IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 15, Issue 4 Ser. I (Jul. – Aug. 2020), PP 26-32
www.iosrjournals.org

An Efficient Optimized Window Function for Designing Low Pass FIR Filter

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Abstract:*In this paper, a modified window function is proposed to design a Finite Impulse Response (FIR) low pass filter. The window function is designed with MATLAB by using the Curvefit Library, Iteration and fvtool. The main aim of this work is to attenuate the sidelobe peak with the same or slightly increase the main lobe width. A window adjusting parameter is defined to incorporate the aim. The proposed window function results in a better performance like minimum side lobe incorporate with equal ripple. The designed window function is compared with Blackman, Hamming, and Hanning window, and the simulation result indicates an improved performance compared to other commonly used windows. Moreover, the low pass finite impulse response filter designed with the proposed window bolster the consequence.*

Key Word*: FIR filter, Iteration, Sidelobe, Sinc function, filter Response, filter kernel.*

Date of Submission: 05-07-2020 Date of Acceptance: 21-07-2020

I. Introduction

The present generation communication system largely depends on the digital filtering process. Though analogue filters show its fast operation, less expensive in value, and broad dynamic range in amplitude and frequency, digital filters have gained its superiority in the level of performance. Accuracy and stability problem also affects the performance of analogue filter compared to digital filters. For these reasons, digital filtering is used for spectrum shaping or frequency-selective filtering, renewal of undesirable noise from desire signal, signal detection, and for performing analysis of the spectra of a variety of signals[1].

Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) are two of the fundamental types of the digital filter. FIR filters are preferable over IIR filter as it has a linear phase, highly stable, non-recursive structure and arbitrary amplitude-frequency characteristic[2]. Among the several techniques implied to design an FIR filter, windowing is one of the most popular procedures. Multiplying the window function to the ideal impulse response generates a filter in the windowing technique. The frequency response of a filter designed with window function approximates the desired frequency response more accurately, and it produces better effects than the frequency sampling technique of filter design. Several window functions have been proposed over time with specific advantages and disadvantages relative to the others. The spectrum of a particular window function slightly differs from other window function.

The desired characteristics of a window function are of smaller ripple ratio and narrower main lobe width. As the ripple ratio and main lobe width are inversely related, there needs a compromise between the main lobe width and sidelobe attenuation[3]. There has been considerable interest in the design of new windows to meet the desired specification for different applications[4, 5, 6]. In this paper, we present a modified Blackman window, which has less sidelobe peak compared to the Hamming, Hanning and Blackman window, while offering equal or slightly larger main lobe width.

The Blackman window technique is more powerful, perfect and useful than Hamming window[6]. As the Blackman window has more terms compared to the Hamming window, it provides more accuracy in the result. The additional cosine terms in Blackman function reduces the sidelobes and hence increases less losing power. For the same specification and filter order, the Blackman window has fewer sidelobes compared to Hamming window. For this reason, in this work, the Blackman window is in use for modifying to enhance the performance.

II. Proposed Window Function

A. Effects of Sinc Functions in Filter Design

An ideal low-pass filter has a flat passband, an infinite attenuation in the stopband, and a small transition bandwidth. The Inverse Fourier Transform of the ideal frequency response is regarded as an ideal filter kernel. This is actually a sinc function defined by[2]:

$$h[n] = \frac{\sin(2\pi\, fcn)}{n\pi} \quad (1)$$

For a low-pass filter, the input signal is convolved with the filter kernel. As sinc function is infinite on both positive and negative side, it creates a problem for computing. We can take two measures to avoid this problem:

i. Same numbers of samples are taken on both sides of the main lobe of the sinc function and truncated it to M+1 points. Here, M is an even number. Samples beyond the M+1 points are set to zero.

ii. The whole sequence is shifted to the right to avoid negative indices. This influenced the sequence to starts from 0 and ends at M. The output is also shifted by the amount of 0.5M for this shifting.

There are problems involved with this truncation process such as reduced stopband attenuation, slow roll-off rate, and passband ripple in frequency response. These problems occur due to the abrupt discontinuity at the end of the truncated sinc function. Increasing the length of the kernel cannot solve the problem[2, 9]. The truncated sinc function is multiplied by a window, e.g. Blackman, Hamming or Hanning, to solve this particular problem. The window function for Hamming, Blackman and Hanning are as follows:

Hamming Window[10, 11]: $w\,[\mathrm{n}] = 0.54 - 0.46\cos\left(\frac{2\pi n}{M}\right)$ (2)

Blackman Window[12]: $w\,[\mathrm{n}] = 0.42 - 0.50\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right)$ (3)

Hanning Window[2]: $w\,[\mathrm{n}] = 0.5 - \cos\left(\frac{2\pi n}{M-1}\right)$ (4)

For windowed sinc function, cutoff frequency (f_c) and length of filter kernel (M) are two essential parameters. The cutoff frequencies defined as a ratio of frequency and sampling rate whose value ranges from 0 to 0.5. The relationship between the length of filter kernel (M) and bandwidth(BW) is defined as,

$$M \cong {}^{4}\!/_{BW}$$

where*BW* is the transition bandwidth which is also a fraction of the sampling frequency[2].

The value of transition bandwidth ranges from 0 to 0.5. Since the time required for convolution is proportional to the length of the signals, the above relation expresses a trade-off between computation time (depends on the value of *M*) and filter sharpness (the value of *BW*).

B. Numerical process for window construction

In this work, a window equation has been assumed that was initially found from the Blackman window equation. The filter visualization tool of MATLAB is used to modify the equation. It provides necessary data of magnitude response, phase response, time delay, phase delay, pole-zero plot, and impulse response. The magnitude response found from the modified Blackman function is needed to be improved. We used the *FREQZ* function of the MATLAB software to manipulate the dataset to obtain the necessary information about the function in the frequency domain. This dataset was helpful to plot the function in the main lobe and side lobe graph.

The side lobes are normalized to reduce their amplitude to a desirable and acceptable level. Several values have been used by the trial and error method to select the normalization factor. The best possible normalization factor is found to be 0.1. After performing array manipulation, the main lobe and the reduced side lobes have been attached together. The resultant frequency domain curve has been further examined. We used the Inverse Fourier Transform function with the aid of the MATLAB to get the time domain form of the equation, and the *CURVEFIT* function of MATLAB to gain a time-domain equation of the window function. The main portion of the plot was a combination of sinc and cosine function, and the initial portion of the curve was mainly a polynomial shown as follows

$$\alpha m^3 + \beta m^2 + \gamma m + \delta \quad when,\ n = 1\ and\ n = m \quad (5)$$

where, m is the length of the window and α, β, γ, and δ are the curve fitting arbitrary constant.

A curve-fitting algorithm is used to find the best match, which gives the ranges of *α*, *β*, *γ*, and *δ* as $\alpha = 5.25e^{-5}$ to $5.25e^{-21}, \beta = 2.15e^{-2}$ to $2.15e^{-16}, \gamma = 8.40$ to $8.40e^{-12}$, and $\delta = 4.10e^{4}$ to $4.10e^{-12}$. The numerical iteration process gives the solution of the equation (5) as m=61, α = 5.25e^{-11}, β = 2.15e^{-8}, γ = 8.40e^{-6} and δ = 4.10e^{-4}.

A numerical iterative process determines the co-efficient and the power of the sinc and cosine function. The values of co-efficient and power are varied in a range of 0.0 to 5.0 by developing a MATLAB function. The performance of the window function is measured in every iteration. It is found that for the sinc function the value of 0.8 for coefficient and a value of 2 for power gives the best possible performance, and for the cosine function, the counter values are 0.499 and 1.

Table no 1: Summary of numerical process to find different constant

Name of the constant	Value		Name of the constant	Value
M	61		*Coefficient of sinc*	0.8
A	$5.25e^{-11}$		*Power of sinc*	2
B	$2.15e^{-8}$		*Coefficient of cosine*	0.499
Γ	$8.40e^{-6}$		*Power of cosine*	1
δ	$4.10e^{-4}$			

III. Results

A. The proposed window function and low pass FIR filter

The proposed window function consists of three main portions, namely the 3rd order polynomial portion, the cosine portion and, the sinc portion. The power of the sinc portion is 2, and it has the angle $2n/(m-1)$; Where n changes from 0 to m-1. The cosine portion has a power of 1.0, and it has the angle $2n/(m-1)$, where n changes from 0 to m-1. The final window function is thus

$$w = (2.5\times(w_1 \times w_2))^r \tag{6}$$

where r is window adjusting parameter and

$$w_1 = \begin{cases} \alpha m^3 + \beta m^2 + \gamma m + \delta, & \text{for n=1 and n=m} \\ 0.499\times\left(\cos\left(2\times n/(m-1)\right)\right), & \text{for n=2 to m-1} \end{cases} \tag{7}$$

$$w_2 = 0.8\times(\text{sinc}(2\times n/(m-1)))^2, \quad \text{for n=1 to m} \tag{8}$$

The time-domain response curves of the proposed window are shown in the fig. 1 for different values of window adjusting parameter, r. From the figure, it can be said that with the increment of the value of the window adjusting parameter, the width of the window function become narrower.

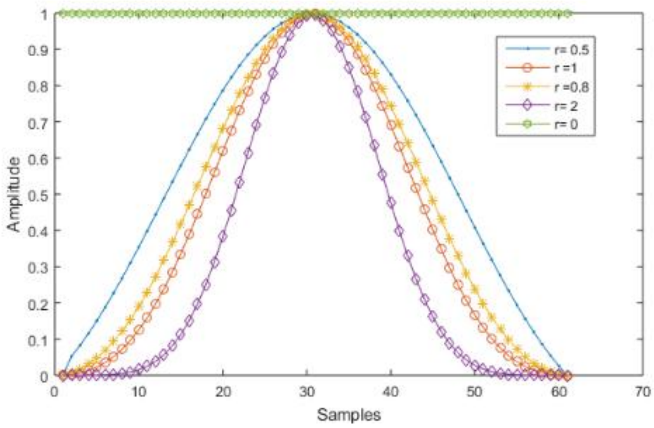

Fig. 1: Time domain response of the proposed window for different r

The frequency-domain response of the window function is shown in fig. 2. From the figure, we can say that with the increase in the value of r the sidelobe attenuation is increased. However, the value of ripple is slightly increased for higher values of r.

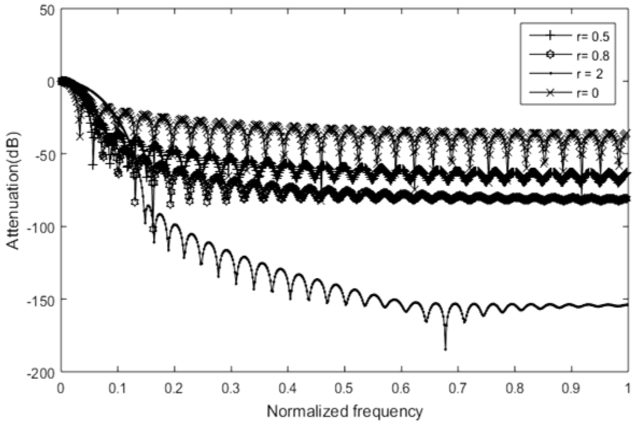

Fig. 2: Frequency domain response of the proposed window for different r

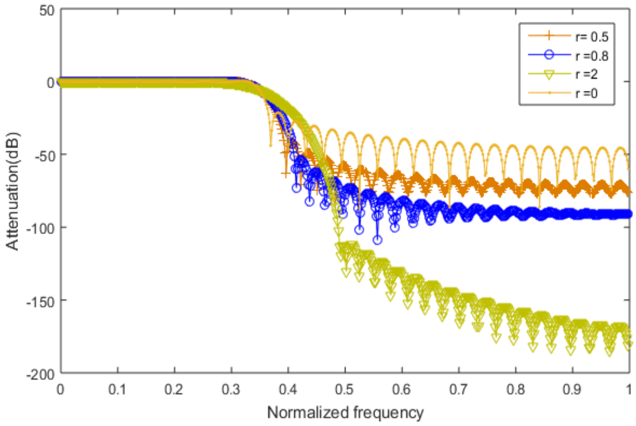

Fig. 3: Performance of the proposed filter for different r

Figure 3 shows the performance of the low pass FIR filter with the proposed window function. It shows that with the value of window adjusting parameter two, the filter exhibits the highest attenuation for sidelobe, and when r=0 exhibits the opposite. It also indicates that the minimum ripple in the sideband is obtained for r=0.8.

The summary of the performance of the proposed window function is described in table 2 and table 3 in a nutshell. It is clear from the tables that for r=2 the aim of the work, i.e. with the highest main lobe width and highest attenuation for sidelobe is achieved fruitfully.

Table 2: Performance of the proposed window at a different value of 'r'

The proposed window for different value of 'r'	Main-lobe width (rad/sample)	Ripple ratio (dB)	Side-lobe roll-off ratio (dB)
r = 0	0.0664	-13.4	22.46
r = 0.5	0.11328	-28.96	29.48
r = 0.8	0.14844	-39.0	33.63
r = 2.0	0.2968	-85.7	42.7

Table 3: Performance of LP FIR filter with modified window function at a different value of 'r'

Window-based low-pass filter for different value of 'r'	Main-lobe width (rad/sample)	Ripple ratio (dB)	Side-lobe roll-off ratio (dB)
r = 0	0.7382	-21.47	24.09
r = 0.5	0.7930	-41.57	29.28
r = 0.8	0.8282	-53.86	37.12
r = 2.0	0.9844	-112.3	55.4

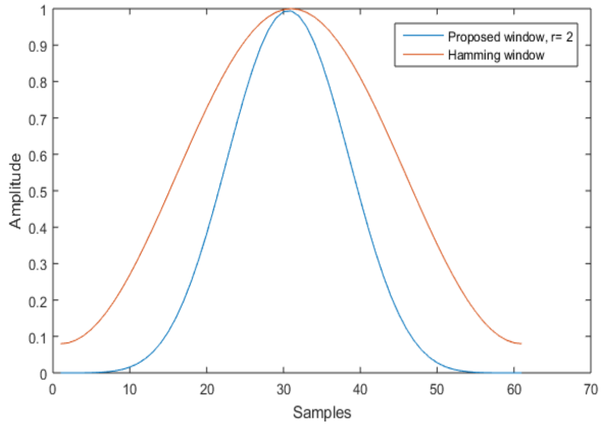

Fig. 4: Comparison between the proposed window and Hamming Window (time domain)

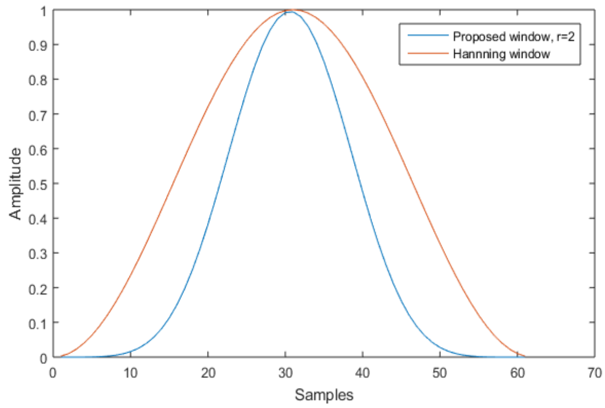

Fig. 5: Comparison between the proposed window and Hanning Window (time domain)

B. Comparison with a traditional filter

Performance of the low pass FIR filter with the proposed window is compared with the known and efficient low pass filter with Blackman, Hamming, and Hanning window. The windows, as mentioned above, are more relevant and latest and have better competence considering other specification points of view.

Figure 4, 5 and 6 compare the time domain response of the proposed window with the Hamming, Hanning and Blackman window. The proposed window represents the minimum window width compared to other windows. This narrow window function ensures the highest attenuation with the same or slightly greater main lobe width, which is required for several applications. As the decrement of the window width incorporates with more significant sidelobe ripple, the window width should be chosen wisely.

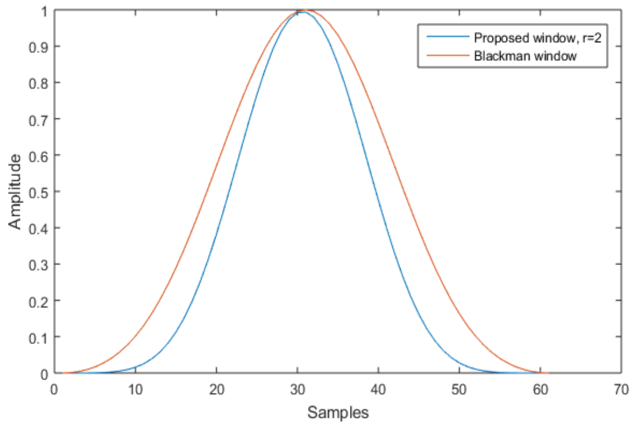

Fig. 6: Comparison between the proposed window and Blackman Window

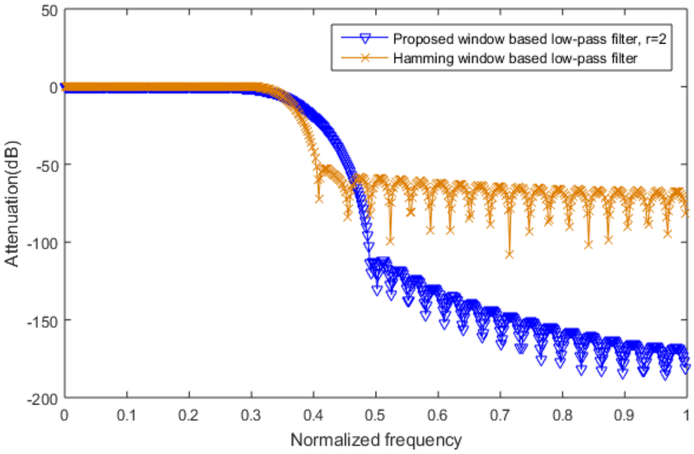

Fig. 7: Comparison between proposed filter and filter designed by Hamming window

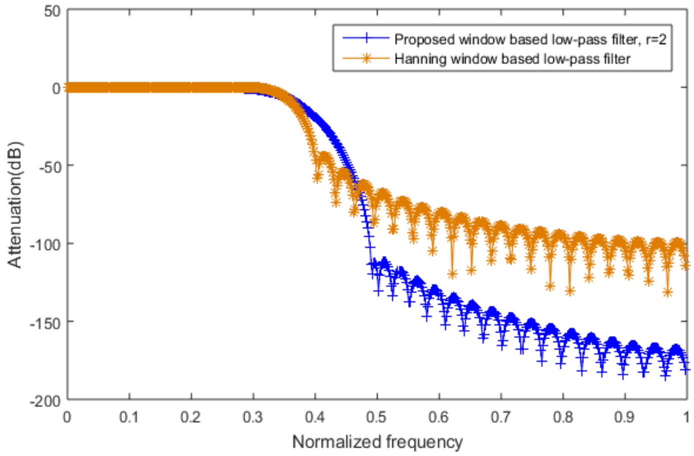

Fig. 8: Comparison between proposed filter and filter designed by Hanning window

The comparison of the frequency response of the filter designed with the proposed window with Hamming, Hanning and Blackman window is shown in figure 7, 8 and 9. In each case, the main lobe width is increased slightly, but the sidelobe attenuation is also increased. Moreover, the low pass FIR filter with the modified window shows less ripple compared to the filter with the Hanning window. The performance of the filters designed with the proposed window, Hamming window, Hanning window and Blackman window is summarized in table 4.

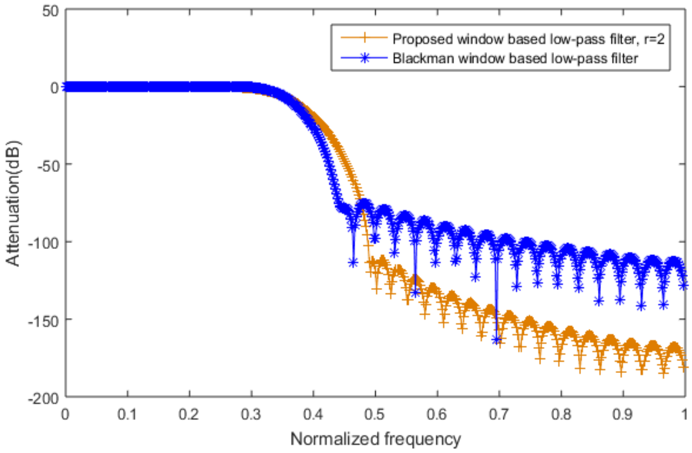

Fig. 9: Comparison between the proposed filter and filter designed by Blackman

Table 4: Performance comparison with other filters

Filter	Main-lobe width (rad/sample)	Ripple ratio (dB)	Side-lobe roll-off ratio (dB)
Proposed (r=2)	0.9844	-112.3	55.4
Blackman	0.9296	-75.29	37.21
Hamming	0.8096	-52.62	14.62
Hanning	0.8046	-43.96	55.71

IV. Conclusion

A low pass filter can perform a great deal in the field of digital signal processing if it is perfectly tuned and used wisely. In this work, the proposed window is a modification of the powerful and efficient Blackman window. The filter designed with this window shows a brilliant increase in the attenuation of side lobe, specifically 40 dB-60 dB compared to other windowed filters with a cost of slightly increased main lobe width. If the ripple of the side lobes can be reduced by further research and development, then this filter would become the most straight forward and cost-effective filter than those which are used nowadays.

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Subrata Bhowmik, et. al. "An Efficient Optimized Window Function for Designing Low Pass FIR Filter." *IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)*, 15(4), (2020): pp. 26-32.