IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 10, Issue 6 Ver. II (Nov – Dec. 2015), PP 52-57
www.iosrjournals.org

Development of a State-Space Thermal Model for High Precision Temperature Control of a Poultry Incubator

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Abstract: *This paper presents a state-space model intended as the basis for the design of a control system for a poultry incubator, where the control objective is to provide optimum temperature for successful incubation. To realize this objective, an accurate model of the thermal system of the incubator is required. Incubation temperature plays a crucial role in the hatching rate of poultry as well as the overall health of hatched chicks. Accurate temperature control often depends on the accuracy of the model used while the ease of realization of the control system depends on the simplicity of the model used. This paper presents a simplified mathematical model of the thermal system of an incubator with electrical heating. First, the constructional features of the incubator is presented, followed by the formulation of a set of first order differential equations describing the setup. Next, the state-space model is obtained. The system is further investigated for stability and then validated through step response simulations.*
Keywords: *Biot number of egg, lumped-parameter model, State-space model of incubator*

I. Introduction

The model of a system is a mathematical description of its dynamic behavior. Models of systems are essential for gaining a deeper insight and understanding of the systems. In this regard, models are often used for system analysis and control [1]. The goal of system analysis is to predict the system's behavior under unforeseen circumstances, that is, to determine how changes in the input to a dynamic system and its parameters affect its output. When models are used as the basis for solving an engineering problem, the quality of the solution often depends, to a large extent, on the accuracy of the models used [1-2]. In the absence of such models, prototypes will have to be built and tested, by several trial and error iterations. This approach is costly, time-consuming and sometimes risky. Therefore, the use of system models is not only justified, but crucial.

A poultry incubator provides the necessary environmental conditions for the incubation of fertile eggs. The most vital of these conditions is temperature. Without optimum or near optimum temperature, the development of the embryo would be adversely hampered. The most favorable temperature for most poultry species is 37.5 ^{o}C. The influence of temperature on hatchability, development of embryo, and fitness of hatched chicks has been presented by Lourens *et al.* [3], Joseph, Lourens and Moran [4] and Wilson [5]. Beside temperature, humidity is another important environmental factor that determines the overall success of incubation. An egg loses water as the embryo develops, this water loss can lead to dehydration. Detail analysis of this moisture loss phenomenon is presented by Meijerhof and van Beek [6].

Though strict adherence to optimum humidity is not critical, the lost water from the egg must be compensated for by adjusting humidity to avoid drying out [7]. The adjustment of humidity influences some thermal parameters of the system. However, to keep the current model simple, the effect of varying humidity on the dynamic behavior of the system is not considered. The impact of humidity is being considered in a separate paper which is currently being worked on.

Various aspects of the thermal characteristics of incubation have been investigated: Turner [8-9] explored the energetics of natural incubation of eggs, Meijerhof and van Beek [6] modeled the influence of climatic conditions on temperature and moisture loss of eggs while Van Brecht *et al.* [10] investigated the effect of air temperature, humidity, air speed, and airflow direction on the heat exchange between the eggshell and its microenvironment. In his work, French [11] developed a thermal model that takes into account the incubator temperature, metabolic heat production of the developing embryo, and thermal conductivity of the egg and surrounding air.

Despite the proliferation of research in this area, most of them have concentrated mainly on the energetics of the incubation process. This paper however, adopts an approach that utilizes the thermal energetics of egg incubation in conjunction with the constructional details of the incubator to model the heat transport between an egg and its microenvironment. The thermal model of the microenvironment is similar to models used for the analysis and control of other HVAC (Heating, Ventilation and Air Conditioning) systems [12-14].

II. Incubator construction

A simplified model of the thermal system comprising an enclosed chamber with an electrical heater is shown in Fig. 1. It has openings in the sides to provide a means of regulating ventilation by adjusting the exposed surface area. More sophisticated ways of controlling the volume of inlet air are discussed in [15]. Air circulating fans are also provided to ensure an even distribution of temperature and humidity in the incubator, as well as replacement of stale air.

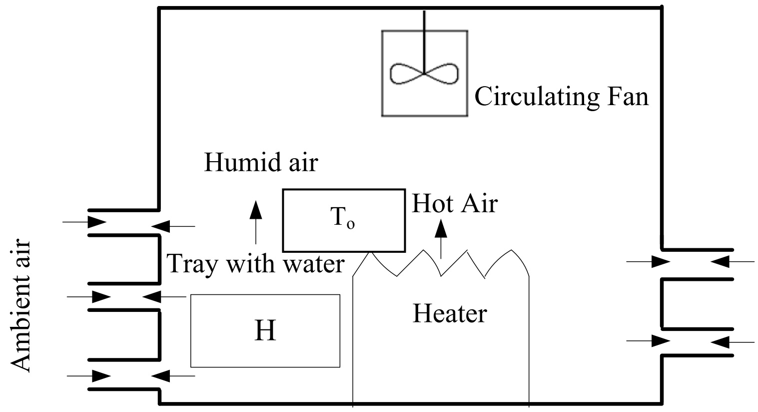


Fig. 1 Simplified diagram of the heating system

In the design, inlet ambient air at temperature T_{i} is heated to a temperature T_{o} by supplying heat energy Q_i to the heater. To provide the needed humidity, a tray of water is placed inside the incubator whose exposed surface area is adjusted to control the amount of humidity. In addition to the output air temperature T_{o}, the internal egg temperature T_{e} is the other measured variable. T_{e} is difficult to measure directly and is in fact approximated to the egg shell temperature which is actually the parameter measured [16]. Internal egg temperature T_{e} is influenced by T_{o} and internal heat, Q_{emb}, produced as a result of metabolic activities by the developing embryo. At the start of incubation, the rate of metabolism is low and Q_{emb} is small. At this stage, T_{e} is less than T_{o} and the egg will be gaining heat from the surrounding air. However, during the second half of incubation, the egg will lose heat as T_{e} increases above $T_{\mathrm{o.}}$ Details of these changes in T_{e} and T_{o} are provided by French [11].

III. Dynamic heat balance equations

Like most thermal systems, the incubator model is a complex system with many parameters. Taking all these variables into account in a model design is quite impractical. Therefore, the lumped-parameter model considered here is an idealized representation of the actual system dynamics. The lumped-parameter model of the system is based on the general heat balance principle [17].

A number of assumptions have been made in the derivation of the lumped parameter model presented here:1) Temperature distribution inside the incubator is uniform, implying constancy of such properties as density and specific heat capacity of air . 2) Temperature distribution throughout the egg is homogeneous. 3) Each eagg is exclusively surrounded by air, with no contact with other eggs or any other material. 4) There is no instantaneous variation of the speed of air in the incubator.

Equation (1), which is based on the principle of conservation of energy and on fundamental heat transfer laws [8, 14], describes the dynamic heat balance equation of the setup. Equations (1.a), (1.b) and (1.c) follow from assumptions 1 through 3, respectively, as stated below.

1. The rate of change of energy stored by the enclosed air is counterbalanced by the heat input and the heat transfer rates between the air and the wall, between the air and the environment and between the air and the egg;
2. The rate of energy storage in the wall equals rate of heat gain by the wall from enclosed air at temperature T_{o} , compensated by heat loss from the wall to the environment;
3. Finally, the rate of change of energy stored in the egg equals sum of heat gain by egg (or heat loss from egg if the egg is warmer than enclosed air) and heat added due to embryonic heat production Q_{emb}.

$$C_o \frac{dT_o}{dt} = Q_i - K_i(T_o - T_w) - K_f(T_o - T_i) - K_e(T_o - T_e) \quad (1.a)$$

$$C_w \frac{dT_w}{dt} = K_i(T_o - T_w) - K_o(T_w - T_i) \tag{1.b}$$

$$C_e \frac{dT_e}{dt} = K_e(T_o - T_e) - Q_{emb} \tag{1.c}$$

where,

T_o - steady-state temperature of outlet air, °C

T_i - steady-state temperature of inlet air, °C

T_w - mean wall temperature of incubator, °C

Q_i - steady-state heat input, W

C_o - thermal capacitance of air contained in the heating chamber, J/K

C_w - lumped thermal capacitance of the structure (wall of the incubator), J/K

K_f - convective conductance ascribed to ventilation, W/K

K_i - convective conductance between the enclosed air and structure node, W/K

K_o - convective conductance between the structure node and the outside air, W/K

K_e - thermal conductance of egg (W /K)

Q_{emb} - embryonic heat production

The thermal conductance K_e comprises the internal conductance, associated with heat flow between the inside of the egg and its surface, and the external conductance associated with heat flow between the egg's surface and the boundary layer of air. In [11], the approximation $K_e = (0.97v^{0.6})m^{0.53}$ was used, where v is air speed (which is estimated as 1.0 cm/s and assumed constant for this study) and m is egg mass (grams).

IV. State-space representation and step response of the system

The system inputs, outputs and states can be identified from (1) and are summarized in Table 1.

Table 1 Interpretation of System Parameters

System parameters	Designation
Inputs	$Q_i(t), T_i(t), Q_{emb}(t)$
Outputs	$T_o(t), T_e(t)$
States	$T_o(t), T_w(t), T_e(t)$

Rearranging (1) and designating the input parameters $Q_i(t)$, $T_i(t)$ and $Q_{emb}(t)$ respectively as $u_1(t)$, $u_2(t)$ and $u_3(t)$ gives

$$\dot{T}_0(t) = \frac{-(K_i + K_f + K_e)}{C_o} T_o(t) + \frac{K_i}{C_o} T_w(t) + \frac{K_e}{C_o} T_e(t) + \frac{1}{C_o} u_1(t) + \frac{K_f}{C_o} u_2(t) \tag{2.a}$$

$$\dot{T}_W(t) = \frac{K_i}{C_w} T_o(t) - \frac{(K_i + K_o)}{C_w} T_w(t) + \frac{K_o}{C_w} u_2(t) \tag{2.b}$$

$$\dot{T}_e(t) = \frac{K_e}{C_e} T_o(t) - \frac{K_e}{C_e} T_e(t) - \frac{1}{C_e} u_3(t) \tag{2.c}$$

Transforming (2) into standard vector-matrix form yields (3).

$$\begin{bmatrix} \dot{T}_0(t) \\ \dot{T}_W(t) \\ \dot{T}_e(t) \end{bmatrix} = \begin{bmatrix} \frac{-(K_i + K_f + K_e)}{C_o} & \frac{K_i}{C_o} & \frac{K_e}{C_o} \\ \frac{K_i}{C_w} & -\frac{(K_i + K_o)}{C_w} & 0 \\ \frac{K_e}{C_e} & 0 & -\frac{K_e}{C_e} \end{bmatrix} \begin{bmatrix} T_o(t) \\ T_w(t) \\ T_e(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{C_o} & \frac{K_f}{C_o} & 0 \\ 0 & \frac{K_o}{C_w} & 0 \\ 0 & 0 & -\frac{1}{C_e} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \tag{3}$$

Since the measured outputs are the air temperature and egg shell temperature T_e, the output equation can be written as

$$z(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_o(t) \\ T_w(t) \\ T_e(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{4}$$

(3) and (4) in the standard **ABCD** form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{Ax}(t) + \boldsymbol{Bu}(t) \tag{5}$$

$$z(t) = \boldsymbol{Cx}(t) + \boldsymbol{Du}(t) \tag{6}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \frac{-(K_i + K_f + K_e)}{C_o} & \frac{K_i}{C_o} & \frac{K_e}{C_o} \\ \frac{K_i}{C_w} & -\frac{(K_i + K_o)}{C_w} & 0 \\ \frac{K_e}{C_e} & 0 & -\frac{K_e}{C_e} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \frac{1}{C_o} & \frac{K_f}{C_o} & 0 \\ 0 & \frac{K_o}{C_w} & 0 \\ 0 & 0 & -\frac{1}{C_e} \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{and } \boldsymbol{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

A Matlab stability test using the command eig(A) returned the vector of eigenvalues $[-3.9685\ -0.0255\ -0.1675]^T$. The system is therefore stable since all the eigenvalues of the system matrix A have negative real parts.

The Simulink state variable model shown in Fig. 2 has been used to analyze the system under different input conditions. The model is constructed from the parameters of the state space matrices of (3) and (4). The parameters were computed from physical data of the incubator. The step response for inlet ambient air temperature T_i = 1°C, supply heat Q_i = 1kJ and embryonic heat Q_{emb} = 0.25J is shown in Fig. 3 (a). It can be seen that in the case with zero input, both output values (incubator air temperature and egg temperature) are zero.

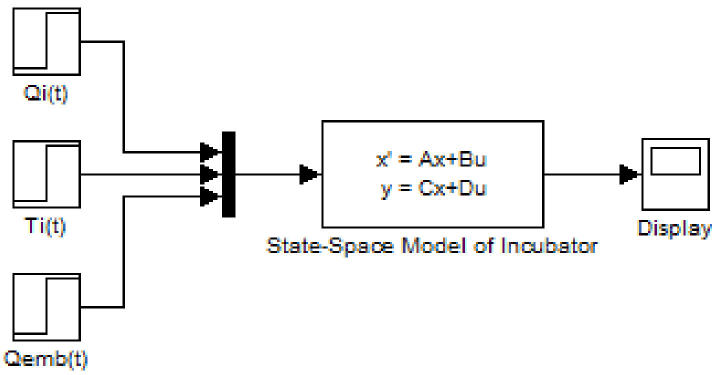


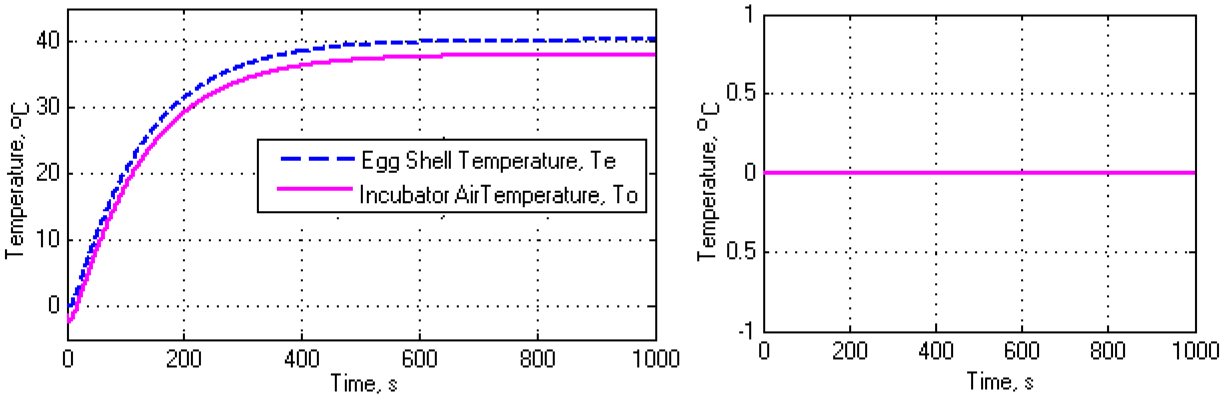
Fig. 2 Simulink State Variable Model of the System

V. Heat Transfer Analyses

For the analyses of heat transfer between an egg and the incubator air, the following Assumptions were made:

1. The egg is approximately spherical in shape with geometric mean diameter D = 4 cm;
2. Heat conduction in the egg is one-dimensional because of symmetry about its centre;
3. The thermal properties of the egg are constant;
4. The heat transfer coefficient is constant and uniform over the entire surface.

Furthermore, embryonic heat production has been ignored in the following analyses.



(b) (b)

Fig. 3 Step Response of the Thermal System with (a) Forced Input and (b) zero input

The temperature trajectory at the center of the egg $T_c(t)$ initially at temperature T_i when placed in the incubator at T_∞ = 37.5 °C can be computed using the approximate equation for a transient one-dimensional conduction in a sphere given by (7) [13, 18-19].

$$\theta = \frac{T_c(t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2 \tau} \tag{7}$$

where θ is a dimensionless quantity that denotes change in temperature, τ= Fo is the Fourier number givent by Fo = $\alpha t/D^2$,with α being thermal diffusivity of water at 37.5 °C and D the characteristic dimension (diameter).

The constants A_1 and λ_1 are functions of the Biot number and are found in transient temperature charts. The Biot number, Bi is given by (8) as

$$Bi = \frac{hD}{k} \tag{8}$$

where h is the convective heat transfer coefficient over the egg's surface, D is its characteristic diameter and k is the thermal conductivity. The egg is approximated to a sphere (of diameter D = 4 cm) filled with water which yields k = 0.6 W/m°C. The convective heat transfer coefficient h (in W/(m^2 °C) is computed as [10]

$$h = 0.336 \times 4.184 \times \left(1.46 + \sqrt{100v}\right) \tag{9}$$

where v is the air speed (in m/s) around the egg. Using these values with speed of air v =1 m/s gives Bi = 1.073 and the corresponding values for A_1 and λ_1 are 1.5708 and 1.2732, respectively.

The centerline temperature $T_c(t)$ given by (7) is plotted for an egg initially at T_i = 28 °C in Fig. 4. We now confirm for this case the validity of the one-term approximate solution for one-dimensional transient conduction in a sphere:

$$\tau = -\frac{1}{\lambda_1^2} \ln\left[\frac{(T_c - T_{air})}{A_1(T_i - T_{air})}\right] = -\frac{1}{1.2732^2} \ln\left[\frac{(37.49-37.5)}{1.5708(28-37.5)}\right] = 2.877 > 0.2$$

Therefore, the one-term approximate solution is applicable.

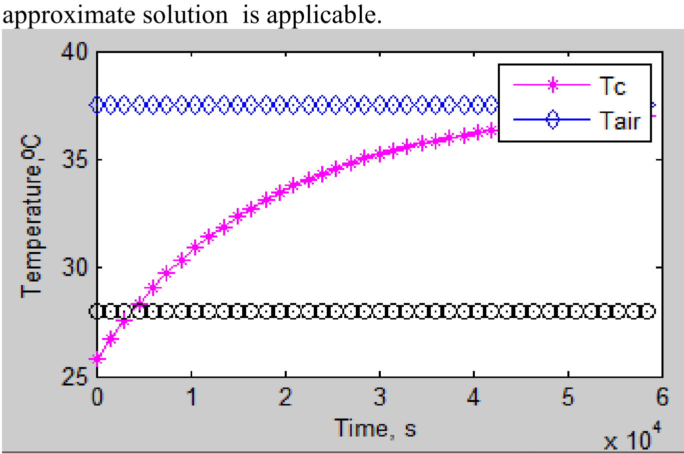


Fig. 4 Heat transfer between center of egg T_c and incubator air T_{air}

The Biot number represented by (8) is a ratio of the egg's internal heat conduction resistance to its surface convection resistance. The lumped system model employed in these analyses requires that the temperature gradients within the egg be negligible, which is the case only when the conduction resistance, and hence the Biot number, is negligible [18, 20]. Thus, the smaller the Biot number, the more accurate the model. From (8), it's seen that the smaller the size of egg and the faster the air (the higher the velocity, *v*), the more accurate the model.

VI. Conclusion

In this paper, a state-space model has been developed to aid the design of a temperature control system for a poultry incubator. This model provides a means of generalizing the results and applying them to solving control and design problems of this particular nature. This model takes into account the thermal parameters of the structure of the incubator and egg as well as the environmental conditions, and is essential for gaining a deeper insight into the dynamic behavior of the system, thereby avoiding needless trial and error in the design process. Important assumptions have aided the simplification of the model, which is essential to its practical realization. Thermal analysis has been conducted to show the transfer of heat between the egg and incubator air at optimum temperature of 37.5 ^{o}C. The stability of the system has also been verified through calculation and step response simulations. The results have shown that the system is stable and behaves as expected. As a result, the model can be used as a basis for the system design.

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