

## A hybrid system based approach to Direct Torque Control (DTC) of induction motors

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**Abstract:** In this paper, we study the Direct Torque Control (DTC) of an Induction Motor coupled to an Inverter (Inv-IM). DTC permits to control directly the stator flux and the torque by selecting the appropriate inverter state. DTC has been introduced because it presents several advantages in comparison to other techniques such as voltage/frequency control, vector control and field control.

In this paper, we first model the DTC of Inv-IM as a hybrid system (HS). Then, we abstract the continuous dynamics of the HS in terms of discrete events. We thus obtain a discrete event model of the HS. And finally, we use Supervisory Control Theory of DES to drive Inv-IM to a desired working point.

**Keywords:** DTC, automaton, DES, Controller, IM, Inv

### I. Introduction

The main advantage of Induction motors (IM) is that no electrical connection is required between the stator and the rotor. Another advantage of IMs is that they have low weight and inertia, high efficiency and a high overload capability [1]. There exist several approaches to drive an IM. The Voltage/frequency (V/f) controller is the simplest technique, but its main disadvantage is its lack of accuracy in both speed and torque. Vector controllers is a technique that can reach a good accuracy, but its main disadvantages are the necessity of a huge computational capability and of a good identification of motor parameters [2]. The method of Field acceleration overcomes the computational problem of vector controllers by achieving some computational reductions [3], [2], [4]. And the technique of Direct Torque Control (DTC), that has been developed by Takahashi [5], [6], [7], [8], permits to control directly the stator flux and the torque by using an appropriate voltage vector selected in a look-up table. The main advantages of DTC are a minimal torque response time and the absence of: coordinate-transform, voltage modulator-block, controllers such as PID for flux and torque. For these advantages, DTC is the control method adopted in this paper. Since the IM is driven through an inverter, the system to be controlled consists actually of the inverter and the IM and will be denoted Inv-IM. The latter is in fact a hybrid system, in the sense that it consists of a discrete component (the inverter) and a continuous component (IM). We propose a three-step method to model the DTC of Inv-IM and then drive Inv-IM to a desired working point.

- In a first step, we model the DTC of Inv-IM as a hybrid system (HS) with: a discrete event dynamics defined by the voltage vectors used to control IM; and a continuous dynamics defined by continuous equations on the stator flux vector ( $\vec{\varphi}_s$ ) and the electromagnetic torque ( $\Gamma$ ).
- In a second step, we abstract the continuous dynamics of the HS in terms of discrete events. Some events are used to represent the entrance and exit of the torque  $\Gamma$  and the amplitude  $\varphi_s$  of  $\vec{\varphi}_s$  in and from a working point region. And some other events are used to represent the passage of the vector  $\vec{\varphi}_s$  between different zones. By this abstraction, the continuous dynamics of the system IM is described as a discrete event system (DES).
- In a third step, we use Supervisory Control Theory (SCT) [9],[18],[19],[20] to drive Inv-IM to a desired working point.

The remainder of the paper is structured as follows. Sect. II presents the inverter and its discrete event dynamics. Sect. III presents the induction motor and its continuous dynamics. In Sect. IV, we propose an abstraction of the continuous dynamics of the IM in terms of discrete events. Sect. V shows how to use SCT to drive Inv-IM to a targeted working point. In Sect. VI, we presents simulation results and we conclude the paper in sect VII.

### II. Inverter And Its Discrete Event Model

The inverter (Fig. 1) is supplied by a voltage  $U_0$  and contains three pair of switches ( $S_i^p, S_i^l$ ), for  $i = a, b, c$ . The input of the inverter is a three-bit value ( $S_a S_b S_c$ ) where each  $S_i$  can be set to 0 or 1. A value 0 of  $S_i$  sets

( $s_i^h, s_i^l$ ) to (close,open), and a value 1 sets it to (open,close). The output of the inverter is a voltage vector  $\vec{v}_s$  that drives IM.

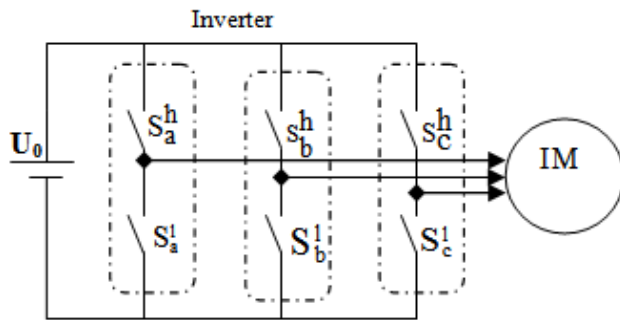


Fig.1. Inverter driving the induction motor

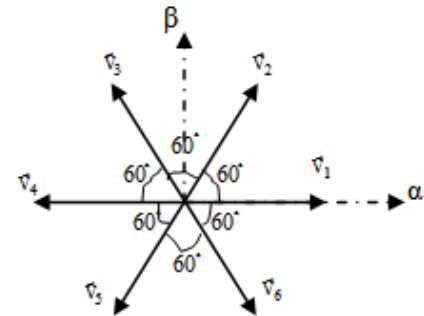


Fig.2. the six non-null voltage vectors in the reference frame fixed to the stator

Equation (1) computes the voltage vector  $\vec{v}_s$  expressed in the  $\alpha$ - $\beta$  axes (i.e. the stationary reference frame fixed to the stator) after Concordia transformation. Note that  $\vec{v}_s$  depends uniquely on  $U_0$  and  $(S_c S_b S_a)$ . Since  $U_0$  is fixed and  $(S_c S_b S_a)$  can have eight different values, we expect to obtain at most 8 voltage vectors. Actually, we have only 7 vectors, because the null vector is obtained for 000 and 111.

$$\vec{v}_s = \sqrt{\frac{2}{3}} U_0 [S_a + S_b e^{j\frac{2\pi}{3}} + S_c e^{j\frac{4\pi}{3}}] \quad (1)$$

The correspondence between  $\vec{v}_k$  and  $(S_c S_b S_a)$  is as follows: ( $\vec{v}_0, 000, 111$ ), ( $\vec{v}_1, 001$ ), ( $\vec{v}_2, 011$ ), ( $\vec{v}_3, 010$ ), ( $\vec{v}_4, 110$ ), ( $\vec{v}_5, 100$ ), ( $\vec{v}_6, 101$ ).

The inverter can be modeled by a 7-state automaton whose each state  $q_k$  ( $k = 0, \dots, 6$ ) means: " $\vec{v}_k$  is the current voltage vector". Since  $\vec{v}_0$  can be obtained from the inputs 000 or 111 of the inverter, the selection between the two inputs can be done according to a specific control strategy. To adopt the terminology of hybrid systems, the term mode will be used as a synonym of state. The transition from any mode  $q_*$  to a mode  $q_k$  occurs by an event  $v_k$  which means "starting to apply  $\vec{v}_k$ ".

### III. Induction Motor And Its Continuous Model

The induction motor is a continuous system because its behavior is modeled by algebraic and differential equations on two continuous variables: the stator flux  $\phi_s$  and the electromagnetic torque  $\Gamma$ . For conciseness, by flux we mean stator flux, and by torque we mean electromagnetic torque.

#### III.1 Model Of Flux And Torque

With DTC, the voltage vector  $\vec{v}_s$  generated by the inverter is applied to the IM to control the flux  $\phi_s$  and the torque  $\Gamma$ . Let us first see how  $\phi_s$  and  $\Gamma$  can be expressed. In a stationary reference frame, the flux vector  $\vec{\phi}_s$  is governed by the differential Eq. (2), where  $R_s$  is the stator resistance and  $\vec{I}_s$  is the stator current vector. Under the assumption that  $R_s \vec{I}_s$  is negligible w.r.t.  $\vec{v}_s$  (realistic if the amplitude of  $\vec{\phi}_s$  is sufficiently high), we obtain Eq. (3) which approximates the evolution of  $\vec{\phi}_s$  from  $\vec{\phi}_{s0}$  after a delay  $t$ .

$$\vec{V}_s = R_s \vec{I}_s + \frac{d\vec{\phi}_s}{dt} \quad (2)$$

$$\vec{\phi}_s = \vec{\phi}_{s0} + \vec{V}_s \cdot t \quad (3)$$

The torque  $\Gamma$  is expressed by Eq. (4), where  $k$  is a constant depending of physical parameters,  $\phi_s$  and  $\phi_r$  are the amplitudes of  $\vec{\phi}_s$  (stator flux vector) and  $\vec{\phi}_r$  (rotor flux vector), and  $\theta_{r,s}$  is the angle from  $\vec{\phi}_r$  to  $\vec{\phi}_s$  [10], [11]. Because the rotor response time is much larger than the stator one, we assume that  $\vec{\phi}_r$  is constant in comparison to the variation of  $\vec{\phi}_s$ . In this case, the torque increases (resp. ecreases) when  $\vec{\phi}_s$  rotates clockwise (resp. counterclockwise) [12], [11].

$$\Gamma = k \phi_s \phi_r \sin(\theta_{r,s}) \quad (4)$$

### III.2. Evolution Of Flux And Torque

Eq. (4) implies that the application of a vector voltage  $\vec{v}_s$  generates a move of the end of  $\vec{\phi}_s$  in the direction of  $\vec{v}_s$ . Note that  $\vec{v}_s$  consists of a radial vector  $\vec{v}_s^1$  (parallel to  $\vec{\phi}_s$ ) and a tangential vector  $\vec{v}_s^2$  (orthogonal to  $\vec{\phi}_s$ ).  $\vec{v}_s^1$  increases (resp. decreases) the flux  $\phi_s$  (i.e., the amplitude of  $\vec{\phi}_s$ ) if it has the same (resp. opposite) direction of  $\vec{\phi}_s$ .  $\vec{v}_s^2$  rotates  $\vec{\phi}_s$  clockwise (resp. counterclockwise) if the angle from  $\vec{\phi}_s$  to  $\vec{v}_s^2$  is  $\frac{-\pi}{2}$  (resp.  $\frac{+\pi}{2}$ ). From Equation (4), we deduce that  $\vec{v}_s^2$  increases (resp. decreases) the torque  $\Gamma$  if the angle from  $\vec{\phi}_s$  to  $\vec{v}_s^2$  is  $\frac{\pi}{2}$  (resp.  $\frac{-\pi}{2}$ ). Figure 3 illustrates the evolution of  $\vec{\phi}_s$  when  $\vec{v}_s^1$  has the same direction as  $\vec{\phi}_s$  and the angle from  $\vec{\phi}_s$  to  $\vec{v}_s^2$  is +90 degrees. Therefore, in this example both the flux  $\phi_s$  and the torque  $\Gamma$  increase.

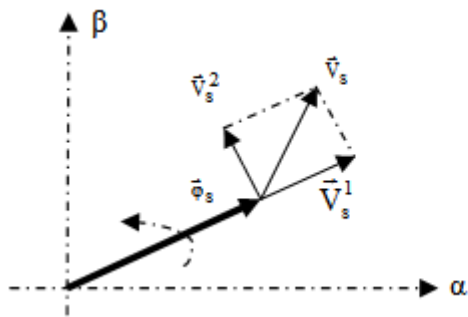


Fig.3. Evolution of the vector  $\vec{\phi}_s$

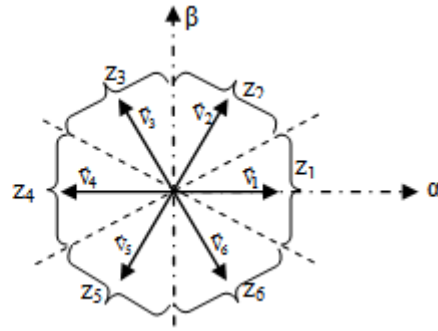


Fig.4. Locus of  $\vec{\phi}_s$  divided into six zones

Takahashi has proposed in [5] to divide the possible global locus of  $\vec{\phi}_s$  into the six zones  $z_1, z_2, \dots, z_6$  of Fig. 4.

Table I shows how the flux magnitude  $\phi_s$  and the torque  $\Gamma$  evolve when  $\vec{\phi}_s$  is in  $z_i$  ( $i = 1 \dots 6$ ) under the control of each of the seven vectors  $\vec{v}_k$  ( $k = 0, i - 2 \dots i + 3$ ), where indices  $i - 2 \dots i + 3$  are defined modulo 6 (from 1 to 6). Symbols  $\uparrow$ ,  $\downarrow$  and  $=$  mean "increases", "decreases" and "is constant", respectively. We see that under the control of  $\vec{v}_{i-2}, \vec{v}_{i-1}, \vec{v}_{i+1}, \vec{v}_{i+2}$  and  $\vec{v}_0$ , the evolution of  $\phi_s$  and  $\Gamma$  is known. But vectors  $\vec{v}_i$  and  $\vec{v}_{i+3}$  are problematic because they can both increase and decrease the torque  $\Gamma$  in the same zone  $z_i$ , depending if  $\vec{\phi}_s$  is in the first or the second 30 degrees of  $z_i$ . This problem will be called nondeterminism of the six-zone division.

	$\vec{v}_{i-2}$	$\vec{v}_{i-1}$	$\vec{v}_{i+1}$	$\vec{v}_{i+2}$		$\vec{v}_i$	$\vec{v}_{i+3}$	$\vec{v}_0$ or $\vec{v}_7$
$\phi_s$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$		$\uparrow$	$\downarrow$	$=$
$\Gamma$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$		?	?	$\downarrow$

Table I: In a six zone division, evolution of  $\phi_s$  and  $\Gamma$  when  $\phi_s$  is in  $z_i$  ( $i=1,2,\dots,6$ ) under the control of  $\vec{v}_k$  ( $k=0, i-2, \dots, i+3$ )

### III.3. Solving The Nondeterminism Of The 6-Zone Division

We propose two approaches to solve the nondeterminism of the 6-zone division. The first approach is based on the observation that the nondeterminism occurs when  $\vec{\phi}_s$  is in a zone  $z_i$  while one of the control vector  $\vec{v}_i$  or  $\vec{v}_{i+3}$  is applied. A solution is to leave nondeterminism as soon as it appears, by applying a control vector  $\vec{v}_k$  different from  $\vec{v}_i$  and  $\vec{v}_{i+3}$ . We suggest to select the control vector to be applied among the four control vectors  $\vec{v}_{i-2}, \vec{v}_{i-1}, \vec{v}_{i+1}, \vec{v}_{i+2}$ , because these four vectors permit to obtain all the combinations of the evolution of  $(\phi_s, \Gamma)$  (see Table I).

A second approach to solve the nondeterminism is to use twelve zones by dividing each of the six zones  $z_i$  into two zones  $z_{i,1}$  and  $z_{i,2}$  comprising the first and the second 30 degrees, respectively [13], [1]. Figure 5 represents the twelve-zone division. In each zone  $z_{i,j}$  and under the control of  $\vec{v}_{i-2}, \vec{v}_{i-1}, \vec{v}_{i+1}, \vec{v}_{i+2}$  and  $\vec{v}_0$ , the evolution of  $\phi_s$  and  $\Gamma$  is thus the one already indicated in Table I for  $z_i$ . Table II shows the evolution of  $\phi_s$  and  $\Gamma$  in zone  $z_{i,j}$  under the control of  $\vec{v}_i$  and  $\vec{v}_{i+3}$ .

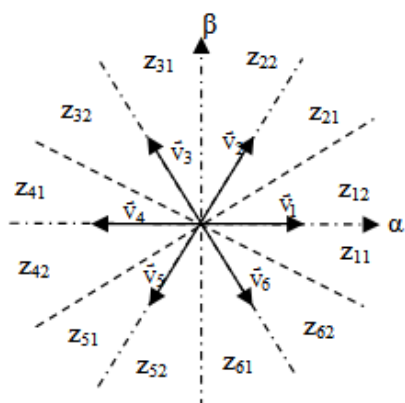


Fig.5. Locus of  $\bar{\varphi}_s$  divided into twelve zones.

	$\vec{v}_i$	$\vec{v}_{i+3}$
$\varphi_s$	↑	↓
$\Gamma$	↑ if j=1	↓ if j=1
	↓ if j=2	↑ if j=2

Table II: Evolution of  $\varphi_s$  and  $\Gamma$  under the control of  $\vec{v}_i$  and  $\vec{v}_{i+3}$ .

#### IV. Im Modeled As A Des By Abstracting Its Continuous Dynamics

Let us show how the continuous dynamics of IM presented in Sect. III is abstracted in terms of discrete events. The first abstraction consists in translating by events the entrance and exit of  $(\varphi_s, \Gamma)$  in and from a working point region. The second abstraction consists in translating by events the passage of the vector  $\bar{\varphi}_s$  between orientation zones.

##### IV.1. Abstracting The Entrance And Exit Of $(\varphi_s, \Gamma)$ In And From A Working Point Region

Let  $\varphi_{wp}$  and  $\Gamma_{wp}$  be the flux magnitude and the torque defining the targeted working point. That is, the aim of control will be to drive IM as close as possible to  $(\varphi_{wp}, \Gamma_{wp})$ . We define a flux interval  $[\varphi_{wp}^-; \varphi_{wp}^+]$  centered in  $\varphi_{wp}$ , and a torque interval  $[\Gamma_{wp}^-; \Gamma_{wp}^+]$  centered in  $\Gamma_{wp}$ . We partition the space of  $(\varphi_s, \Gamma)$  into sixteen regions  $R_{u,v}$ , for  $u, v = 1, 2, 3, 4$ , as shown in Fig. 6. The objective of the control will be to drive IM into the set of regions  $\{R_{u,v} : u, v = 2, 3\}$  and to force it to remain into this set. We define the event  $\varphi_u^+$  that represents a transition from  $R_{u,v}$  to  $R_{u+1,v}$  for any  $v$ , and the event  $\Gamma_v^+$  that represents a transition from  $R_{u,v}$  to  $R_{u,v+1}$  for any  $u$ . Since only transitions between adjacent regions are possible, the unique possible events are the following:  $\varphi_u^{u+1}$  if  $u < 4$ ,  $\varphi_u^{u-1}$  if  $u > 1$ ,  $\Gamma_v^{v+1}$  if  $v < 4$ , and  $\Gamma_v^{v-1}$  if  $v > 1$ .

With the above abstraction, the evolution of  $(\varphi_s, \Gamma)$  can be described by a 16-state automaton, whose states are noted  $(u, v)$  and correspond to the sixteen regions  $R_{u,v}$ ,  $u, v = 1, 2, 3, 4$ . The transitions between states occur with the events defined above:  $\varphi_u^{u+1}$ ,  $\varphi_u^{u-1}$ ,  $\Gamma_v^{v+1}$ ,  $\Gamma_v^{v-1}$ .

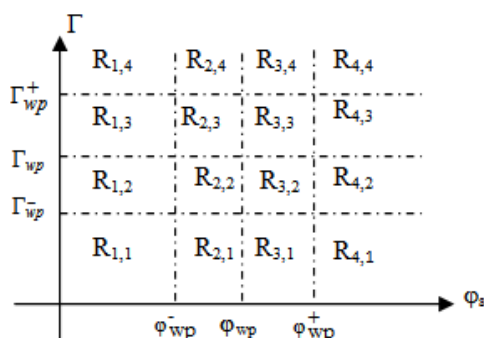


fig.6. Partitioning of the space of  $(\varphi_s, \Gamma)$

##### IV.2. Abstracting The Passage Of $\bar{\varphi}_s$ Between Orientation Zones

In Sects. III-2 and III-3, we have shown how to partition the global locus of  $\bar{\varphi}_s$  into six or twelve zones (Figs. 4 and 5). This partitioning is very relevant, because we have seen that from the knowledge of the current zone occupied by  $\bar{\varphi}_s$ , we can determine the control vector  $\vec{v}_k$  to be applied for obtaining a given evolution of  $(\varphi_s, \Gamma)$  (Tables I and II).

With the 6-zone partition, we define the event  $z_i^i$  that represents a transition from  $z^i$  to  $z^i$ . Since only transitions between adjacent zones are possible, the unique possible events are the following:  $z^{i+1}$ ,  $z^{i-1}$ , where  $i-1$  and  $i+1$  are defined modulo 6. We can thus abstract the evolution of  $\bar{\varphi}_s$  by a 6-state automaton, whose states are noted  $\langle i \rangle$  and correspond to the zones  $z_i$ ,  $i = 1 \dots 6$ . The transitions between states occur with the events defined above:  $z_i^{i+1}$ ,  $z_i^{i-1}$ .

We can use the same approach with the 12-zone partition, by defining the event  $z_{i,j}^{i,j}$  that represents a transition from  $z_{i,j}$  to  $z_{i,j}$ . Since only transitions between adjacent zones are possible, the unique possible events are the following:  $z_{i,1}^{i,2}$ ,  $z_{i,2}^{i,1}$ ,  $z_{i,1}^{i+1,2}$ ,  $z_{i,2}^{i+1,1}$ , where  $i-1$  and  $i+1$  are defined modulo 6. We can thus abstract the evolution of  $\bar{\varphi}_s$  by a 12-state automaton, whose states correspond to the zones  $z_{i,j}$ ,  $i = 1 \dots 12$  and  $j = 1, 2$ . The transitions between states occur with the events defined above:  $z_{i,1}^{i,2}$ ,  $z_{i,2}^{i,1}$ ,  $z_{i,1}^{i+1,2}$ ,  $z_{i,2}^{i+1,1}$ .

### IV.3. Modeling Im As A Des

In Sect. IV-1, we have shown how to abstract the evolution of  $(\varphi_s, \Gamma)$  by a 16-state automaton. In Sect. IV-2, we have shown how to abstract the evolution of  $\bar{\varphi}_s$  by a 6-state or 12-state automaton. In the sequel, we consider uniquely the 6-state automaton because it reduces the state space explosion which is inherent to the use of automata. As we have seen in Sect. III-3, the 6-zone partition necessitates to apply a control vector different from  $\bar{v}_i$  and  $\bar{v}_{i+3}$ , when  $\bar{\varphi}_s$  is in  $z_i$ . We will explain in Sect.V how this requirement can be guaranteed by supervisory control of DES. Let us see how the two automata (16-state and 6-state) are combined into an automaton  $M_k$  that abstracts the behavior of IM when a given control vector  $\bar{v}_k$  is applied by the inverter. A State of  $M_k$  is noted  $\langle u, v, i \rangle_k$  since it is a combination of a state  $\langle u, v \rangle$  (corresponding to  $R_{u,v}$ ) and a state  $\langle i \rangle$  (corresponding to  $z_i$ ).  $M_k$  can therefore have at most  $6 \times 16 = 96$  states  $\langle u, v, i \rangle_k$ , ( $u, v = 1, \dots, 4, i = 1, \dots, 6$ ). By interpreting Table I, we determine the transitions of  $M_k$  as follows, where  $\bar{v}_k$  is the control vector currently applied by the inverter. From state  $\langle u, v, i \rangle_k$  of  $M_k$ :

- The event  $\varphi_u^{u+1}$  can occur when  $u < 4$  and  $\varphi_s$  increases, i.e., when  $k$  is equal to one of the following values:  $i-1, i+1, i$ . This event  $\varphi_u^{u+1}$  leads from  $\langle u, v, i \rangle_k$  to  $\langle u+1, v, i \rangle_k$ .
- The event  $\varphi_u^{u-1}$  can occur when  $u > 1$  and  $\varphi_s$  decreases, i.e., when  $k$  is equal to one of the following values:  $i-2, i+2, i+3$ . This event  $\varphi_u^{u-1}$  leads from  $\langle u, v, i \rangle_k$  to  $\langle u-1, v, i \rangle_k$ .
- The event  $\Gamma_v^{v+1}$  can occur when  $v < 4$  and  $\Gamma$  increases, i.e., when  $k$  is equal to one of the following values:  $i+1, i+2, i, i+3$ . This event  $\Gamma_v^{v+1}$  leads from  $\langle u, v, i \rangle_k$  to  $\langle u, v+1, i \rangle_k$ .
- The event  $\Gamma_v^{v-1}$  can occur when  $v > 1$  and  $\Gamma$  decreases, i.e., when  $k$  is equal to one of the following values:  $i-2, i-1, i, i+3, 0$ . This event  $\Gamma_v^{v-1}$  leads from  $\langle u, v, i \rangle_k$  to  $\langle u, v-1, i \rangle_k$ .
- The event  $z_i^{i+1}$  can occur when  $\bar{\varphi}_s$  rotates counterclockwise, i.e., when  $\Gamma$  increases, i.e., when  $k$  is equal to one of the following values:  $i+1, i+2, i, i+3$ . This event  $z_i^{i+1}$  leads from  $\langle u, v, i \rangle_k$  to  $\langle u, v, i+1 \rangle_k$ .
- The event  $z_i^{i-1}$  can occur when  $\bar{\varphi}_s$  rotates clockwise, i.e., when  $\Gamma$  decreases, i.e., when  $k$  is equal to one of the following values:  $i-2, i-1, i, i+3, 0$ . This event  $z_i^{i-1}$  leads from  $\langle u, v, i \rangle_k$  to  $\langle u, v, i-1 \rangle_k$ .

Due to the nondeterminism related to  $\bar{v}_i$  and  $\bar{v}_{i+3}$  (explained in Sect. III-2 and in the second Table I), the events depending on the evolution of  $\Gamma$  ( $\Gamma_v^{v+1}$ ,  $\Gamma_v^{v-1}$ ,  $z_i^{i+1}$ ,  $z_i^{i-1}$ ) are potential but not certain when  $k$  is equal to  $i$  or  $i+3$ .

## V. Use Of Sct To Drive Im To A Working Point

### V.1. Introduction To Supervisory Control

In this section, we show how to use Supervisory Control Theory (SCT) [9] to drive IM to a desired working point. In supervisory control, a supervisor Sup interacts with a DES (called plant) and restricts its behavior so that it respects a specification [9]. An important study in SCT is to synthesize a supervisor Sup when the plant and the specification are given and defined by two FSA P and S, respectively [9]. Sup observes the evolution of P (i.e., the events executed by the plant) and permits only the event sequences accepted by S. To achieve its task, Sup will disable (i.e., prevent) and force events. The concept of controllable event has thus been introduced, meaning that when an event  $e$  is possible, then Sup can disable it if and only if  $e$  is controllable;  $e$  is said uncontrollable if it is not controllable [9]. We will also use the notion of forcible event, meaning that when an event  $e$  is possible, then Sup can force  $e$  to preempt (i.e., to occur before) any other possible event, if and only if  $e$  is forcible;  $e$  is said unforcible if it is not forcible [14]. A method has been proposed to synthesize Sup automatically from P, S and the controllability and forcibility of every event [9].

**V.2. The Plant Inv-Im Modeled As A Des**

The plant to be controlled is the system Inv-IM (i.e., inverter with IM). In Section II, we have modeled the inverter by an automaton A with 7 states  $q_k$  ( $k = 0, \dots, 6$ ) corresponding to the 7 control vectors  $\bar{v}_k$ , respectively. And in Section IV-3, when a given  $\bar{v}_k$  is applied by the inverter to the IM, we have modeled the evolution of IM by an automaton  $M_k$  that can have at most  $6 \times 16 = 96$  states  $\langle u, v, i \rangle_k$ , ( $u, v = 1, \dots, 4, i = 1, \dots, 6$ ). Therefore, the system Inv-IM can be modeled by replacing in A each mode  $q_k$  by the automaton  $M_k$ . The transition from any state  $\langle u, v, i \rangle_*$  to a state  $\langle u, v, i \rangle_k$  occurs by an event  $V_k$ . The obtained automaton, noted P, can therefore have at most  $7 \times 96 = 672$  states. The initial state is  $\langle 1; 1; 1 \rangle_0$ , that is, initially: the flux and the torque are in Region  $R_{1,1}$ , the flux vector is in zone  $z_1$  and the null control vector  $\bar{v}_0$  is applied. The set of marked states is  $\{\langle u; v; i \rangle_k : u, v = 2, 3\}$ , because the objective of the control is to drive Inv-IM into the set of regions  $\{R_{u,v} : u, v = 2, 3\}$  (i.e., the set of states  $\{\langle u; v; i \rangle_k : u, v = 2, 3\}$ ), and then to force it to remain into this set. For the purpose of control, we define an undesirable event Null meaning that the flux or the torque has decreased to zero, and a state E reached with the occurrence of Null. We will see later how Null and E are necessary. Therefore, the automaton P has actually at most 673 (672 + the state E), and its alphabet  $\Sigma$  is:

$$\Sigma = \{\varphi_u^{u+1}, \Gamma_v^{v+1} : u, v = 1, 2, 3\} \vee \{\varphi_u^{u-1}, \Gamma_v^{v-1} : u, v = 2, 3, 4\} \\ \vee \{Z_i^{i+1}, Z_i^{i-1} : i = 1, \dots, 6\} \vee \{V_k : k = 1, \dots, 6\} \vee \text{null}$$

**V.3. Control Architecture**

We propose the control architecture illustrated in Figure 7. The interaction between the plant and the supervisor is realized through two interfaces A and G:

A : In Sect. II, we have modeled the inverter by an automaton executing the events  $V_k, k = 0, \dots, 6$ . The interface A translates every event  $V_k$  generated by the supervisor into  $(S_a S_b S_c)$ , using the following correspondence (already given in Section II):  $(\bar{v}_0, 000, 111), (\bar{v}_1, 001), (\bar{v}_2, 011), (\bar{v}_3, 010), (\bar{v}_4, 110), (\bar{v}_5, 100), (\bar{v}_6, 101)$ . And the inverter translates  $(S_a S_b S_c)$  into the control vector  $\bar{v}_k$ , which is applied to IM.

G : In Sect. IV, we have modeled IM by an automaton executing the events  $z_i^i, \varphi_u^u, \Gamma_v^v$ . And in Sect. V.2, we have added an event Null in the model of IM. The interface G generates these events as follows:  $z_i^i$  is generated when  $\bar{\varphi}_s$  passes from  $z_i$  to  $z_i'$ ;  $\varphi_u^u$  is generated when  $\varphi_s$  passes from  $R_{u,*}$  to  $R_{u',*}$ ;  $\Gamma_v^v$  is generated when  $\Gamma$  passes from  $R_{*,v}$  to  $R_{*,v'}$ ; Null when  $\varphi_s$  or  $\Gamma$  reaches the value zero.

The system {Inv-IM,A,G} forms the plant modeled by the 673-state automaton P of Sect. V-2. With this architecture:

- Sup observes the events  $z_i^i, \varphi_u^u, \Gamma_v^v, \text{Null}$ . Since these events are generated by IM through G, Sup has no control on them. Hence, these events are uncontrollable and unforcible.
- Sup generates the events  $V_k$ , and thus, has all control on them. Hence, these events are controllable and forcible.

We use the following hypothesis: Hypothesis 5.1: The plant is slow in comparison to Sup, in the sense that Sup can always force an event  $V_k$  to preempt any possible event  $z_i^i, \varphi_u^u, \Gamma_v^v, \text{Null}$ .

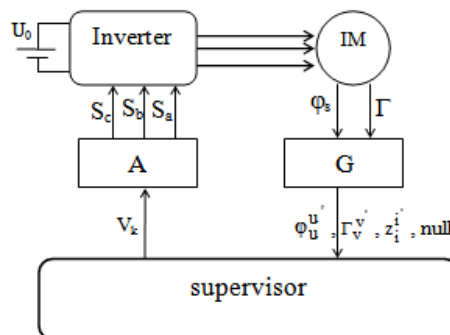


Fig.7. Control architecture: interaction between the plant and the supervisor

**V.4. Specification Of The Desired Behavior**

Our specification S requires to drive the DES {Inv-IM,A,G}, modeled by the 673-state automaton P of Section V-2, into the set of regions  $\{R_{u,v} : u, v = 2, 3\}$  and to force it to remain into this set. S will thus consist of the following three requirements:

1) P, which is initially in the region  $R_{1,1}$ , must enter the set of regions  $\{R_{u,v} : u, v = 2, 3\}$ , i.e., the set of states  $\{(u; v; i)_k : u, v = 2, 3\}$ . Or equivalently: the plant must leave the set of regions  $\{R_{u,v} : (u = 1) \vee (v = 1)\}$ , i.e., the set of states  $\{(u; v; i)_k : (u = 1) \vee (v = 1)\}$ . This requirement is guaranteed by the following one: the event Null must never occur. Indeed, Null can occur only from a region in  $\{R_{u,v} : (u = 1) \vee (v = 1)\}$ , and can be avoided uniquely by driving IM to leave the set of regions  $\{R_{u,v} : (u = 1) \vee (v = 1)\}$ . In fact, the event Null has been introduced as a mean to express this first requirement.

2) After the plant enters a region  $R_{2,v}$  or  $R_{3,v}$ , then it never goes to a region  $R_{1,v}$  or  $R_{4,v}$ . Or equivalently: the events  $\phi_2^4$  and  $\phi_3^4$  must never occur.

3) After the plant enters a region  $R_{u,2}$  or  $R_{u,3}$ , then it never goes to a region  $R_{u,1}$  or  $R_{u,4}$ . Or equivalently: the events  $\Gamma_2^4$  and  $\Gamma_3^4$  must never occur.

To recapitulate, S simply forbids the following five (uncontrollable and unforcible) events Null,  $\phi_2^4$ ,  $\phi_3^4$ ,

$\Gamma_2^4$ ,  $\Gamma_3^4$ . S can be expressed as a single-state automaton with self loops of all the events of the alphabet  $\Sigma$  except the above five events. If we apply a synchronized product of P and S, we have the specification S' obtained from P by "cutting" the above five events.

### V.5. Supervisor Synthesis

The inputs of the supervisor synthesis are:

- Automaton P modeling the plant (Sect. V-2).
- Automaton S modeling the specification (Sect. V-4).
- Controllability and forcibility of each event.

We have justified in Section V-3 that : the events  $z_i^j$ ,  $\phi_u^i$ ,  $\Gamma_v^i$ , Null are uncontrollable and unforcible; and the events  $V_k$  are controllable and forcible. We have applied the synthesis procedure of the software tool TTCT [15]. With TTCT, forcible events can be forced to occur before an event tick that models the passing of one time unit. To be able to use TTCT, we have adapted P and S by preceding every unforcible event by the event tick. This adaptation is consistent with Hypothesis 5.1 in Section V-3. The solution synthesized provides several possible scenarios of control. Here is the simplest one:

- Initially, we are in the initial state  $\langle 1; 1; 1 \rangle_0$ , that is: the flux and the torque are in Region  $R_{1,1}$ , the flux vector is in zone  $z_1$ , and the null control vector  $\bar{v}_0$  is applied.

- When a state  $\langle u; v; i \rangle_k$  such that  $u < 3$  and  $v < 3$  is reached and  $k \neq i+1$ , then Sup generates the event  $\bar{v}_{i+1}$ . Intuitively, when  $\phi_s < \phi_{wp}$  and  $\Gamma < \Gamma_{wp}$ , the control vector  $\bar{v}_{i+1}$  is applied to increase  $\phi_s$  and  $\Gamma$ . Note that this case applies to the initial state.

- When a state  $\langle 3; v; i \rangle_k$  such that  $v < 3$  is reached and  $k \neq i+2$ , then Sup generates the event  $\bar{v}_{i+2}$ . Intuitively, when  $\phi_s > \phi_{wp}$  and  $\Gamma < \Gamma_{wp}$ , the control vector  $\bar{v}_{i+2}$  is applied to decrease  $\phi_s$  and increase  $\Gamma$ .

- When a state  $\langle u; 3; i \rangle_k$  such that  $u < 3$  is reached and  $k \neq i-1$ , then Sup generates the event  $\bar{v}_{i-1}$ . Intuitively, when  $\phi_s < \phi_{wp}$  and  $\Gamma > \Gamma_{wp}$ , the control vector  $\bar{v}_{i-1}$  is applied to increase  $\phi_s$  and decrease  $\Gamma$ .

- When a state  $\langle 3; 3; i \rangle_k$  is reached and  $k \neq i-2$ , then Sup generates the event  $\bar{v}_{i-2}$ . Intuitively, when  $\phi_s > \phi_{wp}$  and  $\Gamma > \Gamma_{wp}$ , the control vector  $\bar{v}_{i-2}$  is applied to decrease  $\phi_s$  and  $\Gamma$ .

Note that Sup never generates an event  $\bar{v}_k$  that leads to a nondeterministic state  $\langle u; v; i \rangle_k$ , i.e., such that  $k = i$  or  $k = i+3$ . Nevertheless, a nondeterministic state can be reached by an event  $z_{i-1}^i$  or  $z_{i+1}^i$ . When such a situation occurs, Sup has not a real control on  $\Gamma$ , because it is not known whether  $\Gamma$  increases or decreases. Sup will quit such a nondeterministic situation by generating one of the four control vectors  $\bar{v}_{i-2}$ ,  $\bar{v}_{i-1}$ ,  $\bar{v}_{i+1}$ ,  $\bar{v}_{i+2}$ , depending on whether each of  $\phi_s$  and  $\Gamma$  must be increased or decreased.

## VI. Simulations Results

### VI.1. Conditions Of Simulation

The simulation is performed in simulation environment MATLAB/SIMULINK/STATEFLOW (2014b). In figure 8, we present the control architecture which consists of the interaction between the plant and the supervisor through two interfaces A and G. The controller receives in its inputs the events generated by interface G and apply a command to the plant via the interface A.

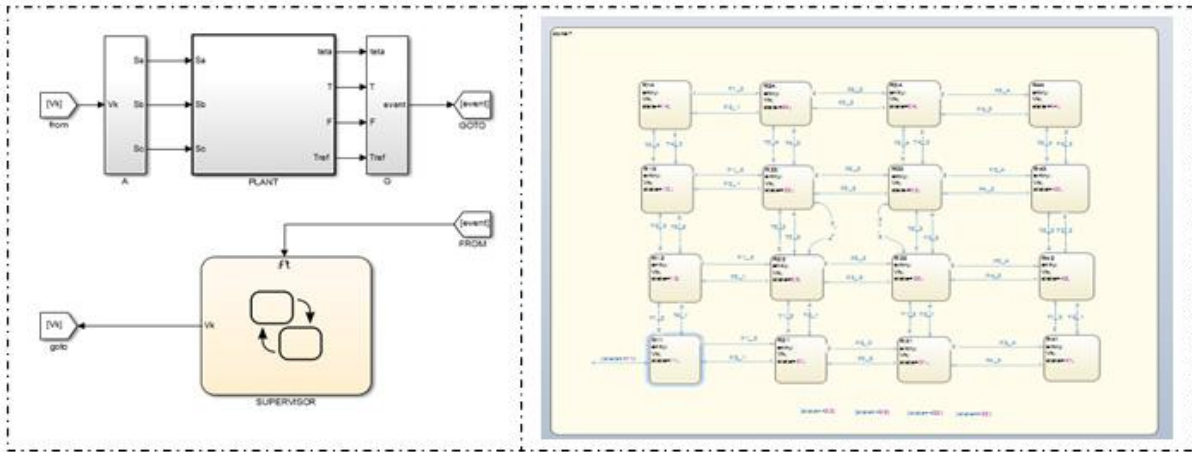


Fig.8. Block diagram simulation

Fig.10. Treatment inside zone zi

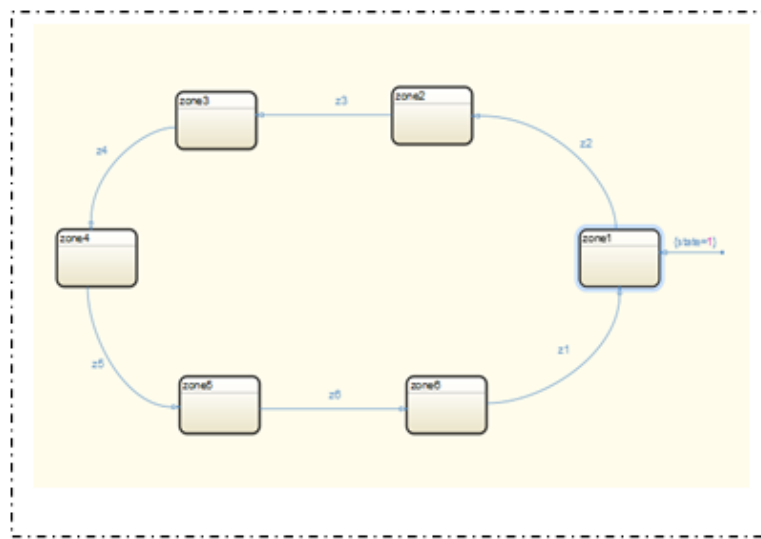


Fig.9. Automaton representing the zones  $z_i \{i=1,2,\dots,6\}$

Parameters of the induction motor used in the simulation:

Characteristics	Symbols	Value
Nominal Voltage	$U_{sn}$	380v
Nominal Flux	$\Phi_{sn}$	1wb
Stator Resistance	$R_s$	$0.63\Omega$
Rotor Resistance	$R_r$	$0.4\Omega$
Moment of Inertia	$J$	$0.22kgm^2$
Coefficient of friction	$f$	0.001
Number of pairs of pole	$p$	2
Stator self	$L_s$	98.3mH
Self rotor,mutual	$L_s, M_{sr}$	90 mH

Table III. Parameters of induction motor

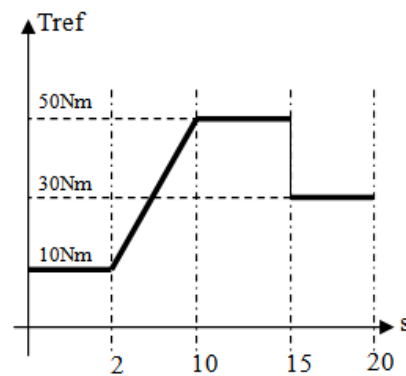


Fig.11. Reference Torque used for the simulation:

VI.2 Simulation Results

We present in this part the numerical simulation results, illustrating the behavior of the structure of direct torque control applied to an induction motor. The simulations are performed for a sampling period  $T_e$  equal to  $1\mu s$ .



Figures 12,13 shows the responses of the electromagnetic torque and the stator currents  $i_{s\alpha}$  and  $i_{s\beta}$  for reference torque which varies according to figure 11, we can note the very good performance of the torque which precisely follows its reference.

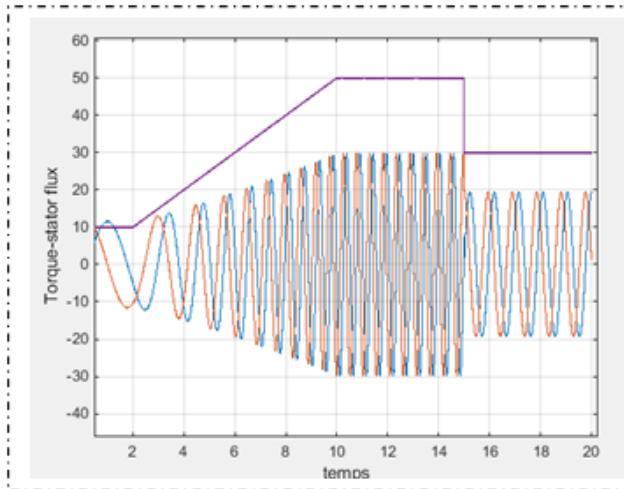


Fig.12. Response of the torque and the stator currents  $i_{s\alpha}$  and  $i_{s\beta}$

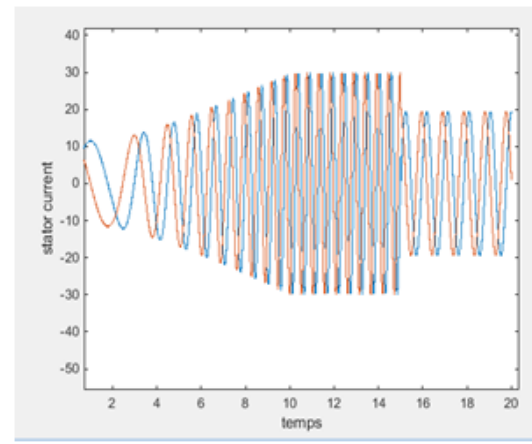


Fig.13. Simulation of the stator currents  $i_{s\alpha}$  and  $i_{s\beta}$

Also, the stator currents  $i_{s\alpha}$  and  $i_{s\beta}$  respond well to the changes imposed on the torque. It is observed that the stator current retains a very close form of sinusoid for all torque variations. We also notice that the stator current is rapidly established in the phase of transition without a great overcoming.

Figure 14 shows the trajectory of the end of the vector stator flux, it may be noted that this trajectory retains a form nearly circular during the entire torque variation.

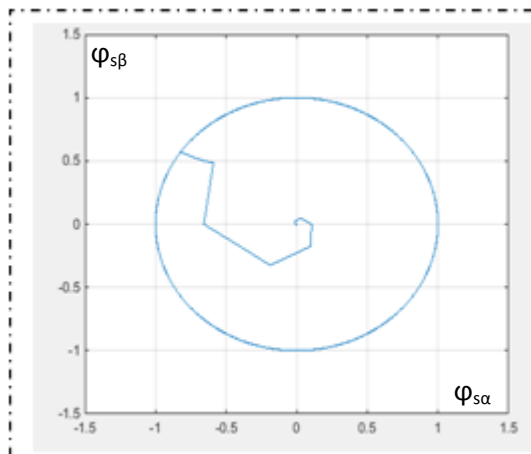


Fig.14. Trajectory of the end of stator flux

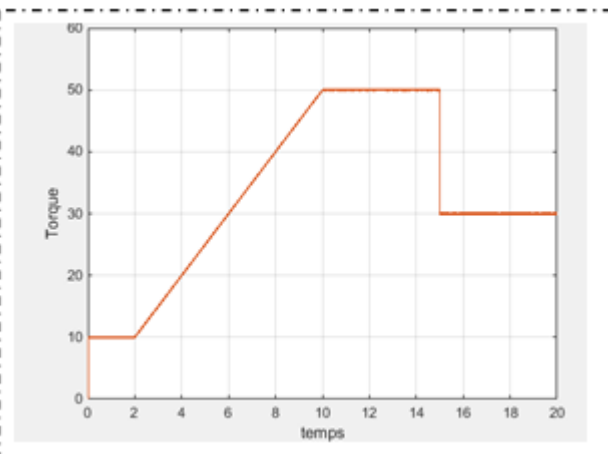


Fig.15 Response of the electromagnetic torque for the reference torque given in figure 11.

On the figures 15 and 19, we can observe the strong dynamic on the torque when we apply a level on the torque. In effect it is noted that the response time is very low and of the order of 0.35ms.

Stator flux  $\varphi_{s\alpha}$  and  $\varphi_{s\beta}$  are represented in figure 16, we can note the forms sinusoidal of stator flux for the entire torque reference, the amplitudes are constants (end of vector flux in a circular crown). It can also be noted the variation of frequency of flux  $\varphi_{s\alpha}$  and  $\varphi_{s\beta}$  when the torque changes its value.

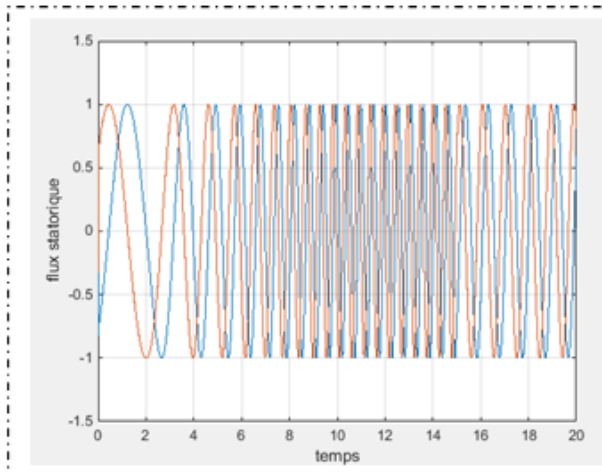


Fig.16 Response of the stator flux  $\phi_{sa}$  and  $\phi_{sb}$

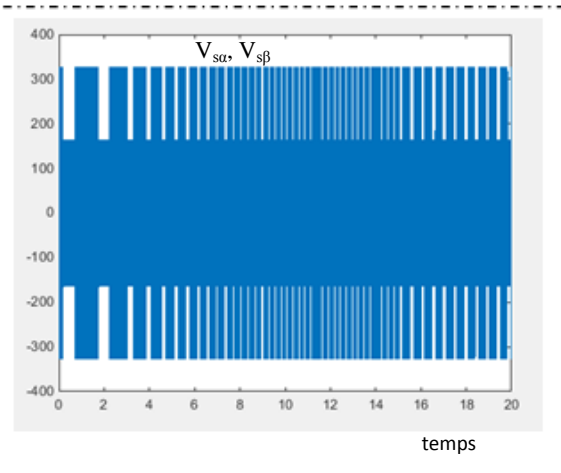


Fig.17. Simulation of the voltage  $V_{sa}$  and  $V_{sb}$

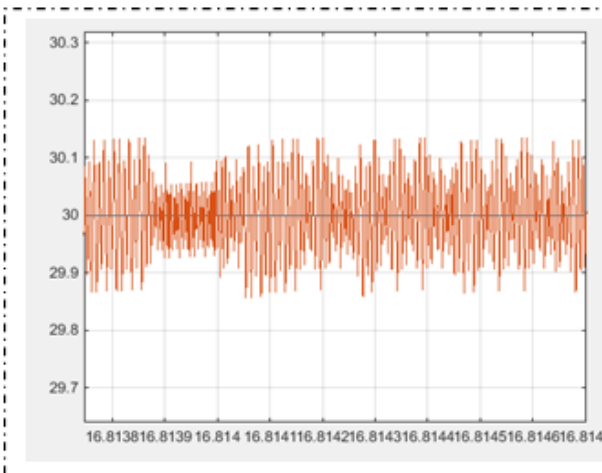


Fig.18. Torque variation around the reference  $T_{ref}=30Nm$

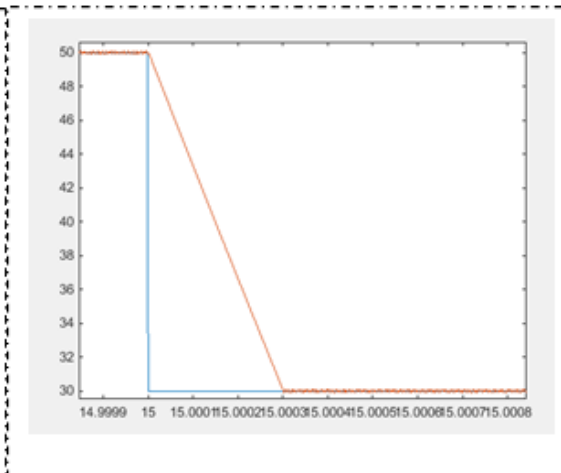


Fig.19. Response time of the torque

## VII. Conclusion

In this paper, we have studied the Direct Torque Control (DTC) of an Induction Motor coupled to an Inverter (Inv-IM). DTC permits to control directly the stator flux and the torque by selecting the appropriate inverter state. Our first contribution is the modeling of Inv-IM as a hybrid system (HS), where the inverter is modeled as a discrete event system (DES) and the induction motor is modeled as a continuous system. Our second contribution is the abstraction of the continuous dynamics of the induction motor as a DES. The HS is thus modeled as a DES. The advantage of this abstraction is that all the rigorous analysis and design methods for DES can be applied for studying DTC. Our third contribution is the use of Supervisory Control Theory (SCT) of DES to drive Inv-IM to a desired working point.

For the sake of clarity, we have based all our study on the fact that the targeted working point is an interval of stator flux and an interval of torque. But our study can be easily adapted for other working points, for example moving the head of the stator flux vector in a circular ring.

An interesting previous work has been done in [16], [17], but the specification was defined with an a priori knowledge of the solution and was not defined as the most permissive solution. Besides, no solution was proposed to the nondeterminism of the 6-zone division. In the present paper, these limitations are solved by taking more advantage of SCT.

As a future work, we intend to improve our control method by using a hierarchical control and a modular control, which are very suitable to take advantage of the fact that the eventbased model of the plant has been constructed hierarchically and modularly. Indeed:

- The hierarchy is in two levels in the construction of the plant. In a first level, the inverter is modeled by an 7-state automaton  $A$ , whose states  $q_k$  correspond to the application of the control vectors  $\vec{V}_k : k=1\dots6$ . In a second level, each state  $q_k$  is replaced by an automaton  $M_k$  modeling the behavior of the induction motor under the control of  $\vec{V}_k$ .

- The modularity is used when constructing each subautomaton  $M_k$  of the plant by combining a 16-state and a 6-state automata;.

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