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Imperatives of Power Factor on the Performance of Electric Power System

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Abstract: *The utmost goal of this study is to X-ray the imperativeness of power factor (p.f) on the performance of electric power system, which include; electric power generator, transmission and reception (load), and the associated cost implications to electric power industries. System's performance parameters such as magnetizing current, armature reaction, leakage reactance, air gap length, machine rating, voltage regulation, efficiency and so on are adversely affected by its power factor (p.f). Power factor changes according to loads on a system. User loads change all the time as equipment is switched on and off. Variations (increasing, decreasing or unity) in power factor have a devastating effect on electric power system lines and machines. As power factor is one amongst the prevailing concerns in electrical industries, a striking balance (corrective measures) of achieving a suitable parity for optimum power factor in electrical power system becomes the Engineer's target and desires.*

Keywords: *Armature reaction, lagging power factor, leading power factor, magnetizing current, phase angle reactive power, unity power factor.*

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I. Introduction

Electrical powers in all alternating current networks are comprised of three components, such as real power, reactive power and apparent power. The real power, also known as true or active power is the work producing component of alternating current power, measured in Watts (W) or kiloWatts (kW). It performs the true work within an electrical network. It is the instantaneous product of voltage and current, which represents the ability of electricity for performing work. It defines the power consumed by the resistive part of a network. As resistance do not produce any phase differences between voltage and current wave form, all the useful power is delivered directly to the resistance and converted to heat (as in heating elements such as boilers, immersion heaters etc), light (as in incandescent lamps etc and work (as in moving systems such as relays etc). All the electric power entering the resistive load is consumed or dissipated and is measured in Watts. As there exists no phase difference between the voltage and the current in such resistive network, the phase shift (angle) between the two wave forms is zero.

Reactive power, on the other hand, does not do any work but indispensably needed to operate equipment. It is also known as wattles power and is consumed in alternating current network that performs no useful work but has an effect on the phase shift between voltage and current wave forms. It is linked to the reactance produced by coils (inductors) and chokes (capacitors) and counteracts the effects of true power. Where reactive loads are present, such as with capacitors and or inductors, energy storage in the loads results in a phase difference between the current and voltage waveforms. Unlike active power, reactive power, usually expressed in voltage amperes-reactive (VAR) or kilo Var (kVAR), takes power away from a network due to the creation of inductive magnetic field and capacitive electrostatic fields, thereby making it difficult for the active power to supply power directly to load. During each cycle of the alternating current voltage, extra energy in addition to any energy consumed in the load is temporarily stored in the load in electric or magnetic fields, then returned to the transmission lines a fraction of the period later (Power factor-wikipedia). Hence, power stored by an inductor (coil) in its magnetic field tries to control the current while the power stored by a capacitor's electrostatic field tries to control the voltage.

Obviously, while real power is dissipated by resistance, reactive power is supplied to a reactance. Hence, there exists a mathematical relationship between the real power and the reactive power known as complex power. Together, they form the complex power expressed as Volt-amperes (VA or kVA), whose magnitude is known as the apparent power. In the absence of harmonics, apparent power, also known as demand power is comprised of (vectorial sum) both real and reactive power. Apparently, the VA and VAR are non SI units, mathematically identical to the Watt, but are used in electrical engineering field instead of the Watt to state what quantity is being expressed.

In electrical engineering and other allied disciplines, power factor of an alternating current electrical power system is an important part of alternating current network, defined as the ratio of the active power taken in by a load to the apparent power flowing in the network. It is generally expressed as either a decimal value or as a percentage. Power factor is a measure of how effectively electrical power is being used, and how much real power electrical equipment utilizes. Power factor in other words is a measure of how much reactive power are real power and consumed in a given apparent power relating. Power factor is a dimensionless parameter in the close interval of **– 1** to **+ 1**. The afore-mentioned three elements which make up power in an alternating current network can be represented graphically by the three sides of a right-angled triangle (power triangle in vector space). The active power extends horizontally in the $\hat{\imath}$ direction (adjacent side), while the reactive power extends vertically in the $\hat{\jmath}$ direction (opposite side). The resulting complex power, and its associated magnitude (Apparent power), representing the hypotenuse, can be calculated, using the vector sum of the active and reactive components.

If we let us suppose that the active and reactive components of the power are represented by $\mathbf{P_a}$ and $\mathbf{P_r}$ respectively, then the apparent power $\mathbf{P_A}$ is given by;

$$P_A = \hat{\imath}\ P_a + \hat{\jmath}\ P_r \qquad 1$$

$$\Rightarrow |P_A| = \sqrt{|P_a|^2 + |P_r|^2} \qquad 2$$

Also power factor (p.f) $= \frac{P_a}{P_A}$ 3

From the power triangle in vector space;

Power factor (p.f) $= \frac{VI\ Cos\ \phi}{V1} = Cos\phi$ 4

Where ϕ (lies between 0^0 - 90^0) is the phase angle difference between P_a and P_A

It should be recalled that in a pure resistive network, the current and voltage wave forms are in phase with each other, so that the phase difference is 0^0. Hence power factor (p.f) in equation (4) becomes unity (as Cos O^0 = 1). The implication of Cos 0^0 = 1 to equation (3) is that the real power (P_a) consumed is the same as the apparent power P_A supplied, which means that all the energy supplied by the source is consumed by the load.

Additionally, in a pure reactive network, the current and voltage wave forms are out-of-phase with each other by 90^0. Hence power factor (p.f) as applicable to equation (4) becomes 0 (as Cos 90^0 = 0). The implication of Cos 90^0 being equal to 0 to equation (3) is that the number of watts consumed is zero but there is still a voltage and current supplying the reactive load. The energy flow is entirely reactive and stored energy in the load returns to the source on each cycle.

Obviously, from the fore-going, reducing the reactive VAR component of the power triangle will correspondingly reduce the phase angle (ϕ), thereby improving the power factor. This is the expected engineer's target and desire in the economics of improving power factor.

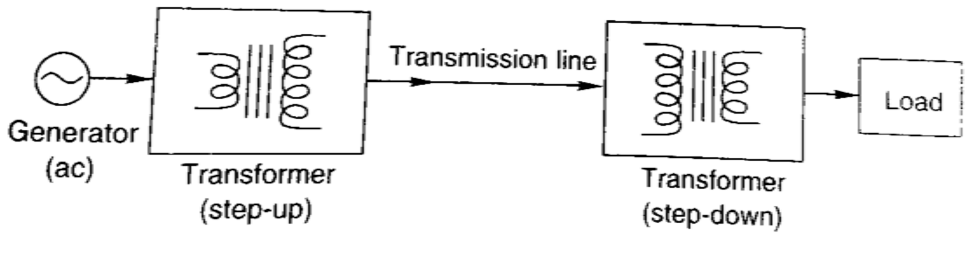

Figure 1 – Simple Electric Power System (D.P Kothari, I.J Nagrath 2007)

II. Effect of Power factor (p.f) on the Performance of Synchronous Generator (Alternator)

A synchronous machine is one of the important types of electric machines, infact all generating machines at power stations are of synchronous type and are known as synchronous generators or alternators (D.P Kothari and I.J. Nagrath 2007).

The alternators produce 3-phase power from mechanical power and are the primary source of all the electrical energy we consume, as they export electrical power to the bus bar. By convention, a synchronous generator operating with a lagging power factor is producing VAR while one operating with leading power factor is consuming VAR. Interestingly; most synchronous generators are designed to operate at lagging power factors.

Plate 1 – Water cool, diesel engine operated alternator.

2.1 – Alternator on no Load

When an alternator is running at no-load, there will be no current flowing through the armature winding. The flux produced in the air-gap will be only due to the rotor ampere-turns. The length of air-gap in synchronous generators is an important design parameter, because its value greatly influences the performance of the machine. Air gap contribution to the no-load magneto motive force (mmf) is quite large. If a synchronous generator is designed with a large air gap, the armature mmf will be large, thus reducing the effect of armature reaction.

2.2 – Alternator on Load

When a synchronous generator is connected to a load, and the load increases, the corresponding armature current (I_a) increases, the field excitation and speed being kept constant, the terminal voltage (V_t) (Phase voltage) of the alternator decreases. This results, due to;

a) Voltage drop I_aR_a due to the armature resistance (R_a) per phase.

b) Voltage drop I_aX_L due to the armature leakage reactance (X_L) per phase

c) Voltage drop due to armature reaction. The load is generally inductive and the effect of armature reaction is to reduce the generated voltage. Since armature reaction results in a voltage effect in a circuit caused by the change in flux produced by current in the same circuit, its effect is of the nature of an inductive reactance. This drop in voltage is due to the interaction of armature and main flux and it is not across any physical element. To include this drop, it is assumed that armature winding has a fictitious reactance which is known as armature reaction reactance (X_{ar}), whose value is such that I_aX_{ar} represents the voltage drop due to armature reaction. The effect of armature flux on the flux produced by field ampere-turns or rotor ampere-turns (main flux) is called armature reaction (V.K Mehta and Rohit Mehta 2000). The effect of the nature of load power factor on armature reaction is given below. When a resistive load (unity power factor) is connected across the terminals of an alternator, the effect of armature reaction is merely to distort the main field; there is no weakening of the main field and the average flux in the air gap practically remains unchanged. Since magnetic flux due to stator current (armature flux) rotates synchronously with the rotor, the flux distortion remains the same for all positions of the rotor. This distorting effect of armature reaction under unity power factor condition of the load is known as cross magnetizing effect of armature reaction.

Additionally, when a pure inductive load (zero power factor lagging) is connected across the terminals of the alternator, current lags behind the voltage by 90^0. All the flux (armature flux) produced by armature current opposes the field flux and, therefore, weakens it. Hence, armature reaction is directly demagnetizing. At zero power factors lagging, the armature reaction weakens the main flux. This brings about a reduction in the generated e.m.f (E_{ph}).

More-still, when a pure capacitive load with zero power factor leading is connected across the terminals of the alternator, the current in the armature winding will lead the induced e.m.f by 90^0. Obviously, the effect of armature reaction will be the opposite for that of pure inductive load. The effect of armature reaction is wholly magnetizing, and consequently, the flux in the air-gap is increased. Thus the armature flux strengthens the main flux (field flux), and the resultant effect is that generated e.m.f is increased.

For intermediate values of load power factor, the effect of armature reaction is partly distorting and partly weakening for inductive load. For capacitive loads, the effect of armature reaction is partly distorting and strengthening. It can now be inferred that the change in terminal voltage of alternator is due to the armature reaction for different kinds of load. In practice, all loads on the alternator are generally inductive. Hence, the reduction of terminal voltage is due to demagnetizing effect of armature reaction.

The overall reactance of the armature winding is the sum of its leakage reactance (X_L) and the fictitious reactance (X_{ar}) which is known as synchronous reactance (X_s).

$$\therefore X_S = X_L + X_{ar} \; \Omega/phase \qquad 5$$

Similarly, the synchronous impedance (Z_s) of armature winding is the complex sum of the effective armature windings resistance (R_a) and synchronous reactance (X_s).

$$\therefore Z_s = R_a + jX_s$$
$$= \sqrt{(R_a)^2 + (X_s)^2} \; \Omega/Phase \qquad 6$$

The synchronous reactance (X_s) depends upon the load and its power factor condition. Therefore, synchronous impedance (Z_s) also depends on the load and its power factor.

2.2.1 – Equivalent Circuit of an Alternator

Figure 2 shows the single line diagram of the equivalent circuit of a loaded alternator for one phase

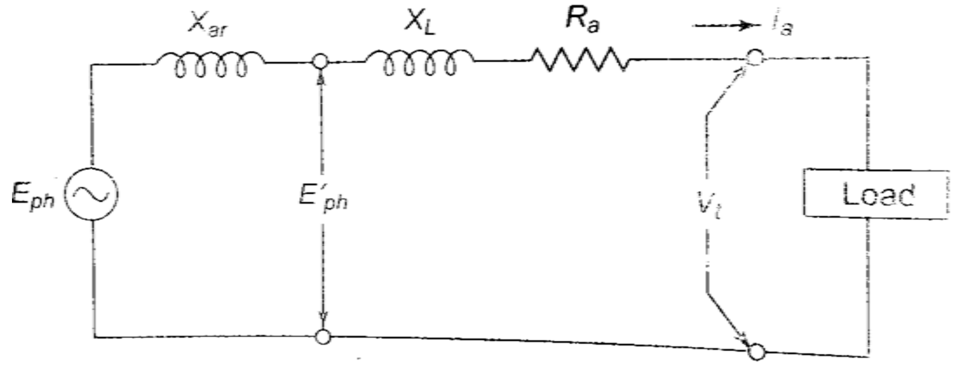

Figure 2 – The per phase equivalent circuit of a loaded alternator

Since the synchronous reactance is a fictitious reactance employed to account for the voltage effects in the armature circuit produced by the actual armature leakage reactance and the change in the air-gap flux caused by armature reaction, combining equations (5) and (6), circuit of figure 2 reduces to the one shown in figure 3.

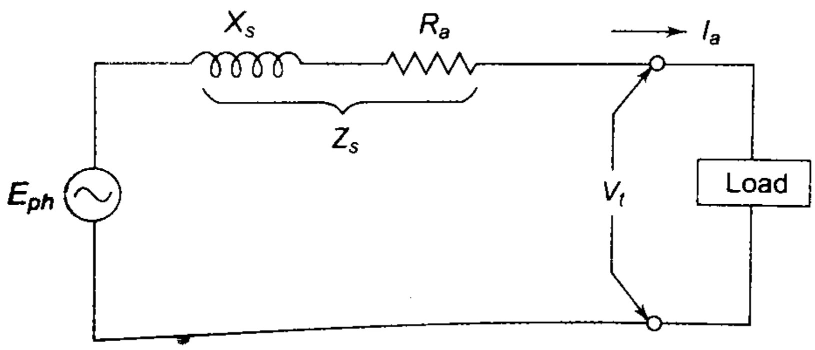

Figure 3- Modified per phase equivalent circuit of a loaded alternator

From figure 2, E_{ph} = No-loaded e.m.f, E'_{ph} = load induced e.m.f. E'_{ph} is the induced e.m.f after allowing for armature reaction. It is equal to phasor difference of E_{ph} and I_aX_{ar}. V_t = Terminal voltage. It is less than E'_{ph} by voltage drop in X_L and R_a.

At no-load, $I_a = 0$, hence, $V_t = E_{ph}$. During the presence of load, E_{ph} will overcome the voltage drops across R_a, X_L and X_{ar}

$\therefore$ The voltage equation of alternator is given by;

$$\bar{E}'_{ph} = \bar{V}_t + \bar{I}_a (R_a + j X_L)$$
$$\text{Also, } \bar{E}_{ph} = \bar{E}'_{ph} + \bar{I}_a (j X_{ar})$$
$$= \bar{V}_t + \bar{I}_a (R_a + j X_s)$$
$$= \bar{V}_t + \bar{I}_a \bar{Z}_s \qquad 8$$

The cosine of the angle between I_a and V_t is called the power factor. Depending on the nature of power of the load, a different phasor diagram of the alternator can be drawn.

2.3- Phasor Diagram of Alternator

The phasor diagrams of an alternator are drawn for lagging, unity and leading power factor respectively.

2.3.1- Lagging power factor

Consider an alternator supplying inductive load. The equipment circuit of the alternator per phase is shown in figure 2. Figure 4 shows the phasor diagram of the alternator for lagging power factor load.

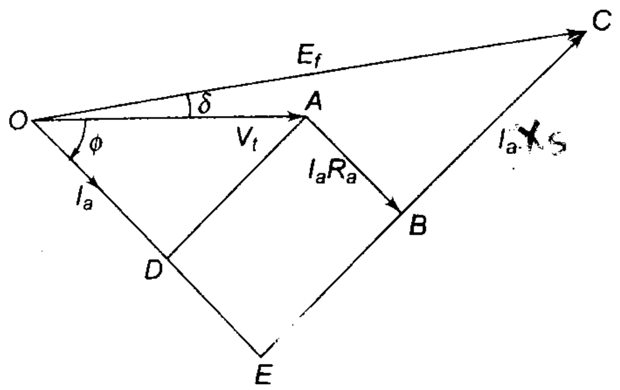

Fig 4 - Phasor diagram of an alternator for the usual case of inductive load (lagging power factor load).

From figure 4, OC = E_{ph} = No-load e.m.f, OA = V_t = Terminal voltage I_a = Armature current, R_a = Effective armature resistance, X_s = synchronous reactance, Cos ϕ = Power factor, δ = Power (torque) angle.

AB = DE = I_aR_a, BC = I_aX_s

Also, $|OC|^2 = |OE|^2 + |EC|^2$

$= |OD + DE|^2 + |EB + BC|^2$

$\therefore |E_{ph}|^2 = |V_t \cos\phi + I_aR_a|^2 + |V_t \sin\phi + I_aX_s|^2$

$$\Rightarrow E_{ph} = \sqrt{|V_t \cos\phi + I_aR_a|^2 + |V_t \sin\phi + I_aX_s|^2} \qquad 9$$

2.3.2- Unity Power Factor

In drawing the phasor diagram for unity power factor as in figure 5, armature current (I_a) has been taken as the reference phasor.

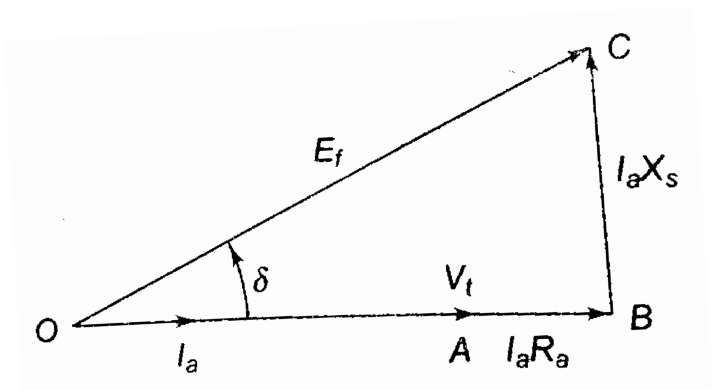

Figure 5: Phasor diagram of an alternator for the case of unity power factor load.

From the phasor diagram of figure 5, we have that;

$$|OC|^2 = |OB|^2 + |BC|^2$$
$$= |OA + AB|^2 + |BC|^2$$

$$\Rightarrow |E_{ph}|^2 = |V_t \cos\phi + I_a R_a|^2 + |I_a X_s|^2$$

$$\therefore E_{ph} = \sqrt{|V_t \cos\phi + I_a R_a|^2 + |I_a X_s|^2} \qquad 10$$

2.3.3 Leading Power Factor

In drawing the phasor diagram for leading power factor, as in figure 6, terminal voltage has been taken as the reference phasor.

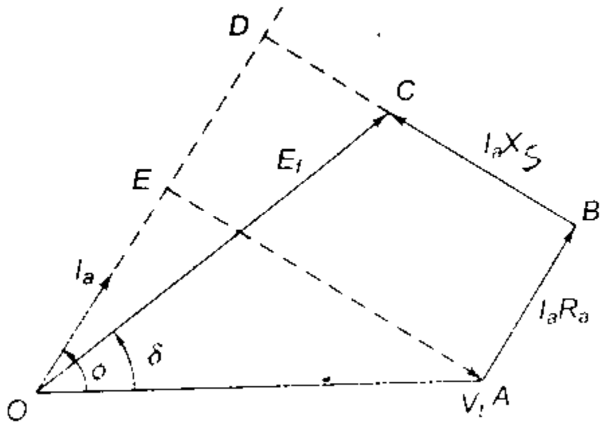

Figure 6 – Phasor diagram of an alternator for the case of leading power factor load.

From figure 6, we can write that;

$$|OC|^2 = |OD|^2 + |DC|^2$$
$$= |OE + ED|^2 + |DB - CB|^2$$
$$\therefore |E_{ph}|^2 = |V_t \cos\phi + I_a R_a|^2 + |V_t \sin\phi - I_a X_s|^2$$

$$\Rightarrow E_{ph} = \sqrt{|V_t \cos\phi + I_a R_a|^2 + |V_t \sin\phi - I_a X_s|^2} \qquad 11$$

2.4- Effect of Power Factor on Voltage regulation of alternator

Constant voltage is the requirement of most domestic, commercial and industrial loads. It is therefore, necessary that the output voltage of an alternator must stay within a narrow limits as the load and its power factor vary. When an alternator is loaded, its terminal voltage (V_t) may not be equal to the induced e.m.f (E_{ph}). It may be less than or greater than the induced e.m.f. At no-load, I_a is zero and hence $V_t = E_{ph}$. If speed and field current remain constant after removal of load at a given power factor, the voltage regulation is given by;

$$\text{Voltage regulation} = \frac{No-load\ voltage - Full\ load\ voltage}{Full\ load\ voltage}$$
$$= \frac{E_{ph} - V_t}{V_t}$$
$$= \frac{E_{ph}}{V_t} - 1 \quad \text{in per unit} \qquad 12$$
$$= \left(\frac{E_{ph}}{V_t} - 1\right) \quad \text{x 100 in percentage} \qquad 13$$

For E_{ph} and V_t, keeping speed and field excitation unchanged;

For lagging power factor load, and seeing from equation (9), E_{ph} increases, and voltage regulation as applicable to equation (12) is positive. Conversely, for leading power factor load, and seeing from equation (11), E_{ph} decreases and voltage regulation as applicable to equation (12) may be negative (Smarajit Ghosh 2007). Since the regulation of an alternator depends on the load and the load power factor, it is pertinent to maintain power factor while expressing regulation.

III. Effect of Power factor (p.f) on the Performance of Power Transformer

Although the static transformer is not an energy conversion device, it is an indispensable component I many energy conversion systems. As a significant component of alternating current power systems, it makes possible electric generation at the most economical generator voltage, power transfer at the most economical transmission line and power utilization at the most suitable voltage for the particular utilization device, see fig 1 (A.E Fitzgerald et al 2003). These are the attributes of transformer's electrical power transformation capacity.

Plate 2 – Power distribution transformer, near K.A.C Lodge Ifite Awka

3.1- Equivalent Circuit of a Practical Transformer

Figure 7, shows a practical transformer having winding resistances (R_1 |R_2) and leakage resistances (X_1 |X_2). These are the actual conditions that exist in a transformer. There is voltage drop in R_1 and X_1, so that primary induced e.m.f (E_1) is less than the applied (excitation) voltage (V_1). Similarly, there is voltage drop in R_2 and X_2, so that secondary terminal voltage (V_2) is less than the secondary induced e.m.f (E_2).

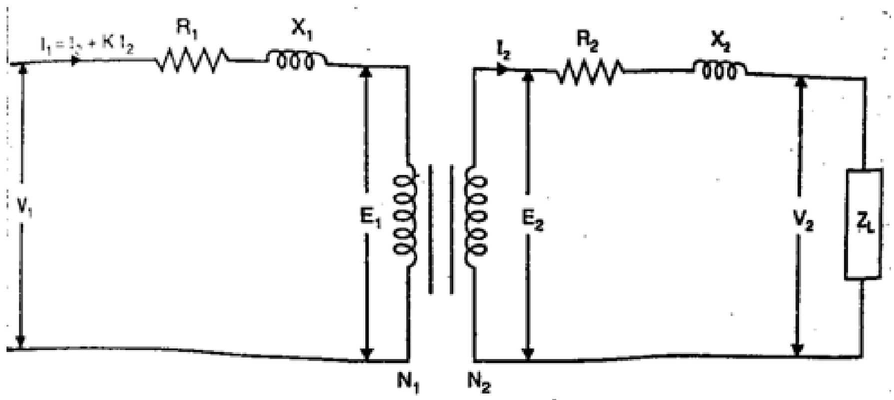

Figure 7 – Per phase equivalent circuit of a practical transformer

3.2 – Phasor diagram of a practical transformer on load

When an inductive load (with a lagging power factor) is connected to the secondary terminals of a power transformer, a secondary current (I_2), which lags behind the secondary voltage (V_2) by phase angle (ϕ_2) is caused to flow as depicted in figure 7. The total primary current (I_1) must satisfy two conditions Viz,

i. I_1 must supply the no-load current (I_0 to meet the iron losses in the transformer and to provide flux in the core

ii. I_1 must supply a current I'_2 (balancing current), necessary to provide a restoring mmf required to counteract the demagnetizing effect of the secondary current I_2 . The magnitude of I'_2 will be such that;

.$N_2 I_2 = N_1 I'_2$, from which

$I'_2 = \left(\frac{N_2}{N_1}\right) I_2 = aI_2$ 14

Hence, the total primary current I_1 is the phasor sum of I'_2 and I_0

$\Rightarrow I_1 = I'_2 + I_0 = I_0 + (-aI_2)$ 15

The negative sign of equation 15 merely indicates that I_2 and I'_2 are in opposite directions to each other. For a power transformer having resistances R_1 and R_2 of primary and secondary windings respectively and leakage reactances X_1 and X_2 of primary and secondary windings respectively as in figure 8, the primary impedance (Z_1) and secondary impedance (Z_2) are given by;

$$\left.\begin{aligned} Z_1 &= R_1 + jX_1 \\ Z_2 &= R_2 + jX_2 \end{aligned}\right\} \quad 16$$

$\therefore$ The applied voltage V_1 on the primary side of the transformer is given by;

$$\left.\begin{aligned} V_1 &= -E_1 + I_1 Z_1 \\ &= -E_1 + I_1 (R_1 + jX_1) \\ &= -E_1 + I_1 R_1 + jI_1 X_1 \end{aligned}\right\} \quad 17$$

Similarly, if V_2 is the secondary terminal voltage during load, the secondary induced e.m.f E_2 is given by;

$$\left.\begin{aligned} E_2 &= V_2 + I_2 Z_2 \\ &= V_2 + I_2 (R_2 + jX_2) \\ &= V_2 + I_2 R_2 + jI_2 X_2 \end{aligned}\right\} \quad 18$$

Note that counter e.m.f that opposes the applied voltage V_1 is - E_1

$$\left.\begin{aligned} &\text{No Load power factor} = \cos\phi_0 \\ &\text{Load power factor} = \cos\phi_2 \\ &\text{Primary power factor} = \cos\phi_1 \\ &\text{Input power } (P_I) \text{ to transformer} = V_1 I_1 \cos\phi_1 \\ &\text{Output power } (P_2) \text{ of transformer} = V_2 I_2 \cos\phi_2 \end{aligned}\right\} \quad 19$$

Using equations (17) and (18) and the power factors of equation 19, the phasor diagram of a practical transformer for the usual case of inductive load (lagging power factor) is shown in figure 8

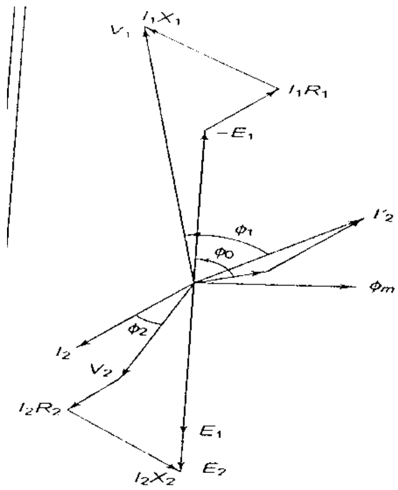

Figure 8: The phasor diagram of a practical transformer on load at lagging power factor.

Similarly, when a resistive load with unity power factor is connected to the secondary terminals of the transformer, both the secondary terminal voltage (V_2) and secondary current (I_2) will be in phase with each other along with the drop $I_2 R_2$ as shown in the phasor diagram of figure 9.

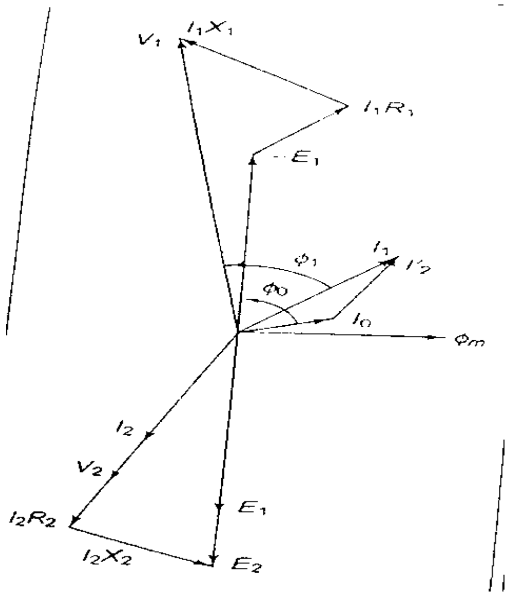

Figure 9 – The phasor diagram of a practical transformer on load at a unity power factor

Additionally, when a capacitive load with a leading power factor is connected across the output terminals of the transformer, the secondary current (I_2) leads the secondary terminal voltage (V_2) as shown in the phasor diagram of figure 10.

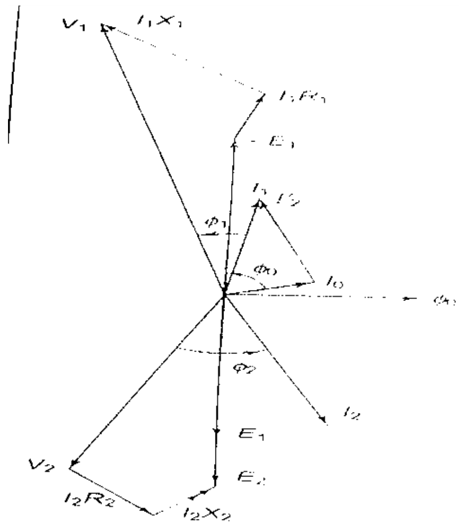

Figure 10: The phasor diagram of a practical transformer on load at a leading power factor

3.3- Effect of Power Factor on the Voltage regulation of Transformer

Constant voltage is the utmost requirement of all electrical loads. This requirement is more stringent in distribution transformers as these directly feed the load centres. The voltage drop in a transformer on load is chiefly determined by its leakage reactance, which must be kept as low as design and manufacturing techniques would permit. The figure of merit which determines the voltage drop characteristic of a transformer is the voltage regulation.

Voltage regulation in transformer is defined as the change in magnitude of the secondary terminal voltage when full load at specified power factor supplied at rated voltage is thrown off, that is reduced to no-load with primary voltage held constant, as percentage of the rated load terminal voltage (D.P Kothari and I.J Nagrath 2004).

If we let us suppose that; V_{2n} = Secondary terminal voltage at no load and V_{2f} = Rated secondary terminal voltage while supplying full load at specified power factor, then there are three kinds of voltage regulation as given below;

i. Inherent voltage regulation $(\mu_i) = V_{2n} - V_{2f}$ 20

ii. Voltage regulation down (μ_D) $= \frac{V_{2n} - V_{2f}}{V_{2n}} = 1 - \frac{V_{2f}}{V_{2n}}$ 21

iii. Voltage regulation up (μ_U) $= \frac{V_{2n} - V_{2f}}{V_{2f}} = \frac{V_{2n}}{V_{2f}} - 1$ 22

It is worthy to note that the secondary terminal is dependent on load current (I_2) and load power factor (Cos ϕ_2). V_{2f} drops steadily with increasing load current. For inductive load with lagging power factor, V_{2f} is less than the secondary induced e.m.f (E_2), (as seen in equation 18), the voltage regulation is positive. For capacitive load with inherent leading power factor, V_{2f} is greater than the secondary induced e.m.f (E_2), hence, the voltage regulation is negative.

However, zero voltage regulation is possible in practical transformers. From the fore-going, for lagging power factor and unity power factor, V_{2n} is greater than V_{2f}. Therefore, we get positive voltage regulation. For leading power factor, V_{2f} begins to increase. At a particular value of leading power factor V_{2n} equals V_{2f} and hence voltage regulation becomes zero. If the load power factor continues to increase, V_{2n} becomes less than V_{2f} and regulation becomes negative.

Hence, for zero voltage regulation, $V_{2n} - V_{2f} = 0$ 23

Based on the information above, equation (20), can be modified to hold for equations 21, 22, and 23 and below,

Inherent voltage regulation (μ_i) = $V_{2n} \pm V_{2f}$

$= I_2 \ (R_{02} \cos\phi \pm X_{02} \sin\phi)$ 24

R_{02} and X_{02} are the equivalent resistance and reactance referred the secondary side of the transformer, + sign is for lagging power factor and – sign is for leading power factor.

From equation (23), if $V_{2n} - V_{2f} = 0$,

$\Rightarrow I_2 \ (R_{02} \cos\phi - X_{02} \sin\phi) = 0$

$\Rightarrow I_2 \ R_{02} \cos\phi = I_2 \ X_{02} \sin\phi$

$\Rightarrow \frac{R_{02}}{X_{02}} = \frac{\sin\phi}{Cos\ \phi} = \tan\phi$

$\Rightarrow \cos\phi = \cos\left(tan^{-1} \frac{R_{02}}{X_{02}}\right)$ 25

Equation (25) gives the leading power factor ($\cos\phi$) at which voltage regulation becomes zero.

It can be seen from equation (24) that the voltage regulation varies with power factor and has a maximum value when;

$\frac{d\mu_D}{d\phi} = 0$ or $\frac{d\mu_D}{d\phi} = 0$ 26

$\Rightarrow \frac{d}{d\phi}\left(\frac{I_2 \ (R_{02} \ Cos\ \phi - X_{02} \sin\phi)}{V_{2n}}\right) = - R_{02} \sin\phi + X_{02} \cos\phi = 0$

$\Rightarrow X_{02} \cos\phi = R_{02} \sin\phi$

$\Rightarrow \frac{\sin\phi}{Cos\ \phi} = \frac{X_{02}}{R_{02}} = \tan\phi$

$\therefore \cos\phi = \cos\left(tan^{-1} \left(\frac{X_{02}}{R_{02}}\right)\right)$

$= \frac{R_{02}}{\sqrt{(R_{02})^2 + (X_{02})^2}}$ 27

Equation (27) suggests that voltage regulation is the maximum when the load power factor (lagging) angle has the same value as the angle of the equivalent impedance.

IV. Effect of Power Factor (p.f) on the Performance of Transmission lines

Transmission line is a system of conductors connecting one point to another and along which electro-magnetic energy can be sent. The conductor system by means of which electric power is conveyed from a generating station to the consumer's premises may, in general, be divided into two distinct parts, that is, transmission system and distribution system. Each part is further sub-divided into two- Primary transmission and Secondary transmission and similarly, Primary distribution and Secondary distribution and then finally the system of supply to individual consumers. An important feature of every transmission line is that it should guide energy from a source at the sending and to a load at the receiving end without loss by radiation (John bird 2010).

Plate 3 – Transmission line, feeding a distribution transformer, near K.A.C lodge Ifite Awka.

Figure 11 shows the case of a short transmission line, with transmission parameters Resistance (R), Inductance (L) and Capacitance (C). When the generator voltage (E_s) is corrected, a current (I_s) flows, which divides between the capacitor (which progressively charges the capacitor) and that which sets up the voltage travelling wave moving along the transmission line (B.L Theraja and A.K Theraja 2009).

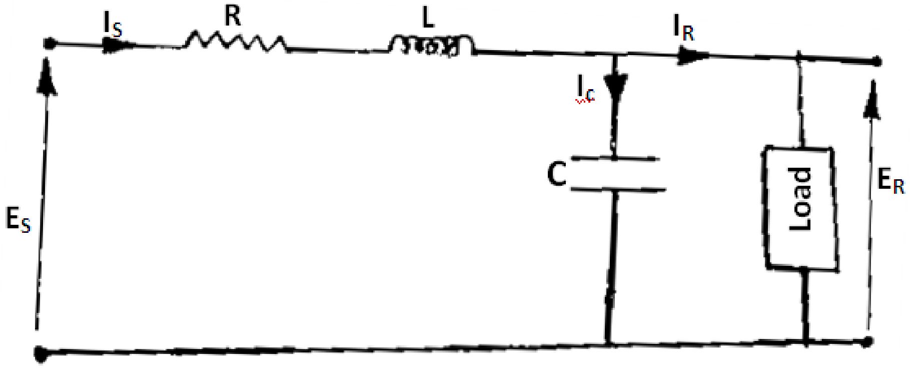

Figure 11 – Short transmission line with transmission parameters.

In figure 11 above, R represents the resistance of the transmission line given by R = $\frac{\rho L}{A}$, where ρ is the resistivity of the conductor material, A is the cross-sectional area of each conductor and L is the length of the conductor. Resistance is stated in Ohms per metre length of the line and represents the imperfection of the conductor.

The inductance (L) is due to the magnetic field surrounding the conductors of a transmission line when a current flows through them. An inductance stated in Henrys per loop metre takes into consideration, the fact that there are two conductors in a particular length of line.

Capacitance (C) on the other hand exists as a result of electrical field between conductors of a transmission line. The capacitance between two conductors is stated in Farads per metre (John Bird 2007).

As power is transferred along a transmission line, it does not consist purely of real power that can do work once transferred to the load, but rather consists of a combination of active and reactive power. The power factor in electric power lines describes the amount of active power transmitted along a transmission line relative to the total apparent power flowing in the line.

It can be recalled from equation 3 that;

Power factor (p.f) $= \frac{P_a}{P_A} = \frac{kW}{kVA}$

$\Rightarrow I = \frac{kW}{V \times Power\ factor}$ 28

We also know that;

Resistance (R) in a transmission line $= \frac{\rho \times L}{A}$ 29

Then from Ohim's law; $I = \frac{V \times A}{\rho \times L}$ 30

Where V = supply voltage, A = conductor's cross-sectional area, ρ = resistivity of the conductor material, L = length of the line. I = current flowing in the line.

Analysis from equation (30) reveals that when power system lines serve a facility that has poor power factor, the lines must be capable of supplying higher current (I) levels to serve a given load. Cables of larger cross-sectional areas (A) are required to carry larger current (I), with a greater increase in transmission loss (I^2R). Hence giving rise to poor voltage regulation with decreased efficiency.

In electric transmission lines, reactive loads cause a continuous "ebb and flow" of non productive power. A network with a low power factor will use a greater amount of current to transfer a given quantity of true power, thus causing increased losses due to resistive heating in power line and requiring the use of higher rated conductors and transformers.

However, as power factor of a transmission line is improved, the total current flow will be reduced; which permits additional loads to be added and served by the existing system.

V. Effect of Power Factor (P.F) on the performance of induction motor

The induction motors are asychrnonous speed machines, operating below synchronous speed. They indeed draw large starting current, typically about six to eight times their full load values and operate with a poor lagging power factor when lightly loaded (Chee-mum ong 1998). Like any electric motor, the induction motor has a stator and a rotor, separated by air-gap, whose length varies, depending on motor design. The presence of air gap between the stator and rotor of induction motor greatly increases the reluctance of the magnetic circuit. Consequently, an induction motor draws a large magnetizing current to produce the required flux in the air-gap.

Power factor isn't supplied to the motor, rather, it is the effect of the motor on the power supply. Most loads such as induction motors are inductive in nature which have power factor less than unity, which implies a part of current and voltage is being used unnecessarily and at the consumer end, one only pays for active power consumption. This power has a say on the motor's design parameters, such as conductor size, air gap length etc, which invariably affect the machine efficiency.

Selection of the right motor for an application can have a major impact on power factor. A lightly loaded induction motor requires little real power and a heavily loaded motor requires more real power. Since reactive power is almost constant, the power factor varies with the loading of a motor.

Plate 4: An induction motor (electrical load) fed with 415V, 3-phase lives and driving a grinding mill (mechanical load). Courtesy: Eke-Awka grinding mill lines, Anambra State.

5.1- Induction motor on no load

When an induction motor is not connected to any load, it runs at almost synchronous speed. The stator draws a very negligible current. The rotor current is fully magnetizing current or lagged by almost 90^0 from the E.m.f induced in the rotor bars; given by $E_r = sE_S$. Where E_r = E.m.f. induced in the rotor bars, s = slip of the induction motor, E_s = E.m.f induced the stator. During no load condition, slips is very low about 0.03, which makes the rotor E.m.f (E_r) very low and armature current is also negligible and lagged 90% from the terminal voltage of rotor (V_2), which causes power factor very low. The amount of magnetizing current drawn by the rotor depends on the length of air gap, hence total ampere turns (mmf) require to overcome the reluctances of air gap. The operating power factor is lesser for motor which draw lighter magnetizing current. Hence, the power factor of an induction motor on no load is low.

5.2- Induction Motor on Load

When we apply mechanical load to the shaft of an induction motor, it will begin to slow down and the rotating flux will cut the rotor conductors at a higher rate. The induced voltage and resulting current in the rotor conductors will increase progressively, producing greater torque. Under load condition of the motor, the slip is higher than no load, the E_r increases and causes more torque developed on the armature bars and speed is decreasing during on load condition. Power factor goes high about 0.7-0.8 lagging. When load on motor increases, the power factor improves as the active power increases for the same amount of reactive power.

The huge load connected to the shaft causes more current to flow through the armature part and makes the motor under excited, this huge current causes more copper losses or variable losses in the armature and makes the increase the input power which reduces the efficiency of the motor.

5.3- Equivalent Circuit of Induction Motor

An induction motor is essentially a transformer with stator forming the primary and rotor forming (the short-circuited) rotating secondary as shown in figure 12. This is so because the transfer of energy from the stator to the rotor of an induction motor takes place entirely inductively with the help of flux mutually linking the two. However, the equivalent circuit of an induction motor is similar to that of a transformer of figure 7. The main difference is that rotor of induction motor rotates and mechanical power is developed.

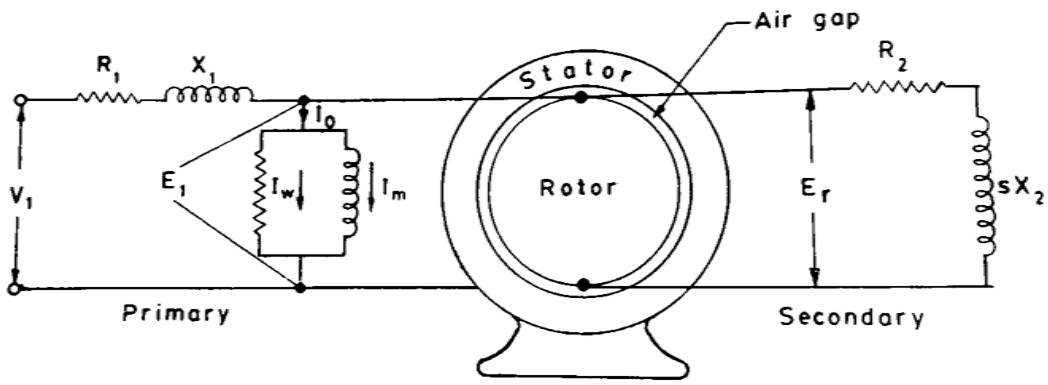

Figure 12. Per phase equivalent circuit of induction motor

5.4- Phasor Diagram of Induction Motor

The total ampere-turns required to overcome the reluctance of air gap is directly proportional to the length of air gap. If induction motor is designed for a large air gap, magnetizing current drawn by the motor would be a large percentage of the full load current. The operating power factor is lesser for induction motor which draws higher magnetizing current. Hence, power factor is reduced when the motor is designed for larger air gap. This will to large extent be detrimental to the generating source and the transmission lines. Due to non-existence of air gap in transformers, the phase angle between stator (primary winding) applied voltage (V_1) and stator current (primary current I_1) is lesser for the case of transformer than the motor, hence, the operating power factor is more in transformer, which draws lesser magnetizing current. Magnetizing current and power factor being extremely important performance parameter for transformer and induction motor, the induction motor should be designed for as small an air gap as mechanically possible.

$$
\left.\begin{aligned}
\text{From figure 12, } V_1 &= E_1 + I_1\,(R_1 + jX_1) \\
&= E_1 + I_1R_1 + jI_IX_1) \\
E_r &= I_2\,(R_2 + jsX_2) \\
&= I_2\,R_2 + jsI_2X_2)
\end{aligned}\right\} \qquad 31
$$

From equation (31), the phasor diagram of induction motor can be drawn as in figure 13

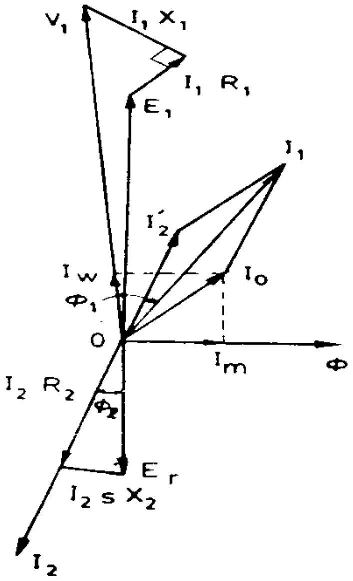

Figure 13 – Phasor diagram of induction motor

VI. Effect of Power Factor (p.f) on the Performance of Synchronous Motor

The synchronous motor is the one type of 3-phase motor which operates at a constant speed from no load to full load. The speed of rotation is tied to the frequency of the source. Since the frequency is fixed, the motor speed stays constant irrespective of the load or voltage of 3 – phase supply. It is similar in construction to alternator in that it has a revolving field which must be separately excited from a D.C source. Hence, it is fundamentally an alternator operated as a motor. The outstanding characteristic of synchronous motor enables it to be used extensively in power houses, sub-stations and factories for power factor improvement/correction, as it can be made to operate over a wide range of power factors (lagging, unity, and leading), by adjustment of its field excitation.

Plate 5: Pictorial view of a Synchronous motor

During operation, the effect of armature reaction in synchronous motors should be seen as the opposite to the effect in a synchronous generators (alternators) as treated before. Hence, the total reactance of the stator (armature) winding is the sum of the leakage reactance (X_L) and the armature reaction reactance (X_{ar}). This sum as noted before is the synchronous reactance (X_s).

$\therefore X_s = X_L + X_{ar}$ $\Omega/_{phase}$ as in equation (5)

A resistance R_a must be considered to be in series with (X_s) to account for the copper losses in the stator (armature) winding. This resistance combines with synchronous reactance and gives the synchronous impedance (Z_s) of the motor.

That is; $Z_s = R_a + jX_s$ as in equation (6)

6.1- Equivalent Circuit of Synchronous Motor

Figure 14 shows the equivalent circuit model of synchronous motor having cylindrical rotor.

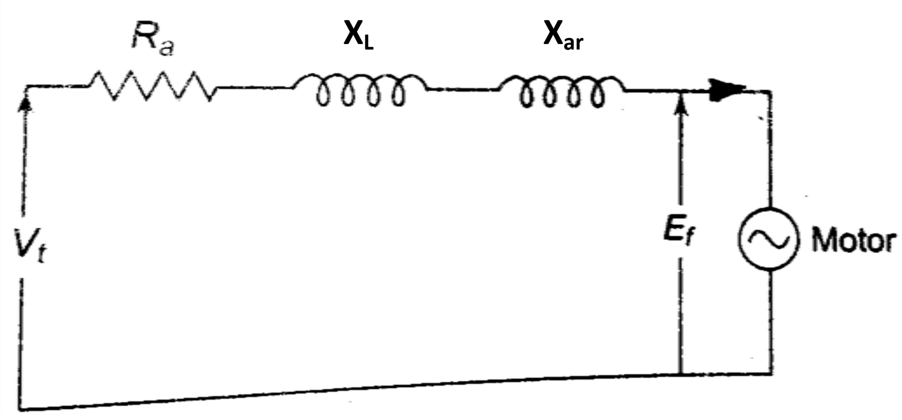

Figure 14- Per phase equivalent circuit model of synchronous motor

Where; V_t = Applied voltage per phase, E_f = Excitation voltage (back e.m.f as in D.C motor) per phase, I_a = Armature current per phase, R_a = Armature resistance, X_L = Leakage reactance per phase, X_{ar} = Armature reaction reactance per phase.

In figure 14, the voltage equation of synchronous motor is given by;

$$\left.\begin{aligned} \bar{V}_t &= \bar{E}_f + \bar{I}_a\,(R_a + jX_L + jX_{ar}) \\ &= \bar{E}_f + \bar{I}_a\,(R_a + jX_s) \\ &= \bar{E}_f + \bar{I}_a \bar{Z}_s \end{aligned}\right\} \qquad 32$$

Due to synchronously revolving field of the rotor, a voltage (E_f) is generated in the stator winding. This generated e.m.f is known as back e.m.f (as in D.C motors) and opposes the stator voltage (V_t). The magnitude of (E_f) is a function of rotor speed (N_r) and rotor flux (ϕ) per pole. Since rotor speed does not change, E_f then depends solely on rotor flux per pole (exciting rotor current).

Then from equation (32);

Net voltage per phase (voltage drop across synchronous impedance per phase) ($\bar{E}_r$) in stator winding $= \bar{V}_t - \bar{E}_f = \bar{I}_a \bar{Z}_s$ 33

$\therefore$ Armature current per phase $\bar{I}_a = \frac{\bar{E}_r}{\bar{Z}_s}$ 34

From equation (33);

If the field excitation is such that $E_r = 0$, that is $E_f = V_t$, then a synchronous motor is said to be normally excited. Similarly, if the field excitation is such that $E_r > 1$, that is $E_f < V_t$, then the motor is said to be under-excited. When $E_f > V_t$, that is $E_r < 1$, the motor is said to be over-excited. Synchronous motors have lagging power factor for both normal and under excitation and leading power factor for over-excitation.

6.2.1- Synchronous Motor on no Load

Let us suppose that an under-excited star-connected synchronous motor ($E_f < V_t$) is supplied with fixed excitation (E_f = constant). When the motor is not loaded, the angular displacement between stator and rotor pole (torque angle $\propto$) is small. As a result, E_f lags behind V_t by a small angle δ (angle between E_f and V_t).

The armature current (I_a) lags behind E_r by θ $\left(tan^{-1} \frac{X_s}{R_a}\right)$
Since $R_a << X_s$, I_a lags E_r by almost 90^0. Therefore the phase angle between V_t and I_a is , so that motor power factor (p.f) is $\cos\phi$.
Hence the input power per phase = $V_t I_a \cos\phi$. 35
The implication of the above statements is that at no load, synchronous motor takes a small power $V_t I_a \cos\phi$ per phase from the supply to meet the losses under no load condition, while it still runs at synchronous speed.

6.2.2- Synchronous Motor on load
When mechanical load is applied to a synchronous motor, the rotor poles fall slightly behind the stator poles, even as it continues to run at synchronous speed. The torque angel (∝) causes the phase of back e.m.f (E_f) to change with respect to the applied voltage (V_t). This brings about increment of E_r in the stator winding. Consequently, I_a increases, so as to withstand the load.

6.3 Effect of Changing Load at Constant Field Excitation
One of the most valuable characteristics of a synchronous motor is that by changing the magnitude of its mechanical load, it can be made to operate from lagging to leading power factor. Also, if the field excitation is kept unchanged, the load current changes with variation of load.

6.4 Phasor Diagram of Synchronous Motor at Variable Load and Constant field excitation
6.4.1 Lagging Power Factor Load
The phasor diagram of a synchronous motor under lagging power factor load is given in figure 15. Here, I_a lags V_t by an angle ϕ.

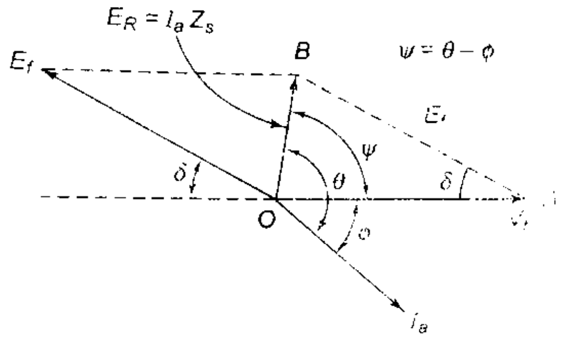

Figure 15-Phasor diagram of synchronous motor with lagging power factor.

From figure 15, $Z_s = R_a + jx_s = |Z_s| < \theta\ \Omega$
$\theta = \left(tan^{-1} \frac{X_s}{R_a}\right)$ 36

Where, I_a lags E_r by an angle ϕ.
The angle between V_t and $E_r = \theta - \phi = \psi$
Also, using cosine rule for ΔOAB, we have that; $|E_f|^2 = |V_t|^2 + |E_r|^2 - |2V_t E_r \mathrm{Cos}\psi|$ 37
By application of sine rule to ΔOAB, we have that;
$$\frac{E_f}{Sin\psi} = \frac{E_r}{Sin\delta}$$
$$\Rightarrow \delta = \sin^{-1}\left(\frac{E_r Sin\psi}{E_f}\right) \quad 38$$
So far, E_r is readily calculated from equation (33), E_f can now be gotten from equation 37, leading to the emergency of δ (the angle between E_f and V_t) of equation (38).

6.4.2- Unity Power Factor load
When the motor mechanical load is varied such that a unity power factor is obtained, its phasor diagram is represented as in figure 16. Here, I_a and V_t are in phase with each other.

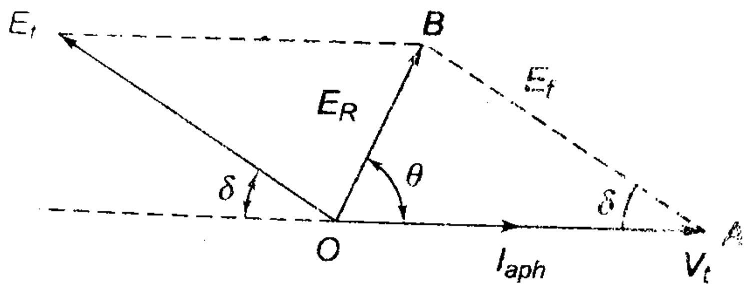

Figure 16- Phasor diagram of synchronous motor with unity power factor load.

From figure 16, it can b obtained; from ΔOAB that;

$|E_f|^2 = |V_t|^2 + |E_r|^2 - |2V_t E_r \text{Cos}\theta|$ 39

By application of sine rule to ΔOAB, as before we have that;

$$\frac{E_f}{sin\theta} = \frac{E_r}{sin\delta}$$

$\Rightarrow \delta = sin^{-1}\left[\frac{E_r sin\theta}{E_f}\right]$ 40

As E_r and E_f can be calculated from equation (33) and (39) respectively, δ can be got from equation (40).

6.4.3 – Leading Power Factor Load

The phasor diagram of a synchronous motor under leading power factor load is shown in figure 17. Here I_a leads V_t by an angle ϕ

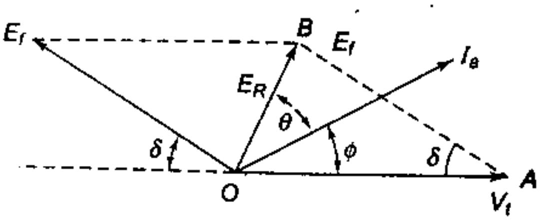

Figure 17 – Phasor diagram of synchronous motor with leading power factor load.

From figure 17, it can be obtained from ΔOAB that;

$|E_f|^2 = |V_t|^2 + |E_r|^2 - |2V_t E_r cos(\theta + \phi)$ 41

Using sine rule in ΔOAB, we have that;

$$\frac{E_f}{sin(\phi + \theta)} = \frac{E_r}{sin\delta}$$

$\Rightarrow \delta = sin^{-1}\left[\frac{E_r Sin\ (\theta+\phi)}{E_f}\right]$ 42

Hence, it can be inferred from the foregoing that, with increase in mechanical load applied to motor, the phase difference (δ) between the applied voltage (V_t) and excitation (E_f) increases because the angle δ increases with increasing load on the synchronous motor, it is known as load angle, power angle, coupling angle, torgue angle or angle of retardation.

6.5 Synchronous Motor Operation of Constant Mechanical Load with varying excitation.

If the mechanical load applied to a synchronous motor is kept constant, while field excitation is varied, the synchronous motor reacts by changing its power factor of operation to give constant output power, which is the most interesting feature of every synchronous motor. Since the mechanical load as well as the speed is

constant, the power input to the motor ($3V_t\ I_a \cos\phi$) is also constant. This means that the in-phase component $I_a \cos\phi$ drawn from the supply will remain constant. If the field excitation is changed, back e.m.f (E_f) also changes.
A correlation between V_t and E_f brings back to sub-section 6.21, where the characteristics of synchronous motor for different values of field excitation were discussed. That is;
(i) At $E_f = V_t$, the synchronous motor is normally excited. The motor draws current I_a from the supply. The power factor of the motor is lagging in nature as shown in figure 18a. The power input (P_i) remains same for constant load demanding the same power output.
That is; input power per phase (P_i) = $V_t I_a \cos\phi$
Or Input Power $P_{in} = \sqrt{3}V_L I_L cos\phi = 3V_{ph} I_{ph} cos\phi$ 43
Since input power is unchanged, the armature current (I_a) must decrease with increase in power factor.
Figure 18(b), (c) and (d) show cases for under excitation (E_f<V_t), over excitation (E_f>V_t) and critical excitation (E_f=V_t).

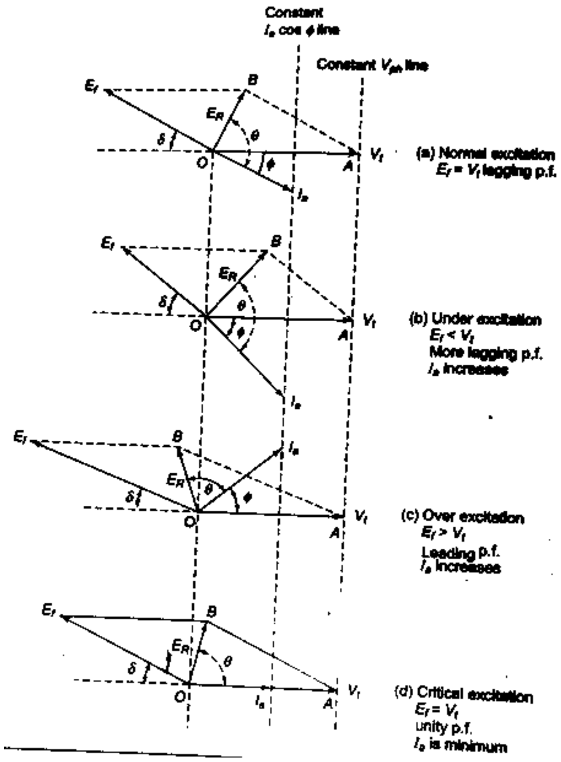

Figure 18: Phasor diagram of synchronous motor at constant mechanical load with varying excitation.

From the discussions, it can be inferred that, for a given mechanical load on a synchronous motor, the power factor is governed by the field excitation; a weak field produces the lagging armature current, while a strong field produces the leading armature current. When the motor is under-excited, it has a lagging power factor. As the excitation is increased, the power factor improves till it becomes unity at normal excitation. Under such conditions, the current drawn from the supply is minimum (critical excitation). If the excitation is further increased (over excitation), the motor power factor becomes leading. The armature current (I_a)is minimum of unity power factor and increases as the power factor becomes poor, either lagging or leading.

Furthermore, a synchronous motor under –over excited condition operates at a leading power factor and as such can be employed in large power installations for improving the overall power factor of the installation. At no-load with losses assumed negligible, a synchronous motor operates at $\delta = 0$, which means that E_f and V_t are in phase. The motor drawn zero power factor leading current (as capacitor variable) (E_f>V_t) or zero power factor lagging current (as inductor variable) (E_f <V_t).

Thus a synchronous motor at no-load (or light load) behaves as a variable condenser or inductor by simply varying its excitation.

A synchronous motor operated under such condition is called a synchronous condenser. It is found useful in large integrated power system for improving the power factor under heavy-load conditions and for depraving the power factor under light-load conditions, thereby controlling the voltage profile of the power system within reasonable limit (D.P Kothari and I.J Nagrath 2007).

Synchronous condenser is normally installed at the receiving end of a supply line when using capacitor bank is found uneconomical. It should always be recalled that an inductor consume reactive power where as a capacitor generates reactive power. If at any point of power system, the generation of reactive power is more than the consumption, then voltage at the point will increase and vice versa. Thus if one has inductive load demanding lagging VAR, then one can connect synchronous condenser to meet the demand as shown in figure 19.

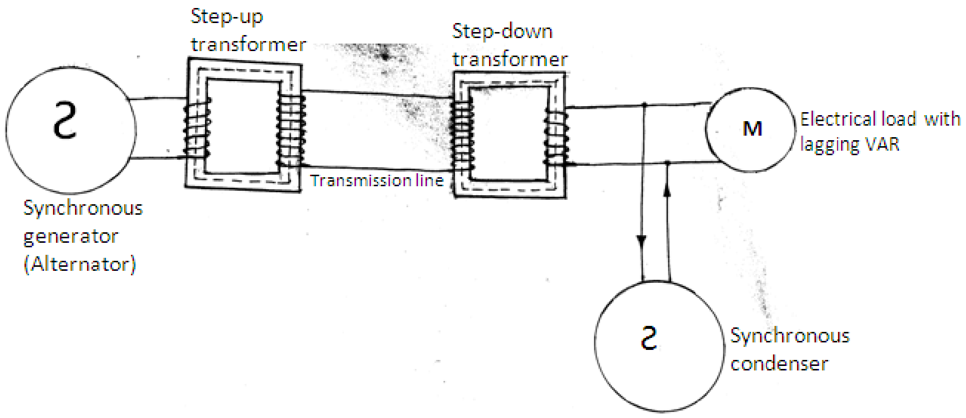

Figure 19 – Synchronous motor (condenser) connected to power systems lines for power factor improvement.

With this system of design /connection, locally, the demand for lagging VAR will be met and the supply system will be relieved from supplying, the lagging VAR and will only supply active power, owing to the fact that the supply system is only providing the active power and no reactive power. Most interestingly, with the insertion of synchronous condenser, the current flowing through the transmission lines, transformers will reduce, thereby leading to saving in energy bill due to drastic reduction in the ohmic losses in transmission line and transformer windings.

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