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Solving An Open Question For Simplification Of A Cubic Root

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Abstract:
We reduce the expression of a radical identity that did not consider its source equation expressed in a cubic polynomial. Our method is applying algebra without using calculus technique to assist practitioners in the future solving their operational research questions from a pure algebraic point of procedure.
***Key Word**: Radical identity; Cubic formula; Reduction of radicals.*

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I. Introduction

More than one hundred papers applied algebraic process to handle operational research problems to help practitioners who are not used to differential equations and calculus. We only mention several of them to show this current of study inclination. Wang and Chen [22] examined Aguaron and Moreno-Jimenez [1], and Yen [30] to show that there are infinite possible solution for the relationship proposed by Yen [30] with respect to the ordering quantity and the backordered quantity. Lin [14] showed that Saaty and Vargas [20], Yen et al. [31], Chu et al. [7], and Lin [13] derived questionable results and then provided a new family of independent vectors. Wang and Chiang [24] verified Filev and Yager [8], and Mandal et al. [16] to obtain revised derivations. Yang and Chen [29] studied Yen [30], and Çalışkan [3, 4] to point out their questionable results that not a real algebraic method. In fact, the "algebraic procedure" proposed by Çalışkan [3, 4] is a tedious expression of calculus that can be replaced by a simple analytic approach. Yang and Chen [29] presented improvements for Yen [30], and Çalışkan [3, 4]. Wang and Lin [25] amended Chang et al. [5] and Lin et al. [15]. Wang and Chen [23] pointed our several questionable findings in Xu [28], Moon and Giri [17], and Glock et al. [10] and provided their revisions. Wu [27] obtained further improvements for Glock et al. [10], Finan and Hurley [9], and Liberatore and Nydick [12]. Wang et al. [26] rectified Chang and Schonfeld [6] to derive an approximated optimal solution, and also improved Aucamp and Kuzdrall [2] and Hua et al. [11]. Based on our above discussion, we know that applying algebraic methods for operational research studies still is a popular research trend. Hence, in the next section, we will consider a mathematical problem with a pure algebraic approach with respect to Sofo [21], Osler [18] and Osler [19].

II. Review of Sofo [21], and Osler [18, 19]

The purpose of this study is to show that

$$\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=1. \tag{1}$$

that did not considering the cubic polynomial

$$t^3+3t-4=0. \tag{2}$$

There are three papers: Sofo [21], Osler [18] and Osler [19] had discussed this problem.

Sofo [21] examined this topic to set that

$$\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=Z, \tag{3}$$

and then apply the algebraic method to cube on both-sides of Equation (3) to obtain that

$$Z^3+3Z-4=0. \tag{4}$$

Because of $Z=1$ is the unique real number answer of Equation (4), Sofo [21] decided that Equation (1) is verified.

Osler [18] claimed that "$x^3+3x=4$ has a solution of $x=1$. He raised a question: How can one algebraically manipulate it into that result? All attempts to do so led the author back to the original cubic from which it started."
In Osler [18], he first derived that

$$\sqrt[n]{a\pm\sqrt{b}}=\frac{x\pm\sqrt{x^2-4c}}{2}. \quad (5)$$

where Osler [18] assumed two auxiliary expressions,

$$x=\sqrt[n]{a-\sqrt{b}}+\sqrt[n]{a+\sqrt{b}}, \quad (6)$$

and

$$c=\sqrt[n]{a^2-b}. \quad (7)$$

To compute and simplify the objective term $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$, Osler [18] assumed that

$$c=-1, \quad (8)$$
$$b=5, \quad (9)$$
$$a=2, \quad (10)$$

and

$$n=3. \quad (11)$$

Osler [18] claimed that "Notice that (22) [Equation (5) in this paper] yields $\sqrt[3]{2\pm\sqrt{5}}=\frac{1\pm\sqrt{5}}{2}$, from which it follows that $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=1$."

We show that according to Equation (5), Osler [18] just found that

$$\sqrt[3]{2+\sqrt{5}}=\frac{x+\sqrt{x^2+4}}{2}, \quad (12)$$

and

$$\sqrt[3]{2-\sqrt{5}}=\frac{x-\sqrt{x^2+4}}{2}, \quad (13)$$

however, Osler [18] cannot show that $x=1$.
If we repeat the proving approach of Osler [18] to indicate that if Osler [18] derived that

$$1=x=\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}} \quad (14)$$

to show that

$$\sqrt[3]{2\pm\sqrt{5}}=\frac{1\pm\sqrt{5}}{2} \quad (15)$$

Based on the findings of Equations (15), Osler [18] may claim that he show that

$$\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}=1. \quad (16)$$

From the above examination, we can claim that Osler [18] had not solved the open problem with respect to Equation (1) under the condition with respect to Equation (2). We recall that the open problem proposed by Osler [18] is to verify Equation (1), but did not use the findings of Equation (2).

Moreover, another further examination, Osler (2002) developed a more generalized system of the closed-form solution for

$$t^3-3ct-2a=0, \quad (17)$$

with the requirement, $b=a^2-c^3\geq 0$. Osler (2002) derived a real number solution as follows,

$$t=\sqrt[3]{a+\sqrt{b}}+\sqrt[3]{a-\sqrt{b}}, \quad (18)$$

by the cubic formula.

We recall that Osler (2002) referred to Equations (17) and (18) to examine Equation (2), under the conditions $c = -1$ and $a = 2$, Osler (2002) derived the solution $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$ such that Osler (2002) proposed the following open question:

Directly using an algebraic approach, to verify that $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}} = 1$?

We call that Osler (2002) claimed that "The reader might try to simplify this difference of two cube roots into the number 1. but all attempts to do this simply lead back to the original cubic $x^3 + 3x - 4 = 0$."

The examination of the cubic expressions in (1) under the real number restriction, without knowing the result of Equation (2) motivates the development of our paper.

III. Our Procedure

In this section, we try to derive a legitimate algebraic method that did not consider the expression of $t^3 + 3t - 4 = 0$, which is denoted as Equation (2), to prove the validation of Equation.

We set the following two auxiliary expressions,

$$\sqrt[3]{2+\sqrt{5}} = \alpha + \beta\sqrt{5}\,, \tag{19}$$

and

$$\sqrt[3]{2-\sqrt{5}} = \alpha - \beta\sqrt{5}\,. \tag{20}$$

The most natural way to solve a cubic root is to perform the cubic operation, therefore, on both sides of Equations (19) and (20), we cube them to show the following,

$$2+\sqrt{5} = \alpha^3 + 3\sqrt{5}\alpha^2\beta + 15\alpha\beta^2 + 5\sqrt{5}\beta^3, \tag{21}$$

and

$$2-\sqrt{5} = \alpha^3 - 3\sqrt{5}\alpha^2\beta + 15\alpha\beta^2 - 5\sqrt{5}\beta^3\,. \tag{22}$$

We execute difference and sum of Equations (21) and (22) to derive that

$$1 = 3\alpha^2\beta + 5\beta^3\,, \tag{23}$$

and

$$2 = \alpha^3 + 15\alpha\beta^2\,. \tag{24}$$

Based on Equations (23) and (24), we multiple the both-sides of Equation (23) by 2, and then we cancel out the constant terms in Equations (23) and (24), to imply that

$$\alpha^3 + 15\alpha\beta^2 - 2\left(3\alpha^2\beta + 5\beta^3\right) = 0\,. \tag{25}$$

We can decompose Equation (25) to show that

$$\alpha^3 - 6\alpha^2\beta + 15\alpha\beta^2 - 10\beta^3 = (\alpha-\beta)\left(\left(\alpha - \frac{5}{2}\beta\right)^2 + \frac{15}{4}\beta^2\right). \tag{26}$$

We observe that in Equation (26), $(\alpha - (5\beta/2))^2$ and $15\beta^2/4$ are two square terms to imply both of them are positive. The above discussion indicates that there only one possible relationship between two unknown variables α and β,

$$\alpha = \beta\,. \tag{27}$$

We substitute Equation (27) into Equation (23) to obtain the following polynomial,

$$\alpha^3 - (1/8) = 0\,. \tag{28}$$

We decompose the result of Equation (28) to show that

$$\alpha^3 - \frac{1}{8} = \left(\alpha - \frac{1}{2}\right)\left(\left(\alpha + \frac{1}{4}\right)^2 + \frac{3}{16}\right). \tag{29}$$

We observe that $(\alpha + (1/4))^2 + (3/16)$ is a positive term to imply that there is a unique real number solution,

$$a = \frac{1}{2}\,. \tag{30}$$

We merge the results of Equations (30) and (27) to show that

$$b = \frac{1}{2}. \tag{31}$$

According to our results of Equations (30) and (31), we show that

$$\sqrt[3]{2+\sqrt{5}} = \frac{1}{2} + \frac{1}{2}\sqrt{5}, \tag{32}$$

and

$$\sqrt[3]{2-\sqrt{5}} = \frac{1}{2} - \frac{1}{2}\sqrt{5}. \tag{33}$$

At last, we add Equation (32) to Equation (33) to show that

$$\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}} = 1, \tag{34}$$

which is our goal of Equation (16).
Finally, we add Equation (31) to Equation (32) to prove the assertion of Equation (1) being true.

IV. Conclusion

We develop an algebraic procedure to derive a cubic radical formula that did not consider the cubic polynomial in the source paper. Our development will help practitioners in the near future to construct their algebraic process to deal with their operational research problems.

References

[1]. J. Aguaron, J. M. Moreno-Jimenez, "Local Stability Intervals In The Analytic Hierarchy Process," European Journal Of Operational Research, Vol. 125, Pp. 113-132, 2000.

[2]. D. C. Aucamp, P. J. Kuzdrall, “Order Quantities With Temporary Price Reductions,” The Journal Of Operational Research Society, Vol. 40, No. 10, Pp. 937-940, 1989.

[3]. C. Çalışkan, "A Derivation Of The Optimal Solution For Exponentially Deteriorating Items Without Derivatives," Computers & Industrial Engineering, Vol. 148, 106675, 2020.

[4]. C. Çalışkan, "Derivation Of The Optimal Solution For The Economic Production Quantity Model With Planned Shortages Without Derivatives," Modelling, Vol. 2022, No. 3, Pp. 54–69, 2022.

[5]. S. K. J. Chang, J. P. C. Chuang, H. J. Chen, Short Comments On Technical Note—The Eoq And Epq Models With Shortages Derived Without Derivatives, International Journal Of Production Economics, Vol. 97, Pp. 241–243, 2005.

[6]. S. K. Chang, P. M. Schonfeld, "Multiple Period Optimization Of Bus Transit Systems," Transportation Research Part B-Methodological, Vol. 25, No. 6, Pp. 453-478, 1991.

[7]. C. H. Chu, K. C. Hung, P. Julian, "A Complete Pattern Recognition Approach Under Atanassov's Intuitionistic Fuzzy Sets," Knowledge-Based Systems, Vol. 66, Pp. 36-45, 2014.

[8]. D. Filev, R. R. Yager, "On The Issue Of Obtaining Owa Operator Weights," Fuzzy Sets And Systems, Vol. 94, Pp. 157–169, 1998.

[9]. J. S. Finan, W. J. Hurley, "The Analytic Hierarchy Process: Can Wash Criteria Be Ignored?," Computers And Operations Research, Vol. 29, Pp. 1025-1030, 2002.

[10]. C. H. Glock, K. Schwindl, M. Y. Jaber, "An Eoq Model With Fuzzy Demand And Learning In Fuzziness," International Journal Services And Operations Management, Vol. 12, No. 1, Pp. 90-100, 2012.

[11]. S. R. Hua, S. Peeta, C. H. Chu, "Identification Of Vehicle Sensor Locations For Link-Based Network Traffic Applications," Transportation Research Part B, Vol. 43, 2009, Pp. 873–894.

[12]. M. J. Liberatore, R. L. Nydick, "Wash Criteria And The Analytic Hierarchy Process," Computers And Operations Research, Vol. 31, Pp. 889-892, 2004.

[13]. S. S. C. Lin, "Analytic Hierarchy Process By Least Square Method Revisit," Mathematical Problems In Engineering, Vol. 2019, Article Id 2797515, 5 Pages, 2019.

[14]. S. C. Lin, "Comparison Of Methods To Derive Relative Weights," Iaeng International Journal Of Applied Mathematics, Vol. 53, No. 2, Pp. 507-517, 2023.

[15]. S. C. Lin, H. W. Tuan, C. T. Tung, P. Julian, "Partition Network Into Communities Based On Group Action On Sets," Physica A, Vol. 491, Pp. 361–376, 2018.

[16]. N. K. Mandal, T. K. Roy, M. Maiti, "Multi-Objective Fuzzy Inventory Model With Three Constraints: A Geometric Programming Approach," Fuzzy Sets And Systems, Vol. 150, Pp. 87–106, 2005.

[17]. I. Moon, B. C. Giri, "Comment On Bose S, Goswami A And Chaudhuri Ks (1995). An Eoq Model For Deteriorating Items With Linear Time-Dependent Demand Rate And Shortages Under Inflation And Time Discounting," The Journal Of The Operational Research Society, Vol. 52, No. 8, Pp. 966-969, 2001.

[18]. Osler, T. J., Cardan Polynomials And The Reduction Of Radicals, Mathematics Magazine, Vol. 47, No. 1, Pp. 26-32, 2001.

[19]. Osler, T. J., An Easy Look At The Cubic Formula, Mathematics And Computer Education, Vol. 36, Pp. 287-290, 2002.

[20]. T. L. Saaty, L. G. Vargas, "Comparison Of Eigenvalue, Logarithmic Least Square And Least Square Methods In Estimating Ratios," Mathematical Modelling, Vol. 5, 309-324, 1984.

[21]. Sofo, A., Generalization Of A Radical Identity, The Mathematical Gazette, Vol. 83, Pp. 274-276, 1999.

[22]. Yu-Lan Wang, And Ming-Li Chen, "Study For Local Stability Intervals In Analytic Hierarchy Process," Iaeng International Journal Of Applied Mathematics, Vol. 53, No. 1, Pp. 194-201, 2023.

[23]. Yu-Lan Wang, And Ming-Li Chen, "Further Study On Goal Programming Models," Iaeng International Journal Of Computer Science, Vol. 50, No. 4, Pp. 1522-1531, 2023.

[24]. Yu-Lan Wang, And Te-Yuan Chiang, "Further Study For The Ordered Weighted Averaging Operator," Iaeng International Journal Of Applied Mathematics, Vol. 53, No. 3, Pp. 810-817, 2023.

[25]. Yu-Lan Wang, And Chiu-Tang Lin, "Intersection-Union Operation To Divide A Network Into Communities," Engineering Letters, Vol. 31, No.2, Pp756-763, 2023.

[26]. Yu-Lan Wang, Chiu-Tang Lin, Junling Zhang, And Xiaolin Li, "An Extension Of Analysis Of Multiple Period Bus Transit System," Iaeng International Journal Of Applied Mathematics, Vol. 53, No. 4, Pp. 1594-1601, 2023.

[27]. Lee-Chun Wu, "Formulated Optimal Solution For Eoq Model With Fuzzy Demand," Iaeng International Journal Of Computer Science, Vol. 50, No. 3, Pp. 890-898, 2023.

[28]. Z. Xu, "Goal Programming Models For Obtaining The Priority Vector Of Incomplete Fuzzy Preference Relation," International Journal Of Approximate Reasoning, Vol. 36, 2004, Pp. 261-270.

[29]. Ting-Chen Yang, And Yu-Chuan Chen, "Solution For Open Questions In Yen (2021) Osler (2001) And Caliskan (2020, 2022)," Iaeng International Journal Of Applied Mathematics, Vol. 53, No. 1, Pp. 48-57, 2023.

[30]. C. P. Yen, "Solving Inventory Models By The Intuitive Algebraic Method," International Journal Of Applied Mathematics, Vol. 51, No. 2, Pp. 341-345, 2021.

[31]. P. C. P. Yen, K. C. Fan, H. C. J. Chao, "A New Method For Similarity Measures For Pattern Recognition," Applied Mathematical Modelling, Vol. 37, Pp. 5335-5342, 2013.