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Fixed-time State Estimation for a Class of Neural Networks with Time Delay

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Abstract: *This paper studies the state estimator for a class of neural networks with time delay. By constructing a suitable Lyapunov function and using the inequality technique, some results are obtained to ensure that the state estimator are consistent with the states of the estimated system in finite time and the time upper bound of fixed-time is estimated. Finally, a numerical simulation shows the effectiveness of the given results.*

***Key words*：** *Neural network, time delay, state estimation, fixed-time stability*

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I. Introduction

Recently, neural networks have received more and more attention because of their extensive applications in various fields such as image processing, computer vision, decision optimization [1]-[4], and so on. Neural networks have great ability to deal with complex nonlinear problems and find optimal solutions at high speed. In practice, owing to the limited speeds of signal processing and transmission between neurons, time-delay inevitably exists in neural networks. Time delay often can affect the stability of dynamic systems and leads to the decline of the performance index of these systems. Therefore, it is necessary to study the stability of delayed neural networks for the satisfactory performance requirement, and there exist some relevant literatures such as [5]-[10]. For example, [5] studied the property of input-to-state stability of the inertial memristor-based neural networks with impulsive effects. The authors investigated the finite-time stability of fractional-order complex valued neural networks with time delay in [6]. By employing Laplace transform and the properties of Mittag-Leffler function, a result on the exponent stability is developed to derive the finite-time stability conditions.

It should be pointed out that, up to now, the research on the finite time stability has made a lot of achievements [11]-[13]. The finite time stability often depends on the initial conditions of the systems, which brings inconvenience to practical application. In order to make up for this deficiency, Polyakov proposed the concept of fixed-time stability in literature [14], and made a theoretical foundation for the fixed time stability of neural networks and provided some relevant theorems. Different from the finite time stability, fixed-time convergence can achieve transient performance faster and control accuracy higher. Thus, the problem of fixed-time stability has been concerned and applied in many fields such as fixed-time fuzzy control of non-singular robot systems [15], distributed formation control for multiple hypersonic gliding vehicles [16], high-precision trajectory tracking control for manipulator systems [17]. [18] studied the fixed-time synchronization problem of discontinuous fuzzy inertial neural network with time-varying delay, and solved the discontinuity of the system by using generalized variable transformation and Filippov theory. [19] proposed a new fixed-time stability theory which provides a lower upper bound for the convergence time, and used these results to solve the synchronization problem of the memory neural networks.

Motivated by the above discussion, we will deal with the problem of fixed-time state estimation for a class of neural networks with time delay in this paper. By constructing appropriate Lyapunov function and by the means of inequality technique, some sufficient conditions are established to ensure the existence of the desired estimators, and the gains of such estimators are given via solving a linear matrix inequality.

The rest of this paper is organized as follows. In section 2, model description and preliminary results are presented. In section 3, the observer guaranteed its states being the same with the states of neural networks with time delay is derived. In section 4, a numerical example is provided to illustrate the effectiveness of our obtained results. Finally, this paper is ended with a conclusion in section 5.

Throughout this paper, the following notations are used. R^n and $R^{n\times m}$, respectively, denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices. For a given vector

$x=(x_1,x_2,...,x_n)^T \in R^n$, $\| x \|=\sqrt{\sum_{i=1}^{n} x_i^2}$ denotes its norm. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X-Y$ is a symmetric semi-definite matrix (respectively, positive definite matrix).

II. Problem Formulation and Preliminaries

Consider the following neural network with time-varying delay

$$\begin{cases} \dot{x}(t) = -Ax(t) + W_0 f(x(t)) + W_1 f(x(t-d(t))) + J + w(t) \\ y(t) = Cx(t) \\ x(t) = \varphi(t),\ t \in [-d_M, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ denotes the neuron state vector, $y(t) \in R^m$ denotes the output, $A = diag\{a_1, a_2, \cdots, a_n\}$, $a_i > 0 (i = 1, 2, ..., n)$, $C \in R^{m \times n}$, $W_0 \in R^{n \times n}$ and $W_1 \in R^{n \times n}$ stand for some known interconnection weight matrices. $J = (J_1, J_2, ..., J_n)^T$ is a known vector, $w(t) \in R^n$ denotes the disturbance and satisfies $\| w(t) \| \leq \varpi$, constant $\varpi \geq 0$ is known. The initial condition $\varphi(t) : [-d_M, 0] \to R^n$ is a continuous function , $d(t)$ denotes the time delay and

$$0 \leq d(t) \leq d_M,\ 0 \leq \dot{d}(t) \leq \tau < 1, \tag{2}$$

where d_M and τ are two known constants. $f(x(t)) : R \to R^n$ is a continuous differential activation function and satisfies Lipschitz condition. For any vector $c = (c_1, c_2, \cdots, c_n)^T$, defined $\| c \|_2^2 = c_1^2 + c_2^2 + \cdots + c_n^2$ and $\| c \|_1 = | c_1 | + | c_2 | + \cdots + | c_n |$.

In this paper, the state estimator is as follows

$$\begin{cases} \dot{\hat{x}}(t) = -A\hat{x}(t) + W_0 f(\hat{x}(t)) + W_1 f(\hat{x}(t-d(t))) + J + D(y(t) - \hat{y}(t)) \\ \qquad + \mu_1^{-1} \cdot C^T sign(y(t) - \hat{y}(t)) \| y(t) - \hat{y}(t) \|_1^{\alpha} \\ \qquad + (\mu_2 \int_{t-d(t)}^{t} (y(\theta) - \hat{y}(\theta))^T (y(\theta) - \hat{y}(\theta)) d\theta)^{\frac{1+\alpha}{2}} \dfrac{C^T (y(t) - \hat{y}(t))}{\| y(t) - \hat{y}(t) \|_2^2} \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{x}(t) = 0,\ t \in [-d_M, 0], \end{cases} \tag{3}$$

where $D \in R^{n \times m}$ is the gain matrix, μ_1 and μ_2 are the constants to be determined in the later, $sign(\cdot)$ is the sign function. Especially, for vector $m = (m_1, m_2, ..., m_n)^T$, $sign(m) = (sign(m_1), sign(m_2), ..., sign(m_n))^T$.

Writing the state error as $e(t) = x(t) - \hat{x}(t),\ \overline{f}(e(t)) = f(x(t)) - f(\hat{x}(t))$, then one gets the following error system

$$\begin{cases} \dot{e}(t) = -(A + DC)e(t) + W_0 \overline{f}(e(t)) + W_1 \overline{f}(e(t-d(t))) + w(t) - \mu_1^{-1} C^T sign(Ce(t)) \| Ce(t) \|_1^{\alpha} \\ \qquad -(\mu_2 \int_{t-d(t)}^{t} e^T(\theta) C^T Ce(\theta) d\theta)^{\frac{1+\alpha}{2}} \dfrac{C^T Ce(t)}{\| Ce(t) \|_2^2} \\ e(t) = \varphi(t),\ t \in [-d_M, 0]. \end{cases} \tag{4}$$

Remark 1. Compared with the fixed-time stability in [12] and [20], the neuron states in (1) does not obtain by measure. The state observer is constructed with the output signal $y(t)$.

In the follows, we give some necessary assumptions and lemmas.

Assumption 1. The activation function $f(x(t))$ satisfies

$$0 \le \frac{f_i(\sigma_1) - f_i(\sigma_2)}{\sigma_1 - \sigma_2} \le l_i,\ \ f_i(0) = 0,\ \ \sigma_1 \ne \sigma_2 \in R,\ \ i = 1, 2, \cdots, n\ , \tag{5}$$

where $l_i (1 \le i \le n)$ are known constants, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$.

In fact, definite $L = diag\{l_1, l_2, \cdots, l_n\}$, for any positive definite diagonal matrix $Z = diag\{z_1, z_2, \cdots, z_n\}$, $z_i > 0 (1 \le i \le n)$, inequality

$$\overline{f}^T(e(t))Z[Le(t) - \overline{f}(e(t))] \ge 0,\ \ \forall e(t) \in R^n \tag{6}$$

holds.

Definition 1[20]. For any initial values $x(0)$ and $\hat{x}(0)$, if there exists a positive constant $T > 0$ such that

$$\lim_{t \to T^-} \| e(t) \|_2 = 0,\ \| e(t) \|_2 = 0,\ \forall t \ge T\ ,$$

then error system (4) is said to be the fixed-time stability.

Lemma 1[21]. If $V(t): R \to R^+$ is a differentiable function, for given positive numbers $0 < \alpha < 1$ and $\beta > 0$, if

$$\dot{V}(t) \le -\beta V^{\alpha}(t)\ , t \ge t_0\ , V(t_0) \ge 0\ , \tag{7}$$

then

$$V^{1-\alpha}(t) \le V^{1-\alpha}(t_0) - \beta(1-\alpha)(t - t_0),\ \ t \in [t_0, t_1]$$

and $V(t) = 0,\ t \ge t_1$, $t_1 = t_0 + \dfrac{V^{1-\alpha}(t_0)}{\beta(1-\alpha)}$.

Lemma 2[22]. For any real number $\theta_i \in R(1 \le i \le n)$, constants $0 < q < 2$, $0 < p \le 1$, then the inequality

$$\Big(\sum_{i=1}^{n} |\theta_i|^q\Big)^{1/q} \ge \Big(\sum_{i=1}^{n} |\theta_i|^2\Big)^{1/2} \tag{8}$$

and

$$\Big(\sum_{i=1}^{n} |\theta_i|^p\Big)^{1/p} \ge \sum_{i=1}^{n} |\theta_i| \tag{9}$$

hold.

The purpose of this paper is to design an appropriate state estimator (3) such that error system (4) is fixed-time stability, so as to ensure that the state of observer (3) is consistent with system (1) in finite time.

III. Main results

Theorem 1. If there exist positive definite diagonal matrix $Z_1 \in R^{n \times n}$ and $Z_2 \in R^{n \times n}$, scalars $\mu_1 > 0,\ \mu_2 > 0$, such that

$$\Upsilon = \begin{bmatrix} -\mu_1(A+DC) - \mu_1(A+DC)^T + \mu_2 C^T C & 0 & \frac{1}{2}L^T Z_1^T + \mu_1 W_0 & \mu_1 W_1 \\ * & -(1-\tau)\mu_2 C^T C & 0 & \frac{1}{2}L^T Z_2^T \\ * & * & -Z_1 & 0 \\ * & * & * & -Z_2 \end{bmatrix} < 0, \tag{10}$$

then system (4) is the fixed-time stability, and $T \le \dfrac{V^{\frac{1-\alpha}{2}}(0)}{\hat{\mu}(1-\alpha)}$, where $*$ is the symmetry elements on the main diagonal symmetry.

Proof. Construct the following Lyapunov function

$$V(t) = \mu_1 e^{\mathrm{T}}(t)e(t) + \mu_2 \int_{t-d(t)}^{t} e^T(s)C^T Ce(s)ds\ .$$

Taking the derivative of $V(t)$ on t, one yields

$$\begin{aligned}\dot{V}(t) &= 2\mu_1 e^{\mathrm{T}}(t)\dot{e}(t) + \mu_2 e^T(t)C^T Ce(t) - \mu_2(1-\dot{d}(t))e^T(t-d(t))C^T Ce(t-d(t)) \\ &\le 2\mu_1 e^{\mathrm{T}}(t)\{-(A+DC)e(t) + W_0\bar{f}(e(t)) + W_1\bar{f}(e(t-d(t))) \\ &\quad - \mu_1^{-1}\cdot C^T \mathrm{s}ign(Ce(t))\|Ce(t)\|_1^{\alpha} \\ &\quad -(\mu_2\int_{t-d(t)}^{t} e^T(\theta)C^T Ce(\theta)d\theta)^{\frac{1+\alpha}{2}} \frac{C^T Ce(t)}{\|Ce(t)\|_2^2}\} \\ &\quad + \mu_2 e^T(t)C^T Ce(t) - (1-\tau)\mu_2 e^T(t-d(t))C^T Ce(t-d(t)).\end{aligned}$$

From (6), for any positive definite diagonal matrix Z_1 and Z_2, there are

$$\bar{f}^T(e(t))Z_1[Le(t) - \bar{f}(e(t))] \ge 0 \tag{11}$$

and

$$\bar{f}^T(e(t-d(t)))Z_2[Le(t-d(t)) - \bar{f}(e(t-d(t)))] \ge 0. \tag{12}$$

From lemma 2, one gets

$$\|Ce(t)\|_2 \le \|Ce(t)\|_1.$$

Noting that

$$\begin{aligned}-2e^{\mathrm{T}}(t)\cdot C^T \mathrm{s}ign(Ce(t))\|Ce(t)\|_1^{\alpha} &= -2(\|Ce(t)\|_1)^{1+\alpha} \\ &\le -2(\|Ce(t)\|_2^2)^{\frac{1+\alpha}{2}} \\ &= -2(e^T(t)C^T Ce(t))^{\frac{1+\alpha}{2}} \\ &\le -2\varepsilon^{\frac{1+\alpha}{2}}(\mu_1 e^T(t)e(t))^{\frac{1+\alpha}{2}}\end{aligned} \tag{13}$$

and

$$\begin{aligned}&-2\mu_1 e^{\mathrm{T}}(t)(\mu_2\int_{t-d(t)}^{t} e^T(\theta)C^T Ce(\theta)d\theta)^{\frac{1+\alpha}{2}} \frac{C^T Ce(t)}{\|Ce(t)\|_2^2} \\ &= -2\mu_1(\mu_2\int_{t-d(t)}^{t} e^T(\theta)C^T Ce(\theta)d\theta)^{\frac{1+\alpha}{2}} \frac{e^{\mathrm{T}}(t)C^T Ce(t)}{e^{\mathrm{T}}(t)C^T Ce(t)} \\ &\le -2\mu_1(\mu_2\int_{t-d(t)}^{t} e^T(\theta)C^T Ce(\theta)d\theta)^{\frac{1+\alpha}{2}}.\end{aligned} \tag{14}$$

Where we use $e^T(t)C^T Ce(t) > \varepsilon\mu_1 e^T(t)e(t)$ for any $\varepsilon > 0$.

Define $\omega(t) = (e^T(t), e^T(t-d(t)), \bar{f}^T(e(t)), \bar{f}^T(e(t-d(t))))^T$ and $\hat{\mu} = \min\{\varepsilon^{\frac{1+\alpha}{2}}, \mu_1\}$, by (10), we have

$$\begin{aligned}\dot{V}(t) &= \omega^T(t)\Upsilon\omega(t) - 2[\varepsilon^{\frac{1+\alpha}{2}}(\mu_1 e^T(t)e(t))^{\frac{1+\alpha}{2}} + \mu_1(\mu_2\int_{t-d(t)}^{t} e^T(\theta)C^T Ce(\theta)d\theta)^{\frac{1+\alpha}{2}}] \\ &\le \omega^T(t)\Upsilon\omega(t) - 2\hat{\mu}[(\mu_1 e^T(t)e(t))^{\frac{1+\alpha}{2}} + (\mu_2\int_{t-d(t)}^{t} e^T(\theta)C^T Ce(\theta)d\theta)^{\frac{1+\alpha}{2}}] \\ &\le -2\hat{\mu}[V(t)]^{\frac{1+\alpha}{2}}.\end{aligned}$$

According to lemma 1, system (4) is the fixed-time stable and $T \le \dfrac{V^{\frac{1-\alpha}{2}}(0)}{\hat{\mu}(1-\alpha)}$.

In system (1), if taking $d(t) \equiv 0$, there is

$$\begin{cases}\dot{x}(t) = -Ax(t) + W_0 f(x(t)) + J + w(t) \\ x(t) = \varphi(t),\ t \in [-d_M, 0].\end{cases} \tag{15}$$

For this system, the following conclusions can be obtained.

Corollary 1. If there exist positive definite diagonal matrices $Z_1 \in R^{n\times n}$ and $Z_2 \in R^{n\times n}$, positive numbers $\mu_1 > 0$ and $\mu_2 > 0$, such that

$$\begin{bmatrix} -\mu_1(A+DC)-\mu_1(A+DC)^T+\mu_2 C^T C & L^T Z_1^T+\mu_1 W_0 \\ * & -Z_1 \end{bmatrix} < 0, \tag{16}$$

then system

$$\begin{cases} \dot{e}(t) = -(A+DC)e(t) + W_0 \overline{f}(e(t)) + w(t) - \mu_1^{-1} C^T \operatorname{sign}(Ce(t)) \| Ce(t) \|_1^{\alpha} \\ e(t) = \varphi(t),\ t \in [-d_M, 0] \end{cases}$$

is the fixed-time stable.

IV. A Simulation Example

Consider the time delay neural networks (1) with the following parameters, where

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ W_0 = \begin{bmatrix} 3 & -5 & 6 \\ -2 & 1 & 1 \\ -2 & 2 & 4 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -2 & 4 & 1 \\ 3 & -5 & 1 \\ -2 & 4 & -5 \end{bmatrix},\ L = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},\ J = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.7 \end{bmatrix},$$

$$f(x) = 5\tanh(x),\ w(t) = (\sin t, \cos t, \sin 2t)^T,\ d = 1,\ \tau = 0.5.$$

According to Theorem 2 and using the LMI toolbox in MATLAB, the feasible solutions satisfying inequality (10) are obtained as follows:

$$D = \begin{bmatrix} 5.5131 & -0.2395 & -0.5539 \\ -0.2395 & 4.4559 & 0.1134 \\ -0.5539 & 0.1134 & 3.6272 \end{bmatrix},\quad Z_1 = \begin{bmatrix} 0.0603 & 0 & 0 \\ 0 & 0.1174 & 0 \\ 0 & 0 & 0.1012 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 1.4621 & 0 & 0 \\ 0 & 1.5658 & 0 \\ 0 & 0 & 1.4796 \end{bmatrix},\quad \mu_1 = 2,\quad \mu_2 = 2.3620.$$

Setting the initial states as $x_0 = (2,-3,-4)^T$ and $\hat{x}_0 = (-3,5,3)^T$, the state curve of system (1) is shown in Figure 1, the state curve of observer (3) is shown in Figure 2, the state curve of error system (4) is shown in Figure 3, respectively. These results show that observer (3) can be consistent with neural network (1) in fixed time.

Figure 1. State curve of system (1)

Figure 2. State curve of observer (3)

Figure 3. State curve of error system (4)

V. Conclusions

This paper investigates a state estimator for a class of neural networks with time delay. By constructing an appropriate Lyapunov function and using inequality techniques, some conditions have been obtained to ensure that the state estimator is consistent with the state of the estimated system in a fixed time, and the upper bound have been estimated. Finally, the effectiveness of the results has been proved by numerical simulation.

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