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A Comprehensive Investigation on Investigation of System with Sparse Co-Prime Sampling

Ms. Apoorva Aggarwal, Mr. Amit Kr. Gautam, Ms. Neelam
[1](*Innovation in Information Technology, Cluster Innovation Centre/ University Of Delhi, India)*
[2](*Innovation in Information Technology, Cluster Innovation Centre/ University Of Delhi, India)*
[3](*Master of Technology, Signal Processing, Ambedkar Institute of Technology, Delhi, India)*

ABSTRACT*:Considering a continuous time LTI system, with impulse response* $h_c(t)$*, the uniformly spaced samples* $h_c(nT)$ *can be identified by using an impulse train input with an arbitrarily small rate 1/NT and sampling the system output with an arbitrarily small rate 1/MT, provided M and N to be coprime for any chosen time spacing T. It is shown that different LTI filters can be identified using the sparse coprime sampling and from input-output measurements. It is also shown that the pattern of random noise corrupting the transmitting signal in the channel can be identified.*

I. INTRODUCTION

Consider Fig. 1 where, $x(n)$ is a sequence which is passed through a discrete to continuous converter,

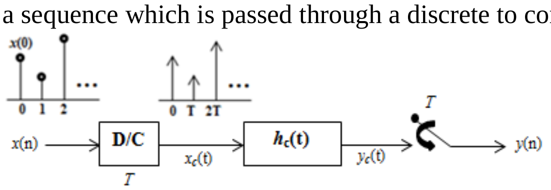

Figure 1: System Identification Problem.

transforming$x(n)$ into an impulse train

$$x_c(t) = \sum_{m=-\infty}^{m=\infty} x(m)\,\delta(t - mT) \qquad (1)$$

with sample spacing T and is transmitted through a continuous time LTI system with impulse response $h_c(t)$. The system output is given by

$$y_c(t) = \sum_{m=-\infty}^{m=\infty} x(m)\,h_c(t - mT) \qquad (2)$$

This output is sampled with spacing T to obtain

$$y(n) = \sum_{m=-\infty}^{m=\infty} x(m)\,h_d(n - m) \qquad (3)$$

where $h_d(n) = h_c(nT)$. Thus the discrete-time equivalent of the system in Fig. 1 is an LTI system with impulse response $h_d(n)$. Since $H_d(z) = Y(z)/X(z)$, it is clear that $h_d(nT)$ can be identified from a knowledge of appropriately designed $x(n)$ and $y(n)$.

$$x_c(t) = \sum_{m=-\infty}^{m=\infty} x(m)\,\delta(t - mNT) \qquad (4)$$

Here, we show that the sampled impulse response $h_c(nT)$, with sample spacing T, is identified by transmitting an impulse train at an arbitrarily small rate 1/NT and taking samples of the received signal at another arbitrarily small rate 1/MT. (The adjective “arbitrarily small” is used here because M and N can be arbitrarily large.) This is schematically shown in Fig. 2.

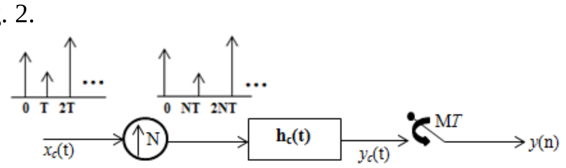

Figure 2: Input stream transmitted at a lower rate 1/NT for system identification. Receiver also performs under-sampling by a factor of M[1].

Such identification is possible if and only if the integers M and N are coprime[1]. This scheme is referred to as coprime sensing method for system identification. The proof of the main result, presented in Section II, is based on a simple connection to fractional sampling rate alteration systems in multirate signal processing theory[2]. Simulations and results are shown in Section III. Finally, the concluding remarks along with the future aspects of the scheme are stated in Section IV.

II. LTI SYSTEM IDENTIFICATION

Consider the Discrete time representation of the system for the Fig. 1, shown in Fig. 3. The output signal $y(n)$ is given by the convolution sum of $h(n)$ and input $x(n)$:

$$y(t) = \sum_{m=-\infty}^{m=\infty} x(m)\, h_c(t - mT) \qquad (5)$$

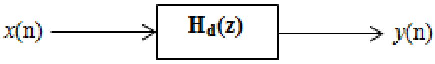

Figure 3: Discrete time LTI System.

Now consider the discrete time representation of the system sensing scheme for the Fig. 2, shown in Fig. 4. The output signal $y_c(t)$ is given by the convolution sum of the $h(n)$ and the input $x(n)$:

$$y_c(t) = \sum_{m=-\infty}^{m=\infty} x(m)\, h_c(t - mNT) \qquad (6)$$

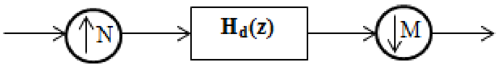

Figure 4: Discrete time LTI system sensing scheme[1]

Here ↓M and ↑N represent the M-fold decimator and N-fold expander respectively, as defined in [2].The M - fold under sampled output $y(n)$ becomes

$$y(n) = y_c(nMT)$$
$$= \sum_{m=-\infty}^{m=\infty} x(m)\, h_c(nMT - mNT) \qquad (7)$$

Defining the desired-rate samples of the system:

$$h_d(n) = h_c(nT)$$

The discrete time model for the system of Fig. 2 is given by

$$y(n) = \sum_{m=-\infty}^{m=\infty} x(m)\, h_d(nM - mN) \qquad (8)$$

Therefore, with the scheme of Fig. 4 where an input stream $x(n)$ is transmitted with uniform spacing NT and the LTI system output is uniformly sampled with spacing MT to obtain $y(n)$. Assuming the sampled impulse response $h_d(n) = h_c(nT)$ is FIR, then $h_d(n)$ can be identified from the received signal $y(n)$ (for an appropriately designed finite duration input $x(n)$) if and only if M and N are coprime.
Stated equivalently, the FIR system $H_d(z)$ in Fig. 4 can be identified from a finite-duration observation of $y(n)$ for appropriately designed input $x(n)$, if and only if M and N are coprime.
The argument of $h_d(.)$ in the (8), has the form

$$i = nM - mN \qquad (9)$$

By Euclid's theorem, coprimality implies that every integer can be expressed in the above form, for an appropriate integer pair (m,n), say, (m_i,n_i):

$$i = Mn_i - Nm_i$$

Then, output of the discrete time model for the system of Fig. 4, is given by

$$y(n) = \sum_{p=0}^{L} x(m)\, h_d(Mn_i - Nm_i) \qquad (10)$$

Now, the integer *i* can be rewritten as:

$$i = M(n_i + Nk_i) - N(m_i + Mk_i)$$

for any integer k_i. Thus, for fixed *i* and k_i the output $y(n_i + Nk_i)$ has the term $h_d(i)x(m_i + Mk_i)$. For each *i* suppose we have identified one initial (m*i*,n*i*) pair. Suppose we modify (m*i*,n*i*) to:

$$m_i' = m_i + Mk_i, \qquad n_i' = n_i + Nk_i$$

for some set of integers k_i and construct an input $x(m)$ which is nonzero only at the points $m_i + Mk_i, 0 \le i \le L$. Then the output at $n_i' = n_i + Nk_i$ is given by:

$$y(n_i') = \sum_{p=0}^{L} x(m_p')\, h_d(Mn_i' - Nm_p'), 0 \le i \le L \quad (11)$$

Note that Mn_i' in the right hand side is independent of p. Since the initial set $\{m_i\}$ is fixed and the above equation holds for any choice of the integers $k_0, k_1, k_2, \ldots, k_L$, we can always choose them such

$$Nm_p' > Nm_{p-1}' + L, \qquad 1 \le p \le L$$

Or we can say,

$$Nm_p' - Nm_{p-1}' > L, \qquad 1 \le p \le L.$$

where L is the order of $H_d(z)$. Here $H_d(z)$ is assumed to be FIR which is defined as:

$$H_d(z) = \sum_{i=0}^{L} h_d(i) z^{-i} \qquad (12)$$

Then the term $h_d(Mn_i' - Nm_p')$ in (11), cannot be nonzero for more than one value of p. Hence,

$$y(n_i') = x(m_i') h_d(Mn_i' - Nm_i') = x(m_i') h_d(i), \qquad 0 \le i \le L \qquad (13)$$

Since $x(m_i')$ and $y(n_i')$ are known, we can identify $h_d(i)$ from this, for each iin $0 \le i \le L$. In this case, we can design with its nonzero samples sufficiently spaced apart, so that any output sample is affected by at most one input sample.

III. SIMULATION AND RESULTS

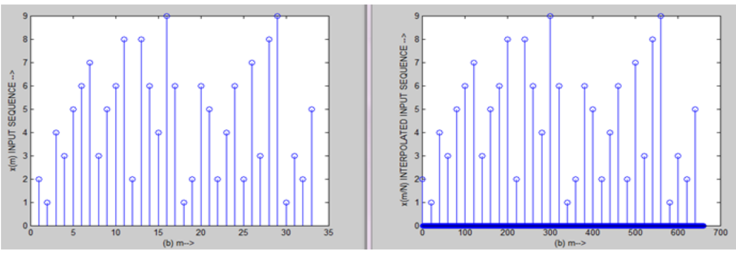

Figure 5: I/p Sequence [$x(m)$] and Interpolated I/p Sequence [$x(m/N)$], N=20.

First, considering the identifying system to be FIR filters and then, to be different random noise patterns generated by MATLAB.

System - Fir Filters: The system under consideration for the problem of system identification is the FIR Filter. The FIR Filter is designed with the help of FDATool in MATLAB. Considering 3 different filters: Low pass, High pass, Band pass, results of each filter are presented.

Firstly we designed 6th order, band-pass, high-passand low-pass filters. Fig. 5 shows the input sequence $x(m)$ and the sequence obtained by interpolating $x(m)$ i.e. $x(m/N)$, with N=20.

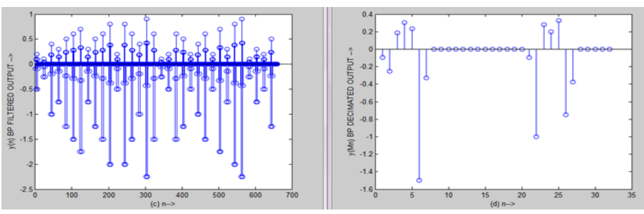

Figure 6: BP Filtered O/p Sequence [$y(n)$] & Decimated BP O/p Sequence [$y(Mn)$], M=21.

Fig. 6 shows the sequence of samples obtained when interpolated input is filtered using a band-pass filter and this sequence is decimated with a factor of M=21, where every 21st sample of output is considered and rest of the samples are discarded.

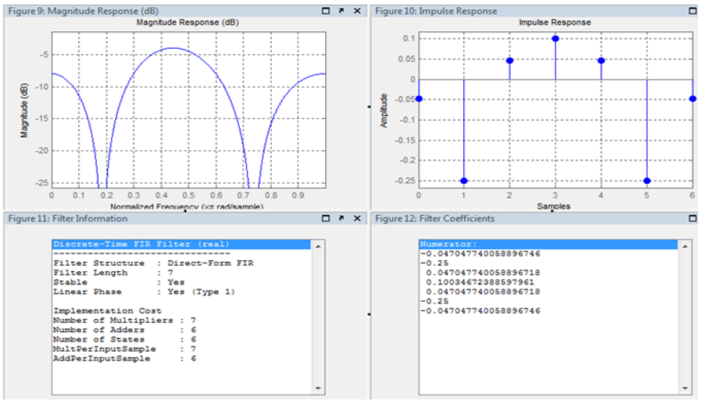

Figure 7: FVTool window providing four results: Magnitude Response of identified BP Filter, Impulse Response, Filter Information and Filter Coefficients.

The results for the identification of band-pass filter, are shown in Fig. 7. Here FVTool plots Magnitude Response of identifiedfilter, which is the measure of magnitude of filter coefficients as a function of frequency. In second part, it produces the ImpulseResponse, which is the plot of amplitudes of filter coefficients in form ofimpulses. Third it provides the information of identified filter, such as the filter type, order, stability and the number ofcomputational blocks. In the fourth part, it displays all the filter coefficients which the program is tuned to identify.

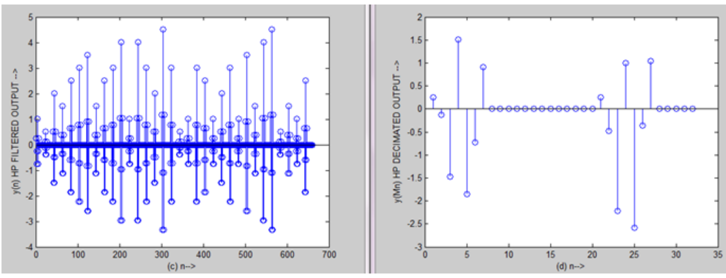

Figure 8: HP Filtered Output Sequence [y(n)] and Decimated HP Output Sequence [y(Mn)], with M=21.

Fig. 8 shows the sequence of samples obtained when interpolated input is filtered using a high-pass filter. This sequence is decimated with a factor of M=21 in similar way as above. Output sequence of Low-pass filter is shown in Fig. 10. The results for the identification of high-pass and low-pass filter are plotted in Fig. 9 and Fig. 11 respectively.

These results validate that the theoretical results stated in Section II and in [1], are accurate and valid for identification of any FIR filter from the knowledge of its input and output sequence. Also they confirms that filter response can be identified from the received signal y(n) (for an appropriately designed finite duration input x(n)) if and only if M and N are coprime.

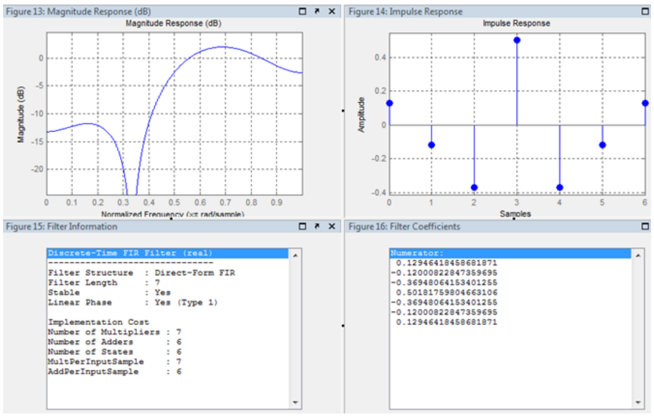

Figure 9: FVTool window providing four results: Magnitude Response of identified HP Filter, Impulse Response, Filter Information & Filter Coefficients.

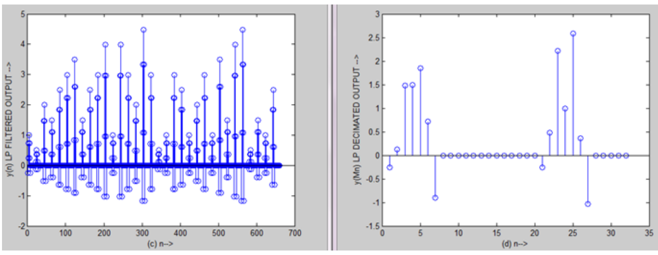

Figure 10: LP Filtered Output Sequence [y(n)] and Decimated LP Output Sequence [y(Mn)], with M=21.

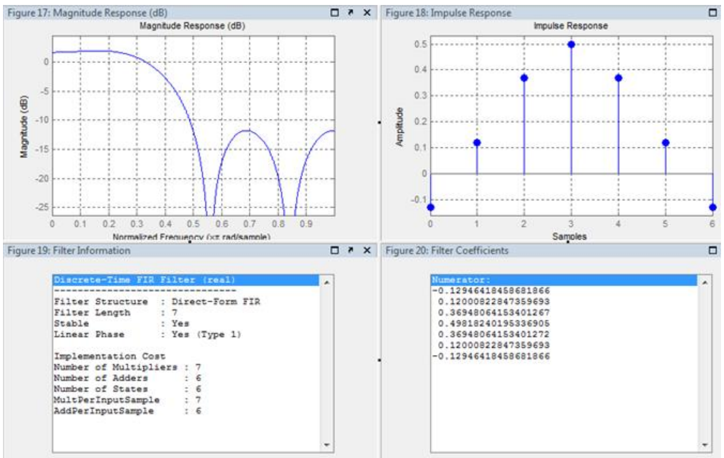

Figure 11: FVTool window providing four results: Magnitude Response of identified LP Filter, Impulse Response, Filter Information & Coefficients.

System - Random Noise: Firstly, we create different kinds of noise using the command for generating random noise. Fig. 12 demonstrates all these random noise. In this, the impulse response of all noise patterns are plotted which finally will be identified.Fig. 13 plots four impulse response: First denotes the input sequence which is interpolated with a factor N=20, second is the output from corrupted noise, third displays the decimated output sequence of samples with decimation factor, M=21. Now, selected samples of this decimated output and the input are considered to calculate the exact samples of noise pattern 1 which is the pseudorandom values drawn from the standard uniform distribution and obtain its impulse response.Next, similar technique is applied to obtain Fig. 14 in which results for the identification of noise pattern 2 are plotted. This noise consists of a normal distributed random pattern of length 15. It shows the input sequence, corrupted output sequence, decimated output and finally, the identified noise pattern 2 which is verified with the noise2 plot in Fig. 12.This same procedure is applied again for identification of noise pattern 3(sequence of random permutation of integers ranging from 1 to 15) and pattern 4(sequence of uniformly sampled random values with integers ranging from 1 to 20), shown in Fig. 15 and Fig. 16 respectively.

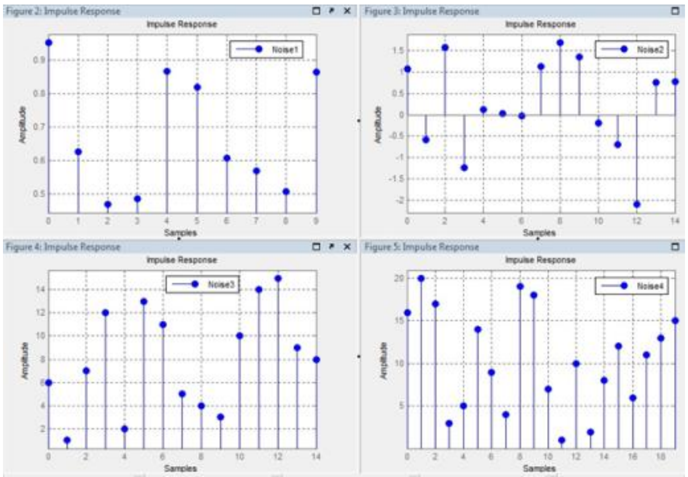

Figure 12: FVTool window providing impulse response of four noise patterns.

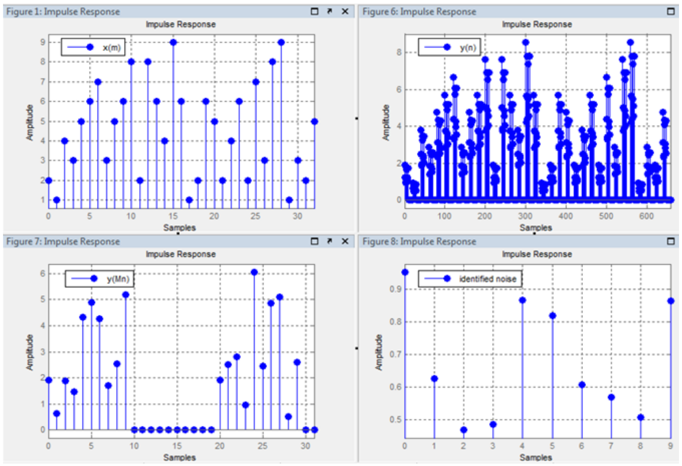

Figure 13: Impulse Response of the input sequence [$\mathbf{x(m)}$], output containing noise1 [$\mathbf{y(n)}$], the decimated output [$\mathbf{y(Mn)}$, with M=21], the identified samples of noise pattern 1 [$\mathbf{h(i)}$].

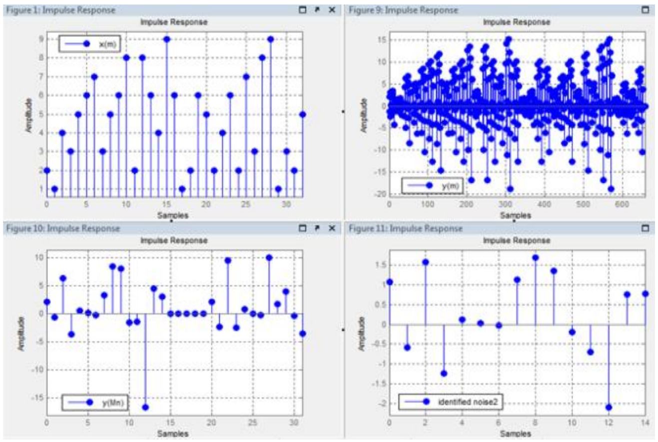

Figure 14: Impulse Response of the input sequence [**x**(**m**)], output containing noise2 [**y**(**n**)], the decimated output [**y**(**Mn**), with M=21], the identified samples of noise pattern 2 [**h**(**i**)].

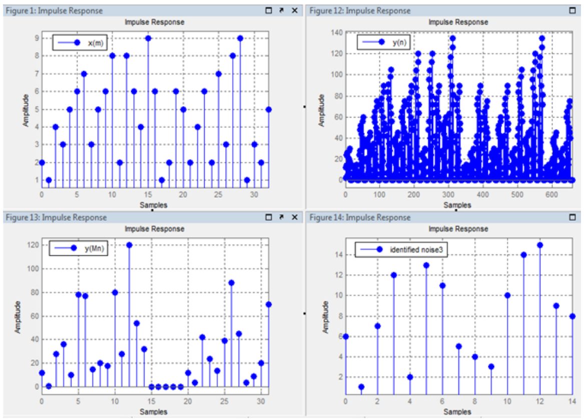

Fig. 15 : Impulse Response of the input sequence [x(m)], output containing noise3 [y(n)], the decimated output [y(Mn), with M=21], the identified samples of noise pattern 3 [h(i)].

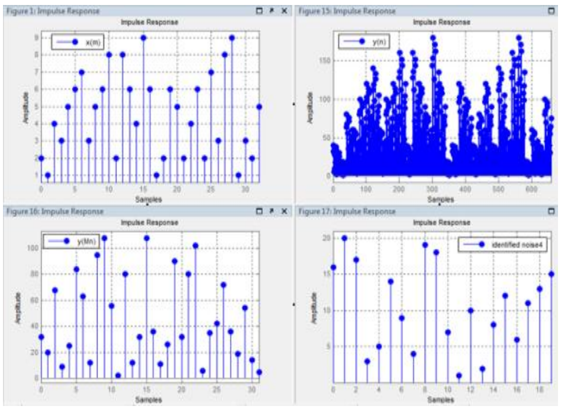

Figure 16: Impulse Response of the input sequence [x(m)], output containing noise4 [y(n)], the decimated output [y(Mn), with M=21], the identified samples of noise pattern 4 [h(i)].

IV. CONCLUSION

The results presented in [1] were quite basic, and remain valid in any situation where a linear time invariant system has to be identified by "sounding out" the system with an impulse train. With the simulation of practical implementation of this defined system sensing scheme, we were successful in proving the scheme to be valid by identifying different types of filters and different noise patterns whose results are presented in Section III. Further applications of the result include channel identification, in which case additive channel noise should also be taken into account. Another potential application is in the identification of target signature in an active sensing scenario. This problem is more sophisticated because of the presence of signal driven interference such as clutter. It will be interesting to explore these applications in greater detail.

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