IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676, p-ISSN: 2320-3331, Volume 18, Issue 2 Ser. I (Mar. – Apr. 2023), PP 09-13
www.iosrjournals.org

Corollaries of Point Spread Function with Co-sinusoidal, Gaussian, Hanning Filters

Padamuttum. Anitha[1] , S. Vidya Rani[2] , N. Sabitha[3],D. Karuna Sagar[4]
[1]Lecturer in Physics GDC. Nelakondapalli, Khammam-507160, Telangana, India.
[2] SV Govt. Arts &Science College Palem, Nagarkarnool, Telangana, India.
[3] Optics Research Group, Department of Physics, Hyderabad-500007, India.
[4]Professor, Head Dept. of Physics, Osmania University -Hyderabad -500007. Telangana, India.

ABSTRACT
The impact of apodized Point spread function along with its parameters like Encircled energy, zonal energy increment, displaced energy, and relative encircled energy have been studied for a symmetric diffraction-limited optical system using different filters including Co-sinusoidal, Gaussian, and Hanning filters. From our investigation, arranging these filters at the appropriate zone can further improve their effectiveness in suppressing side lobes of the point spread function.
KEYWORDS*: Point spread function, Encircled energy, Zonal energy increment, Displaced energy.*

Date of Submission: 09-03-2023 Date of Acceptance: 22-03-2023

I. INTRODUCTION

Encircled energy is the most significant corollary of the point spread function(PSF). It is to determine the energy distribution of the Fraunhofer diffraction of a point object in the different rings. Encircled energy is a smoother function than the point spread function. Encircled energy measures the percentage of the PSF's total energy, and is a parameter that can be used to characterize how the point source diffraction image behaves when integrated. [1-3]. Lord Rayleigh, who developed formulas for and initially emphasized the significance of the corollaries in the diffraction pattern, also created a formula to estimate the illuminations in the various rings of the diffraction pattern. earlier studies on the encircling energy Lord Rayleigh [4] established the mathematical formula for a circular aperture without apodization in a diffraction-limited Point Spread Function.

Lansraux and Boivin's study in 1961[5] focused on the effects of spherical aberration on encircled energy. Spherical aberration is a type of optical aberration that occurs when light rays passing through the edges of a lens or mirror focus at different points than those passing through the center. This results in a blurred image, as the different parts of the image are not in focus at the same time.

In 1976, Rao, Mondal, and Seshagiri Rao conducted a study on the encircled and excluded energies for Straubel apodization filters. They investigated the effects of different filter parameters, such as filter size and shape, on the encircled and excluded energies [6]. K. Surendar and P.K. Mondal are researchers who have studied the corollaries of Point Spread Function (PSF) apodized with Lanczos and Hanning amplitude filters [7-9].

In this investigation, the Hanning amplitude filter was placed at the outer zone and the Co-sinusoidal amplitude filter at the first zone. This arrangement resulted in the simultaneous achievement of low side-lobes and a steep principal maximum. The Hanning amplitude filter at the outer zone helped to reduce the side lobes, while the Co-sinusoidal amplitude filter at the first zone and Gaussian amplitude filter at the middle zone, helped to achieve a sharp principal maximum.

II. MATHEMATICAL EXPRESSION

The encircled energy within a circle of radius δ, about the axial point in the image plane, can be mathematically represented by integrating the intensity of the PSF over the circle:

$$EE(\delta) = \frac{\int_0^{\delta} |G_P(\emptyset d,z)|^2 z dz}{\int_0^{\infty} |G_P(\emptyset d,Z)|^2 z dz} \quad (1)$$

Using this expression, we can calculate the encircled energy (EE) within a given radius δ as:

Where $G_P(Ød,z)$ represents the complex amplitude in the image plane at point Z units away from the diffraction head in the given plane of observation and subscript p indicates that optical imaging system means that the optical system is apodised with a given filter. The amplitude of the diffraction light in the image field region associated with a rotationally symmetric pupil function that is apodized by a combination of co-sinusoidal, Gaussian, and Hanning filters of a three-zone aperture can be expressed as:

$$\mathbf{G_p(\varphi d, Z)} = 2\int_0^1 (f(x) J0(Zx) x dx \quad (2)$$

The intensity of the diffraction light in the area of the image field is linked to the Co sinusoidal filter, Gaussian filter, Hanning filter, and phase change of (-π/4) and(π/3), complex pupil function is given by:

$$\mathbf{G_p(\varphi d, Z)} = 2\left[\int_0^a (f1(x) J0(Zx) x dx + \int_a^b (f2(x) J0(Zx) x dx + \int_b^1 (f3(x) J0(Zx) x dx\right] \quad (3)$$

$$\mathbf{A_P(Ø, Z) = 2}$$
$$\left[\int_0^a 1 + \beta\cos(\pi r^{\wedge}2)/1 + \beta\, J0(Zx) x dx - -\pi/4 \int_a^b \exp.(-\beta r/2x^{\wedge}2) J0(Zx) x dx + \pi/3 \int_b^1 \cos(\pi\beta x) J0(Zx) x dx\right] \quad (4)$$

Where f1(x) is Co-sinusoidal, f2(x) is Gaussian, and f3(x) is Hanning mask pupil functions for the amplitude apodization of the pupil transmission. And Z is the reduced dimensionless diffraction coordinate in the image plane.

And, Haning function -f1=cos(πβx) (5)

Gaussian function -f2=exp.(-βr/2x^2) (6)

C0-sinusoidal function -f3= (1+βcos(πr^2)/1+β) (7)

A quality criterion called zonal energy increment (ZEI), which looks at the specifics of energy redistribution in a degraded PSF, has been established. The ZEI calculates the energy that is added to or withdrawn from the ideal PSF (Airy) in the region between radii δ_1

Similarly, to calculate the relative encircled energy, one typically measures the light flux within a circular aperture of radius δ in the image plane for both the apodized pupil and the Airy pupil. The ratio of the fluxes gives the EE(δ)rel. value.

$$\mathbf{EE(\delta)_{relative}} = \frac{\int_0^{\delta} |G_P(Ød,z)|^2 z dz}{\int_0^{\infty} |G_A(Ød,Z)|^2 z dz} \quad (8)$$

Where A and P are the amplitudes for Airy case of free aperture and apodization by pupil section.

Displaced energy refers to the energy that is lost or displaced from the central peak of an optical system due to the presence of an apodization function. or

The displaced energy is the difference of the encircled energy of a perfect optical system and that of the given apodized system. It thus compares the energy distribution in the apodized system, with its perfect counterpart, namely, the Airy pattern. The displaced energy is given by

$$\mathbf{DE(\delta) = EE_A(\delta) - EE_P(\delta)} \quad (9)$$

Where $EE_A(\delta)$ is the encircled energy for the perfect lens (Airy case) in which β= 0. $EE_P(\delta)$ is the corresponding quantity for the optical system apodized with amplitude filters. δ is the radial distance in the observational plane. Integration is performed over a radius δ.

The ZEI metric measures the amount of energy that is added or subtracted from the ideal PSF (Airy pattern) in the region between two radii, typically radii δ1 and δ2. These radii correspond to the central lobe of the Airy pattern and a surrounding region where the energy is distributed. Thus

$$\mathbf{ZEI(\Delta\delta) = [EE_P(\delta_2) - EE_P(\delta_1)] - [EE_A(\delta_2) - EE_A(\delta_1)]} \quad (10)$$

From the above equations (10) ($EE_A\delta_2$) represents the encircled energy for a perfect lens (Airy case) in which β=0 and $EE_P(\delta_1)$ is the apodized optical system for which β≠0. Here δ_2 is greater than δ_1 and $(\Delta\delta) = \delta_2 - \delta_1$, this is the zone between radii δ_2 and δ_1. In effect, ZEI measures a scatter function, which accounts for both the origin and destination of the energy being scattered. When a zone's ZEI is negative, there is less energy present there than in the Airy situation, in another case the ZEI is positive, and there is greater energy present there than in the Airy situation.

III. RESULTS AND DISCUSSIONS

The improved resolution of the point spread function with amplitude filters and proper thickness of the zones has been evaluated in terms of energy corollaries for all variations

. However, we reported the majority of the results for decreasing encircled energy. The results of studies on the impact of amplitude filter apodization on the PSF corollaries of optical imaging systems are presented in equations (1), (8), (9), and (10). With the help of the Mat lab program to study the effect of encircled energy with suitable order of apodization parameter β, defocus, and also different filters wit sutable thickness of aperture. By introducing the Gaussian quadrate numerical integration method to compute the envelope energy for different values of δ ranging from 0 to 10 in steps 0.2. with these values of δ was evaluated for apodization β with 0 to 1 in steps 0.25, and defect of defocus ranging from 0-2π with the difference of π/2. The encircled energy increases EE(δ) for the Airy case. By increasing apodization as well as defocus values, and the order of the filter at the appropriate zone, the encircled energy decreases EE(δ), this can be observed in Fig 1, and Fig 2 both for the apodization β= 1and defocus φd=2π with the thickness of 0.7 first zone, 0.7-0.9 second zone and 0.9-1 outer zone From Fig 1, the encircled energy gradually reaches unity with the increase of the value δ, whereas in the case of Fig 2 which is an F1F2F3 combination, the encircled energy reaches unity in a disordered manner with the variation of δ.

Fig 3, and Fig 4. Shows the zonal energy increment -ZEI(Δδ) variation with δ for apodization β= 1, and defocus φd=2π for the combination of amplitude filters placed as Cosinusidal filter at first zone(a=0.7), Gaussian filter at second zone(b=0.9), and Hanning filter in the outer zone for Fig 1, and in case of Fig 2 Hanning filter at first zone, Gaussian filter at second zone, and Cosinusidal filter at the outer zone. Zonal energy increment was evaluated from the equation [10]. This corollary was useful to study the detailed redistribution of the energy for the case of apodized with suitable filers and a defocused optical system which explains Fig 1 where the energy distribution gets altered as compared to the second case, that is Fig 2.

It is observed that for ZEI negative value indicates that the energy is scattered out of the zone. Positive values indicate that the energy is scattered into the optical system. However, the first combination is the best case to achieve good results.

Fig 5 and Fig 6 depict the variation of displaced energy, and apodized with suitable filters, for different values of defocus, and with the variation of δ ranging from 0-10, for both combinations. This is one of the important aspects of our investigation that is for the combination of F3F2F1 the DE is positive for all values of δ, whereas, in the case of the F1F2F3 combination, there are positive and negative results. This indicates that the outward displacement of energy for the first case that is F3F2F1 also has more encircled energy compared to the other case.

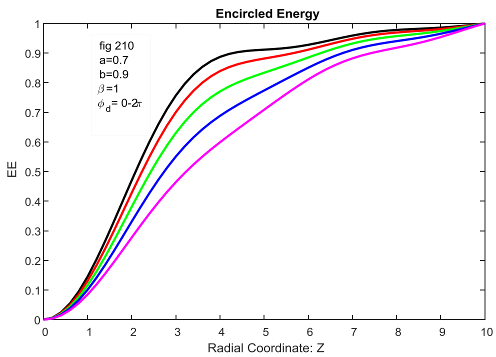

Fig 1
Variation of Encircled energy in case of **F3F2F1** combination
With apodization β=1.

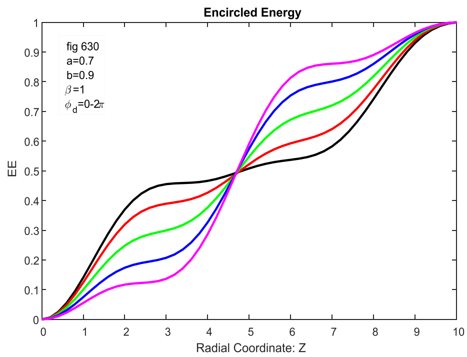

Fig 2
Variation of Encircled energy in case of **F1F2F3** combination
with apodization β=1

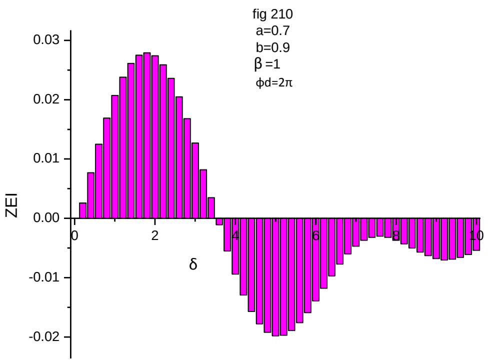

Fig 3
Variation of Zonal Energy Increment for the case **F3F2F1**

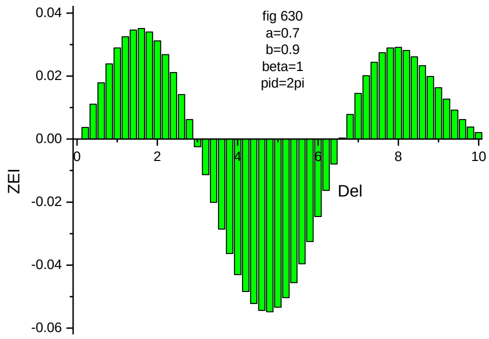

Fig 4
Variation of Zonal Energy Increment for the case **F1F2F3**

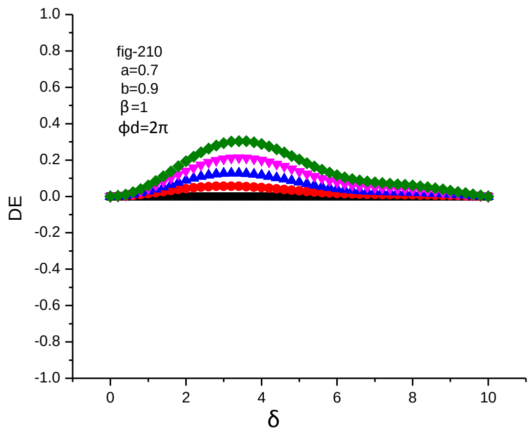

Fig 5
Variation Displaced Energy of the case **F3F2F3.**

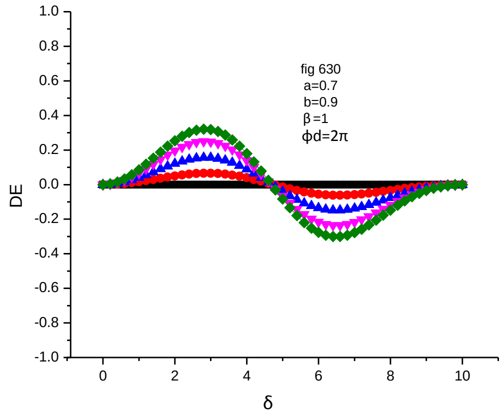

Fig 6
Variation Displace Energy for the combination of **F1F2F3**

IV. CONCLUSIONS

The present investigation found that when an optical system is apodized with amplitude filters, it can modify the resolution of the PSF and affect the amount of encircled energy. For larger radii, the situation is more complex. If an apodization function is applied to a three-zone aperture the PSF can be modified in a controlled way. When the suitable apodization function reaches its maximum value that is β=1for the F3F2F1 case, the resulting PSF can have higher encircled energy than the unsuitable case that is F1F2F3, and it can also have a symmetrically apodized profile with almost vanished lower-order optical sidelobes. However, the exact behavior of the PSF depends on the details of the suitable apodization function and the aperture thickness. However, this approach achieves symmetric apodized PSF with suppressed sidelobes of the optical system.

References

[1]. Sundus Yaseen Hasan Kufa University/ Education College for Girls/ Physics Department sundus. Journal University of Kerbala , Vol. 15 No.3 Scientific . 2017
[2]. Alety. Srisailam*, Vendavetla. Dharmaiah, Ramanamurthy M.V and Pranab Kumar Mondal. www.pelagiaresearchlibrary.com
[3]. A. NARESH KUMAR REDDY, M. KESHAVULU GOUD AND S. LACHA GOUD International Journal of Theoretical and Applied Physics (IJTAP) ISSN: 2250-0634, Vol.1, No. I (November 2011), pp. 109-126
[4]. Rayleigh, Lord, Phil Mag., 11, 214, (1881).
[5]. G. Lansraux and G., Can. J. Phys., 1961, 39, 158-189.
[6]. Rao, K.P., Mondal, P.K., Seshagiri Rao, T., Pramana, 7, 9 (1976).
[7]. K.Raju, V.Rao, S.L.Goud, K.Surendar, J. Pure Appl. Phys., 6(2), 41 (1994).
[8]. K.Surendar, S.L.Goud, P.K.Mondal, Acta Ciencia Indica, 18(1), 91 (1992).
[9]. K.Surendar, Ph.D. thesis, 1992

[10]. Dr. R.T.Borlibghaus, Confocal optical sectioning, Leica micro systems, 2011
[11]. W.Wetherell, Appl. Opt. Engineering, 8, 1980.
[12]. A.Naresh Kumar Reddy, R.Komala, M.Keshavulu Goud and S.L.Goud, AIP Conf. Proc., 1391, 341 (2011).

Padamuttum. Anitha, et. al. "Corollaries of Point Spread Function with Co-sinusoidal, Gaussian, Hanning Filters." *IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)*, 18(2), 2023, pp. 09-13.