. In the case of a higher dimensional dynamic system, huge numbers of sensors are needed for sensing the state variables of large-scale systems for the feedback controller’s design. Due to this, series controllers are preferable over feedback controllers.

Based on the provided real-time system specification, a reference system M(s) is constructed to yield the expected performance so that the reference model's response roughly matches the closed-loop operation of the controlled system with unity feedback. According to [29, 54], the specific algorithm for deriving the reference system from the provided specification is detailed in. Assume that the compensator's transfer function, which provides the necessary closed-loop performance, is given as

$$G_c(s) = \frac{K(1+K_1 s)}{s(1+K_2 s)} \quad (24)$$

To design the controller by using a lower order model, the open loop reference system ($\widetilde{M}(s)$) is obtained from the given (or computed from the given specification) closed loop reference model ($M(s)$)

$$\widetilde{M}(s) = \frac{M(s)}{1-M(s)} \quad (25)$$

The response of the open loop-controlled model is compared to that of the open loop reference system to determine the unknown controller parameters.

$$G_c(s)G(s) = \widetilde{M}(s) \quad (26)$$

$$G_c(s) = \frac{\widetilde{M}(s)}{G(s)} = \frac{\sum_{i=0}^{2} e_i s^i}{s} \quad (27)$$

The power series expansion coefficients about s=0 is denoted by e_i (i=0, 1, 2), which are derived using the moment generating technique described by [19]. G(s) represents the transfer function of the original plant, and it can be swapped out with a well-approximated reduced order system to cut down on simulation time and mathematical calculations during controller design. Equations (24) and (27) can be compared to determine the unknown controller parameters.

$$\frac{K(1+K_1 s)}{s(1+K_2 s)} = \frac{e_0+e_1 s+e_2 s^2}{s} \quad (28)$$

By solving equation (28), the controller with the necessary structure can be obtained. Once the controller settings have been acquired, the closed loop transfer function can be acquired as

$$G_{cl}(s) = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} \quad (29)$$

Numerical experiments

In order to compare the effectiveness of different model reduction techniques, the integral square error (ISE), relative integral square error (RISE), integral absolute error (IAE), and integral time weighted absolute error (ITAE) are defined as the following error indices that are computed between the transient portions of the actual and reduced models (Prajapati and Prasad, 2018d; Sikander and Prasad, 2015; Tiwari and Kaur, 2018).

$$\begin{cases} ISE = \int_0^\infty [y(t) - y_r(t)]^2 dt \\ RISE = \int_0^\infty [y(t) - y_r(t)]^2 dt \,/ \int_0^\infty [\hat{y}(t)]^2 dt \end{cases} \quad (30)$$

$$\begin{cases} IAE = \int_0^\infty |y(t) - y_r(t)| dt \\ ITAE = \int_0^\infty t|y(t) - y_r(t)| dt \end{cases} \quad (31)$$

Where $\hat{y}(t)$ is the original system's impulse response and y(t) and $y_r(t)$ are the unit step responses of the higher order and reduced order systems, respectively. These error indices are computed for several reduced systems that are obtained using the suggested technique in addition to other well-known MOR approaches that may be found in the literature.

Example 1: In this SISO sixth-order system, the proposed MOR method is illustrated, and this system has been recently considered by different researchers (Jamshidi, 1983; Soloklo and Farsangi, 2015; Tiwari and Kaur, 2018).

$$G(s) = \frac{2s^5+3s^4+16s^3+20s^2+8s+1}{2s^6+33.6s^5+155.94s^4+209.46s^3+102.42s^2+18.3s+1} \quad (32)$$

$G(s)$ can be written in terms of time moments by Taylor series expansion about $s = 0$ as

$$G(s) = 1 - 10.3s + 106.1s^2 - 1079.6s^3 + \cdots \quad (33)$$

To obtain a second order reduced order system, poles of the original system are grouped into two clusters. The original system has six poles as below:

$$-0.1, -0.2, -0.5, -1, -5, -10$$

These poles are placed into two clusters based on their dominancy

Cluster-1 has poles： $-0.1, -0.5, -5$

Cluster-2 has poles: $-0.2, -1, -10.$

The cluster centers of these groups are obtained by using the formula discussed in (12)– (14) as

$$c_{1p} = -\left\{\left(\left[\frac{1}{(0.1)^X}\right] + \left[\frac{1}{(0.5)^X}\right] + \left[\frac{1}{(5)^X}\right]\right)/3\right\}^{-(1/X)}$$

$$c_{2p} = -\left\{\left(\left[\frac{1}{(0.2)^X}\right] + \left[\frac{1}{(1)^X}\right] + \left[\frac{1}{(10)^X}\right]\right)/3\right\}^{-(1/X)}$$

By using the above cluster centers, denominator of the second order reduced system is obtained as

$$D_2(s) = (s - c_{1p})(s - c_{2p}) \quad (34)$$

By using the different values of X, different denominator polynomials of lower order system will be obtained after that the numerator polynomial determined by using the factor division scheme. The lower order system calculated by the proposed method with $X = 50$, is

$$R_2(s) = \frac{0.0913s+0.0209}{s^2+0.3066s+0.0209} \quad (35)$$

$$= 1 - 10.3s + 103.3s^2 - 1022.1s^3 + \cdots \quad (36)$$

The derived reduced order model from equations (33) and (36) maintains the full order system's first "r" (r=2, or the order of the reduced order system) time moments. As a result, the suggested method maintains the factor division technique's basic function.

The original and lower order model's time responses are shown in Figure 1. The given technique yields a model whose reaction is closely matched to that of the entire order system. The frequency responses of the reduced and original models are displayed in Figure 2, and the suggested method's response is almost the same as that of several common model reduction techniques [9, 22, 60]. Table 1 presents the quantitative comparison between the original system and second order reduced models in terms of different performance error indices. It is also clearly demonstrated that the performance error indices obtained by the suggested approach are least as compared to the well-known model reduction methods. It is also evident, though, that the suggested method does not provide the least error indices for X=10, but it does for X=50. Therefore, when X is increased, the error indices will decrease, and the resultant model's time response will converge toward the original system's response.

Table 1. Quantitative analysis of various MOR Techniques with respect to ISE, RISE, IAE and ITAE.

System reduction method	Reduced order model	ISE	RISE	IAE	ITAE
Kumar and Tiwari (2012), Vishwakarma and Prasad (2008)	$\frac{-0.5076s + 0.1209}{s^2 + 0.7377s + 0.1209}$	12.653	8.5517	52.0168	689.369
Kumar, Nagar and Tiwari (2013)	$\frac{1512s + 360}{2458s^2 + 2196s + 360}$	12.0636	8.1533	49.9090	657.9140
Gutman (1982)	$\frac{576s + 360}{2458s^2 + 2196s + 360}$	11.4012	7.7057	58.0571	853.9372
Gu (2005)	$\frac{0.0492s + 0.0896}{s^2 + 0.9811s + 0.09526}$	2.4924	1.6845	47.8283	2741.6
Moore (1981), Safonov and Chiang (1989)	$\frac{0.0961s + 0.0042}{s^2 + 0.1342s + 0.0046}$	2.3754	1.6054	40.2052	2777.9
Komarasamy, Albhonso and Gurusamy (2011), Sikander and Prasad (2017), Vishwakarma (2011),	$\frac{0.0105s + 0.0404}{s^2 + 0.4266s + 0.0404}$	1.0487	0.7087	17.5677	305.0096
Shamash (1975), Singh, Prasad and Gupta (2006)	$\frac{5.6402s + 1}{87.3752s^2 + 15.9402s + 1}$	0.7712	0.5212	18.1392	429.4817
Krishnamurthy and Seshadri (1978)	$\frac{7.1064s + 1}{87.3752s^2 + 15.9402s + 1}$	0.6967	0.4708	18.6800	543.6731
Huang (2013)	$\frac{0.0961s + 0.0046}{s^2 + 0.1342s + 0.0046}$	0.6091	0.4116	20.4007	772.3096
Shamash (1981)	$\frac{8s + 1}{102.42s^2 + 18.3s + 1}$	0.2871	0.1941	11.2768	284.5266
Tiwari and Kaur (2018)	$\frac{-0.4809s + 0.6396}{s^2 + 6.1070s + 0.6393}$	0.2700	0.1825	8.0013	136.8471
Chen and Chang (1979), Chen, Chang and Han (1980), Sikander and Prasad (2015)	$\frac{8s + 1}{101.0101s^2 + 18.3s + 1}$	0.2506	0.1694	10.4825	263.2632
Prajapati and Prasad (2018d)	$\frac{0.0868s + 0.0046}{s^2 + 0.1342s + 0.0046}$	0.1487	0.1005	9.1923	302.7152

Proposed method with X=10	$\frac{0.0783s + 0.0249}{s^2 + 0.3348s + 0.0249}$	0.1023	0.0691	5.5975	105.6542
Prajapati and Prasad (2018b, 2018c), Prasad (200)	$\frac{0.0880s + 0.011}{s^2 + 0.2012s + 0.011}$	0.0659	0.0445	5.5113	156.9259
Hutton and Friedland (1975)	$\frac{0.0879s + 0.011}{s^2 + 0.2012s + 0.011}$	0.0626	0.0423	4.8481	114.097
Proposed method with X=50	$\frac{0.0913s + 0.0209}{s^2 + 0.3066s + 0.0209}$	0.0210	0.0142	2.2612	37.7357

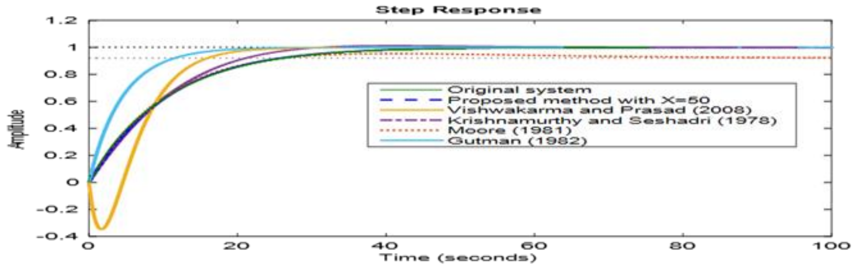


Figure 1: Comparative analysis of time responses between complete order and lower order systems

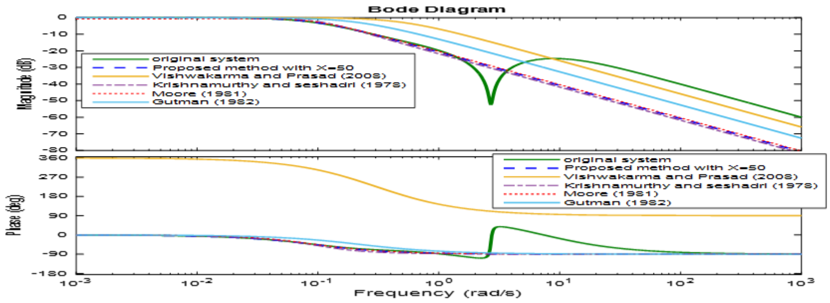


Figure 2. Comparison of frequency responses between complete order and lower order models

Example 2: Now consider a MIMO sixth-order system which has been considered by several researchers (Prajapati and Prasad, 2018e; Sikander and Prasad, 2015; Tiwari and Kaur, 2018) as below:

$$G(s) = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \quad (35)$$

$$= \frac{1}{D(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix}$$

Where,

$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

and

$$A_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$A_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$A_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$A_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

The complete order system has the poles as : $-1, -2, -3, -5, -10, -20$ and these poles are grouped into two clusters as below:

Cluster-1 has poles : $-1, -3, -10$

Cluster-2 has poles: $-2, -5, -20$.

The cluster centers of these clusters are obtained by using equations (12)-(14) and the denominator of the reduced model is obtained as

$$D_2(s) = (s - c_{1p})(s - c_{2p}) \quad (36)$$

The various denominator polynomials of lower order model will be obtained for different values of X. For the value of X, the numerator polynomial is determined by using factor division method. For $X = 50$, the transfer matrix of second order reduced model is obtained as equation (37).

$$[R_2(s)] = \frac{\begin{bmatrix} 1.1858s+2.0898 & 0.8505s+0.8359 \\ 0.5406s+1.0449 & 1.6734s+2.0898 \end{bmatrix}}{s^2+3.0666s+2.0898} \quad (37)$$

Table 2. Comparison of various MOR methods in terms of ISE

Reduction technique	Reduced system	$r_{11}(s)$	$r_{12}(s)$	$r_{21}(s)$	$r_{22}(s)$
Parmar, Mukherjee and Prasad (2007)	$\frac{\begin{bmatrix} 6.0429s+8.4707 & 3.9419s+3.3883 \\ 2.8097s+4.2354 & 8.0195s+8.4707 \end{bmatrix}}{s^2+13.6666s+8.4707}$	0.225	0.0682	0.0613	0.6780
Sikander and Prasad (2015)	$\frac{\begin{bmatrix} 0.7938s+0.6181 & 0.4273s+0.2472 \\ 0.37952s+0.309 & 0.93382s+0.6181 \end{bmatrix}}{s^2+1.34952s+0.6181}$	0.1672	0.0958	0.0312	0.2004
Narwal and Prasad (2015)	$\frac{\begin{bmatrix} 0.8930s+0.6181 & 0.4517s+0.2472 \\ 0.4314s+0.3091 & 1.0579s+0.6181 \end{bmatrix}}{s^2+1.34952s+0.6181}$	0.1615	0.0897	0.0296	251.3574
Parmar, Prasad and Mukherjee (2007)	$\frac{\begin{bmatrix} 0.8503s+0.6171 & 0.4617s+0.2466 \\ 0.4093s+0.3086 & 0.9977s+0.6171 \end{bmatrix}}{s^2+1.34952s+0.6181}$	0.1471	0.0884	0.0258	0.1598
Prajapati and Prasad (2018 b, c)	$\frac{\begin{bmatrix} 0.9098s+0.7091 & 0.4916s+0.2836 \\ 0.4373s+0.3545 & 1.0753s+0.7091 \end{bmatrix}}{s^2+1.548s+0.7091}$	0.0765	0.0595	0.0115	0.0808
Proposed method X=10	$\frac{\begin{bmatrix} 1.106s+2.4914 & 0.8909s+0.9966 \\ 0.4907s+1.2457 & 1.6874s+2.4914 \end{bmatrix}}{s^2+3.3483s+2.4914}$	0.0157	0.0003	0.0024	0.0381
Vishwakarma and Prasad (2009)	$\frac{\begin{bmatrix} 1.1816s+3.6508 & 1.0466s+1.4603 \\ 0.4982s+1.8254 & 1.6911s+3.6508 \end{bmatrix}}{s^2+4.3374s+3.6508}$	0.0151	0.0078	0.0030	0.0469
Narwal and Prasad (2016)	$\frac{\begin{bmatrix} 1.3276s+3.0962 & 1.0447s+1.2444 \\ 0.6116s+1.5480 & 1.7815s+3.0960 \end{bmatrix}}{s^2+4.0965s+3.0965}$	0.0093	0.0040	0.0008	247.8492
Proposed method X=50	$\frac{\begin{bmatrix} 1.1858s+2.0898 & 0.8505s+0.8359 \\ 0.5406s+1.0449 & 1.6734s+2.0898 \end{bmatrix}}{s^2+3.0666s+2.0898}$	0.0089	0.0002	0.0008	0.0377

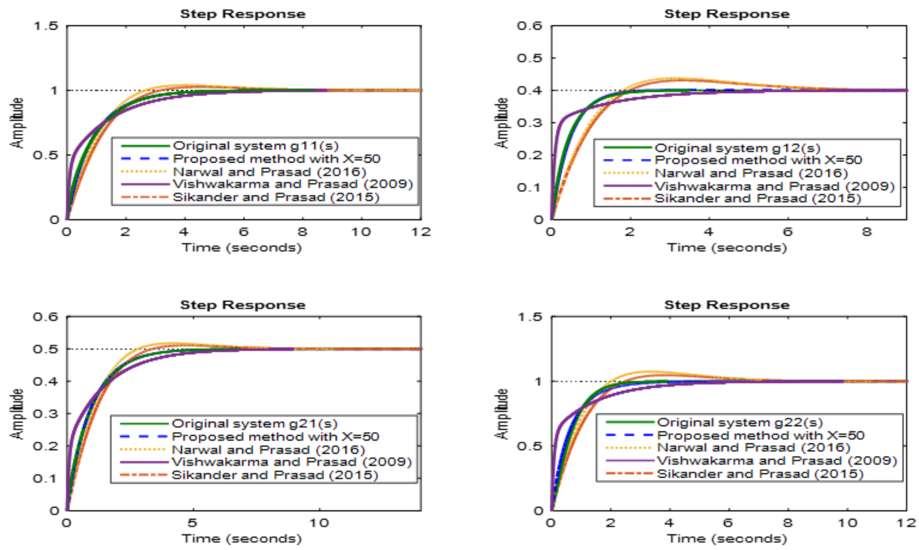
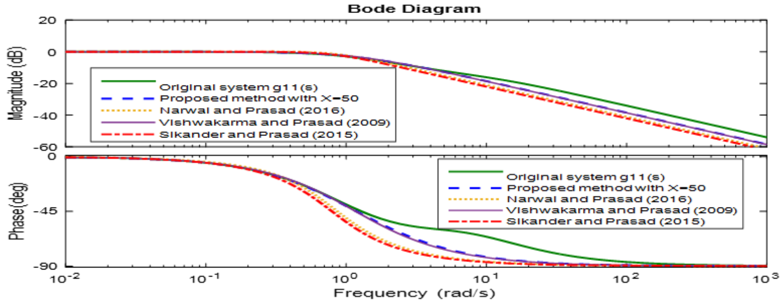


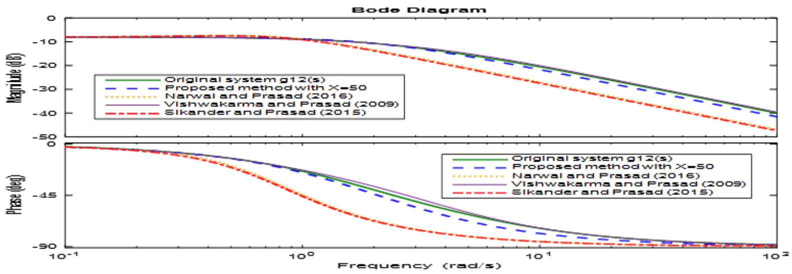
Figure 3. Qualitative comparison of the suggested approach and other existing model reduction techniques

The comparison of the step responses for the entire order and lower order systems is shown in Figure 2. When compared to some other conventional methods, it is evident that the response of the reduced model computed by the suggested scheme is substantially closer to the response of the given model. Additionally, it may be seen in Figure 4's frequency response characteristic. Table 2 presents a tabular comparison between the lower order system produced by the suggested technique and the other model reduction schemes found in the literature. It is unambiguously shown that the reduced system produced by the used technique closely resembles the specifications of the full order system with the lowest ISE value. The suggested approach yields the least ISE when X is taken to be 10, but it produces the least ISE when X is increased to X=50. Hence, a greater value for X can be chosen in order to improve the suggested method's accuracy.

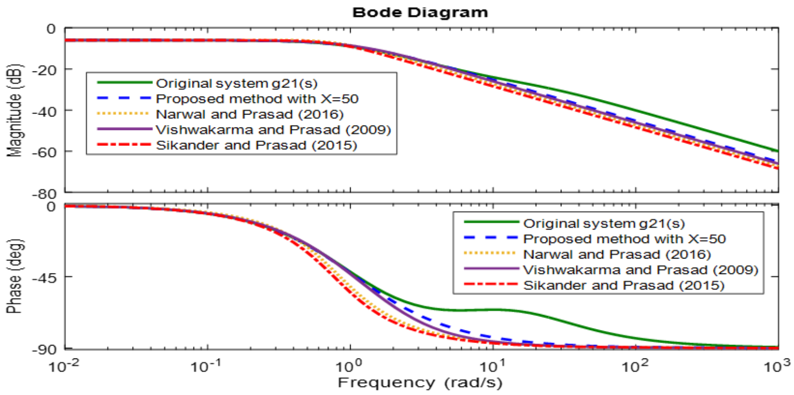
(a)



(b)



(c)



(d)

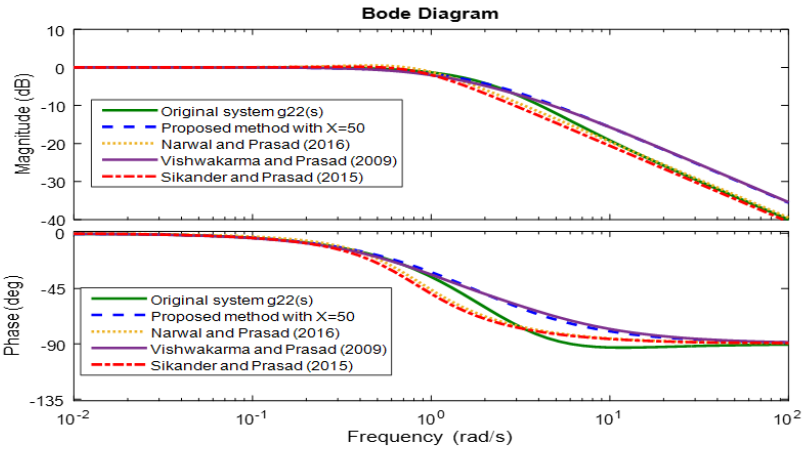


Figure 4: Frequency response comparison of (a) g11(s), (b) g12(s), (c) g21(s) and (d) g22(s).

Example 3: A fourth order regulator problem with its reference model given in [31] for the design of compensator controller is considered and expressed as

$$G(s) = \frac{s^3+12s^2+54s+72}{s^4+18s^3+97s^2+180s+100} \quad (40)$$

$$M(s) = \frac{4.242s+25}{s^2+7.07s+25} \quad (41)$$

Reference model of open loop is expressed as

$$\widetilde{M}(s) = \frac{M(s)}{1-M(s)} = \frac{4.242s+25}{s(s+2.828)} \quad (42)$$

Using the original system, the parameters of controller are obtained as follows

$$G_c(s) = \frac{\widetilde{M}(s)}{G(s)} = \frac{e_0+e_1s+e_2s^2}{s} = \frac{K(1+K_1s)}{s(1+K_2s)} = \frac{12.278+10.6337s-1.3782s^2}{s} \quad (43)$$

Hence, $K = 12.278, K_1 = 0.9957, K_2 = 0.1296$

$$R_{cl}(s) = \frac{G(s)G_c(s)}{1+G(s)G_c(s)} = \frac{12.23s^4+159s^3+807.5s^2+1543s+884}{0.1296s^6+3.333s^5+42.8s^4+279.3s^3+1000s^2+1643s+884} \quad (44)$$

The reduced order model obtained by proposed method with $X = 50$ is given follows

$$R_2(s) = \frac{0.6357s+1.48}{s^2+3.042s+2.056} \quad (45)$$

By using the above reduced system, the parameters of controller are obtained as follows

$$G_{cr}(s) = \frac{\widetilde{M}(s)}{R_r(s)} = \frac{e_0+e_1s+e_2s^2}{s} = \frac{K(1+K_1s)}{s(1+K_2s)} = \frac{12.2777+10.6334s-1.1369s^2}{s} \quad (46)$$

Hence, $K = 12.2778, K_1 = 0.8866, K_2 = 0.0206$. The close loop transfer of the original model with the compensator calculated by the obtained reduced system is as:

$$R_{cl}(s) = \frac{G(s)G_{cr}(s)}{1+G(s)G_{cr}(s)} = \frac{10.89s^4+142.9s^3+735.2s^2+1447s+884}{0.02057s^6+1.37s^5+30.88s^4+243.6s^3+917.2s^2+1547s+884} \quad (47)$$

The time responses of the original closed-loop plant with compensators are compared in Figure 3. Lower order systems and the original system are used to compute these compensators. The results of the simulation demonstrate that the produced compensators function well in both transient and steady state responses. Additionally, Figure 6's frequency response makes this evident. The time domain characteristics of the closed loop systems with compensators are displayed in Table 3. This table shows that the specifications of the closed loop models with compensators designed using the original system roughly match the time domain specifications of the closed loop system with the compensator computed using lower order systems. The compensator design that employs a lower order system is somewhat simpler than the compensator design that uses a higher dimensional system. Additionally, this table shows that the specifications of the needed reference system and the closed loop models with compensators are similar in the time domain. Therefore, the compensator design may be done using the suggested method to get the dynamical systems to operate as needed.

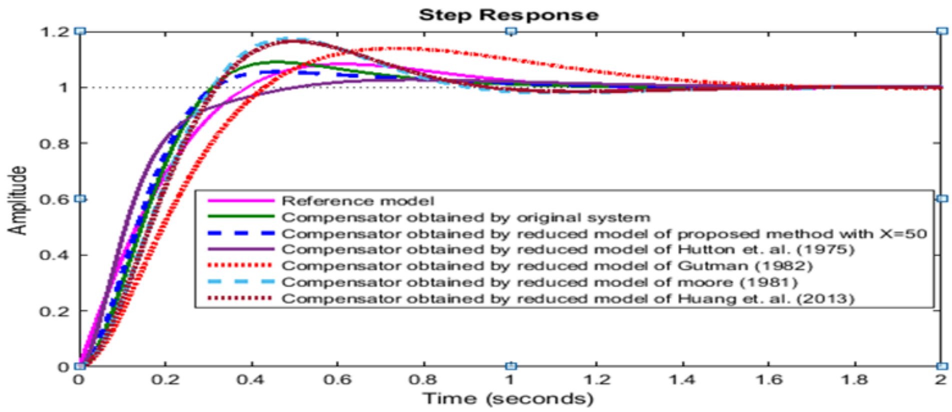


Figure 5: Comparison of time responses of reference model and closed system with compensator

Table 3. Comparing the closed loop system's time domain the variables with the compensator

Reduced techniques	Reduced model	Compensator (K, K_1, K_2)	Rise time (Second)	Settling time (Second)	Peak overshoot	Peak time (Second)
---	Reference model	---	0.2817	1.0338	1.0833	0.6123
---	Original system	12.278, 0.9957, 0.1296	0.2057	0.8896	1.0903	0.4641
Proposed method (X=50)	$\frac{0.6357s+1.48}{s^2+3.042s+2.056}$	**12.2777, 0.973, 0.1069**	**0.2027**	**0.9331**	**1.0556**	**0.4609**

Hutton and Friedland (1975)	$\frac{0.6207s + 0.8276}{s^2 + 2.069s + 1.149}$	12.2733, 0.9197, 0.0530	0.2308	1.0138	1.0270	0.7696
Gutman (1982)	$\frac{216s + 864}{194s^2 + 1080s + 1200}$	12.278, 0.5848, 0.1188	0.293	1.4439	1.1394	0.7315
Shamash (1974)	$\frac{0.6779s + 0.5169}{s^2 + 1.695s + 0.718}$	12.2794, 0.9939, 0.1286	0.2057	0.8921	1.0888	0.4645
Prajapati and Prasad (2018 b, c)	$\frac{1.774s + 8.719}{s^2 + 15.18s + 12.11}$	12.278, 1.1653, 0.2993	0.2393	1.5561	1.2646	0.5790
Moore (1981)	$\frac{1.002s + 8.931}{s^2 + 15.18s + 12.11}$	11.9868, 1.1562, 0.1988	0.2116	0.8381	1.1741	0.4914
Huang et al. (2013)	$\frac{1.002s + 8.7193}{s^2 + 15.18s + 12.11}$	11.9868, 1.1562, 0.1988	0.2086	1.1452	1.1803	0.4844
Gu (2005)	$\frac{3.07s + 70.46}{s^2 + 107.9s + 95.62}$	11.9959, 1.0908, 0.1901	0.217	0.869	1.1649	0.4945

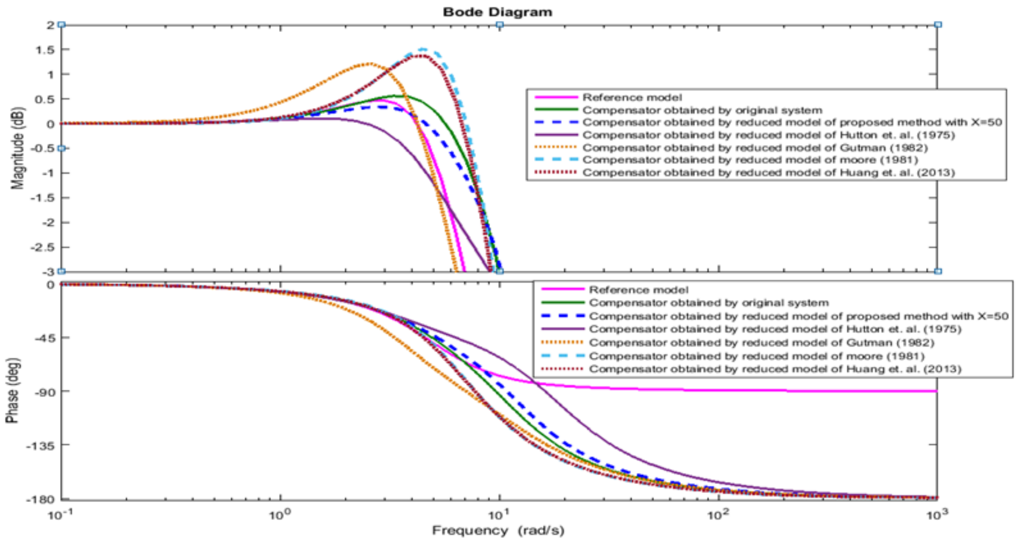


Figure 6. Comparison of frequency responses of reference model and closed system with compensator.

IV. Conclusion

This paper presents a new way for minimizing the complexity of linear dynamic systems of higher order using MOR. Factor division approach and generalized pole clustering are used to obtain the lower order model. If the full order model is stable, this straightforward computational method guarantees the stability of the reduced order systems. Additionally, the suggested plan guarantees that the first few time moments of the entire order system are preserved in the reduced order model, maintaining the basic behavior of the factor division approach. The temporal response comparison shows that the suggested approach provides a more accurate approximation of the large-scale model. Furthermore, Tables 1-3 comprise the various performance indices that have been determined for the purpose of validating the suggested technique and comparing it to other reduction techniques that are currently in use. This comparison shows that the suggested method surpasses a few other well-known model reduction strategies in terms of performance. The techniques for creating compensators for large-scale systems using a reduced order model are also provided in this study. The accuracy of the suggested methodology is demonstrated and validated with three standard examples.

References

[1] Antonio L And Umberto V, "An Improvement In The Routh-Padé Approximation Techniques". Int. J. Control 36(4): 643-661, 1982.

[2] Aoki M., "Control Of Large-Scale Dynamic Systems By Aggregation". Ieee Trans. Automat. Control 13 (3): 246-255, 1968.

[3] Bentbib A H Jbilou K And Kaouane Y., "A Computational Global Tangential Krylov Subspace Method For Model Reduction Of Large-Scale Mimo Dynamical Systems". Journal Of Scientific Computing 75(3): 1614–1632, 2018.

[4] Chen T. C., Chang C. Y. And Han K. W., "Model Reduction Using The Stability-Equation Method And The Padé Approximation Method. J. Frankl. Institute 309(6): 473-490, 1980.

[5] Chen T C Chang C Y And Han Kw., "Reduction Of Transfer Functions By The Stability-Equation Method". J. Franklin Institute 308(4): 389-404, 1979.

[6] Davison E J (1966) A Method For Simplifying Linear Dynamic Systems. Ieee Trans. Automat. Control 11(1): 93-101.

[7] Gautam R K Singh N Choudhary N K And Narain A (2018) Model Order Reduction Using Factor Division Algorithm And Fuzzy C-Means Clustering Technique. Transactions Of The Institute Of Measurement And Control: 1–8. Doi: 10.1177/0142331218762605

[8] Gu G (2005) All Optimal Hankel-Norm Approximations And Their Error Bounds In Discrete-Time. Int. J. Control 78(6): 408–423.

[9] Gutman P Mannerfelt C And Molander P (1982) Contributions To The Model Reduction Problem. Ieee Trans. Automat. Control 27(2): 454-455.
[10] Haider S Ghafoor A Imran M And Fahad M M (2018) Time-Limited Gramians-Based Model Order Reduction For Second-Order Form Systems. Transactions Of The Institute Of Measurement And Control, 1-9, Doi: 10.1177/0142331218798893
[11] Huang C Zhang K Dai X And Tan W (2013) A Modified Balanced Truncation Method And Its Application To Model Reduction Of Power System. Proceedings Of Ieee Power And Energy Society General Meeting, Vancouver, Bc, Canada.
[12] Hutton M F And Friedland B (1975) Routh Approximations For Reducing Order Of Linear, Time-Invariant Systems. Ieee Trans. Automat. Control 20(3): 329-337.
[13] Fernando K And Nicholson H (1982) Singular Perturbational Model Reduction Of Balanced Systems. Ieee Trans. Automat. Control 27(2): 466 – 468.
[14] Jamshidi M (1983) Large-Scale Systems: Modeling, Control And Fuzzy Logic (Pp. 66-98). New York: Prentice-Hall, Inc.
[15] Kokotovik P V Malley R E O And Sannuti P (1976) Singular Perturbation And Order Reduction In Control Theory-An Overview. Automatica, 12: 123-132.
[16] Komarasamy R Albhonso N K And Gurusamy G (2011) Order Reduction Of Linear Systems With An Improved Pole Clustering. Journal Of Vibration And Control 18(12): 1876–1885.
[17] Kumar D K Nagar S K And Tiwari Jp (2013) A New Algorithm For Model Order Reduction Of Interval Systems. Bonfring International Journal Of Data Mining, 3(1): 6-11.
[18] Kumar V And Tiwari Jp (2012) Order Reducing Of Linear System Using Clustering Method Factor Division Algorithm. International Journal Of Applied Information Systems 3(5): 1-4.
[19] Lal M And Mitra R (1974) Simplification Of Large System Dynamics Using A Moment Evaluation Algorithm. Ieee Trans. Automat. Control 19(5): 602-603.
[20] Langholz G And Feinmesser D (1978) Model Order Reduction By Routh Approximations. Int. J. Systems Sci. 9(5): 493-496.
[21] Lucas T N (1983) Factor Division: A Useful Algorithm In Model Reduction. Iee Proceedings, Control Theory And Applications 130(6): 362-364.
[22] Moore Bc (1981) Principal Component Analysis In Control System: Controllability, Observability, And Model Reduction. Ieee Trans. Automat. Control 26(1): 17-36.
[23] Narain A Chandra D And Singh Rk (2014) Model Order Reduction Using Fuzzy C-Means Clustering. Transactions Of The Institute Of Measurement And Control 36(8): 992–998.
[24] Narwal A And Prasad R (2016) Optimization Of Lti Systems Using Modified Clustering Algorithm. Iete Technical Review 34(2): 201-213.
[25] Narwal A And Prasad R (2015) A Novel Order Reduction Approach For Lti Systems Using Cuckoo Search Optimization And Stability Equation. Iete Journal Of Research 62(2): 154-163.
[26] Parmar G Mukherjee S And Prasad R (2007) System Reduction Using Factor Division Algorithm And Eigen Spectrum Analysis. Applied Mathematical Modelling 31: 2542–2552.
[27] Parmar G Prasad R And Mukherjee S (2007) Order Reduction Of Linear Dynamic Systems Using Stability Equation Method And Ga. International Journal Of Electrical And Computer Engineering 1(2): 236-242.
[28] Paul S And Chang J (2018) Fast Numerical Analysis Of Electric Motor Using Nonlinear Model Order Reduction. Ieee Transactions On Magnetics, 54(3): 1-4.
[29] Peterson Wc And Nassar Ah (1978) On The Synthesis Of Optimum Linear Feedback Control Systems. J. Frankl. Institute, 306(3): 237-256
[30] Prasad R (2000) Padé Type Model Order Reduction For Multivariable Systems Using Routh Approximation. Comput. Electr. Eng. 26(6): 445–459.
[31] Prasad R Pal J And Pant Ak (1987) Controller Design Using Reduced Order Models. Proceedings Of 14th National Systems Conference (Nsc-87) Kurukshetra, India.
[32] Prajapati A K And Prasad R (2018a) Failure Of Padé Approximation And Time Moment Matching Techniques In Reduced Order Modelling. Proceedings Of The 3rd Ieee International Conference For Convergence In Technology (I2ct -2018), Pune, India.
[33] Prajapati A K And Prasad R (2018b) Reduced Order Modelling Of Lti Systems By Using Routh Approximation And Factor Division Methods. Circuit, System And Signal Processing, 1-12. Doi: 10.1007/S00034-018-1010-6
[34] Prajapati A K And Prasad R (2018c) Order Reduction Of Linear Dynamic Systems By Improved Routh Approximation Method. Iete Journal Of Research, 1-14. Doi:10.1080/03772063.2018.1452645
[35] Prajapati A K And Prasad R (2018d) Model Order Reduction By Using The Balanced Truncation Method And The Factor Division Algorithm. Iete Journal Of Research, 1-16. Doi:10.1080/03772063.2018.1464971
[36] Prajapati A K And Prasad R (2018e) Reduced Order Modelling Of Linear Time Invariant Systems Using The Factor Division Method To Allow Retention Of Dominant Modes. Iete Technical Review, 1-14. Doi:10.1080/02564602.2018.1503567
[37] Redmann M And Kürschner P (2018) An Output Error Bound For Time-Limited Balanced Truncation. Systems And Control Letters 121: 1-6.
[38] Rosenzweig P Kater A And Meurer T (2018) Model Predictive Control Of Piezo-Actuated Structures Using Reduced Order Models. Control Engineering Practice 80: 83-93.
[39] Sabir A And Ibrir S (2018) Induction Motor Speed Control Using Reduced-Order Model. Automatika Journal For Control, Measurement, Electronics, Computing And Communications 59(3): 275-286.
[40] Samar R Postlethwaite I And Gu D W (1995) Model Reduction With Balanced Realizations. Int. J. Control 64(1): 33-34.
[41] Safonov M G And Chiang R Y (1989) A Schur Method For Balanced-Truncation Model Reduction. Ieee Trans. Automat. Control 34(7): 729-733.
[42] Shamash Y (1974) Stable Reduced-Order Models Using Padé-Type Approximations. Ieee Trans. Automat. Control 19(5): 615–616.
[43] Shamash Y (1981) Truncation Method Of Reduction: A Viable Alternative. Electron. Letter 17(2): 97-98.
[44] Shamash Y (1980) Failure Of The Routh-Hurwitz Method Of Reduction. Ieee Trans. Automat. Control 25(2): 313-314.
[45] Shamash Y (1975) Model Reduction Using The Routh Stability Criterion And The Padé Approximation Technique. Int. J. Control 21(3): 475-484.
[46] Sikander A And Prasad R (2017) A New Technique For Reduced-Order Modelling Of Linear Time-Invariant System. Iete Journal Of Research 63(3): 316-324.
[47] Sikander A And Prasad R (2015) Linear Time-Invariant System Reduction Using A Mixed Methods Approach. Applied Mathematical Modelling 39: 4848–4858.
[48] Singh J Vishwakarma C B And Chatterjee K (2016) Biased Reduction Method By Combining Improved Modified Pole Clustering And Improved Padé Approximations. Applied Mathematical Modelling 40(2): 1418-1426.

[49] Singh N Prasad R And Gupta H O (2006) Reduction Of Linear Dynamic Systems Using Routh Hurwitz Array And Factor Division Method. Iete Journal Of Education 47(1): 25-29.
[50] Sinha A K. And Pal J (1990) Simulation Based Reduced Order Modelling Using A Clustering Technique. Computers And Electrical Engineering 16(3): 159–169.
[51] Soloklo H N And Farsangi Mm (2015) Model Order Reduction By Using Legendre Expansion And Harmony Search Algorithm. Majlesi J. Electr. Eng. 9(1): 25–35.
[52] Tiwari S K And Kaur G (2018) Improved Reduced-Order Modeling Using Clustering Method With Dominant Pole Retention. Iete Journal Of Research, 1-11. Doi:10.1080/03772063.2018.1465365
[53] Tiwari S K And Kaur G (2017) Model Reduction By New Clustering Method And Frequency Response Matching. J. Control Autom Electr Syst. 28(1): 78-85.
[54] Towill D R (1970) Transfer Function Techniques For Control Engineers. (Illiffebooks Ltd.) London.
[55] Vishwakarma C B And Prasad R (2014) Time Domain Model Order Reduction Using Hankel Matrix Approach. J. Frankl. Institute 351(6): 3445-3456.
[56] Vishwakarma C B (2011) Order Reduction Using Modified Pole Clustering And Padé Approximations. International Journal Of Electrical, Computer, Energetic, Electronic And Communication Engineering 5(8): 998-1002.
[57] Vishwakarma C B And Prasad R (2009) Mimo System Reduction Using Modified Pole Clustering And Genetic Algorithm. Modelling And Simulation In Engineering 2009(1): 1-5.
[58] Vishwakarma C B And Prasad R (2008) Clustering Method For Reducing Order Of Linear System Using Padé Approximation. Iete Journal Of Research 54(5): 326-330.
[59] Yang J M And Jiang Y L (2018) Krylov Subspace Approximation For Quadratic-Bilinear Differential System. Int. J. Systems Sci. 49(9): 1950-1963.
[60] Krishnamurthy V And Seshadri V (1978) Model Reduction Using The Routh Stability Criterion. Ieee Trans. Automat. Control 23(3): 729-731.