

SYSTEM IDENTIFICATION USING ADAPTIVE FILTER ALGORITHMS

S.D.Shinde¹, Mrs.A.P.Patil², Mrs.P.P.Belagali³

¹(Dept. of Electronics & Telecommunication Engg, JJMCOE Jaysingpur, India.)

^{2,3}(Dept. of Electronics Engg, JJMCOE Jaysingpur, India.)

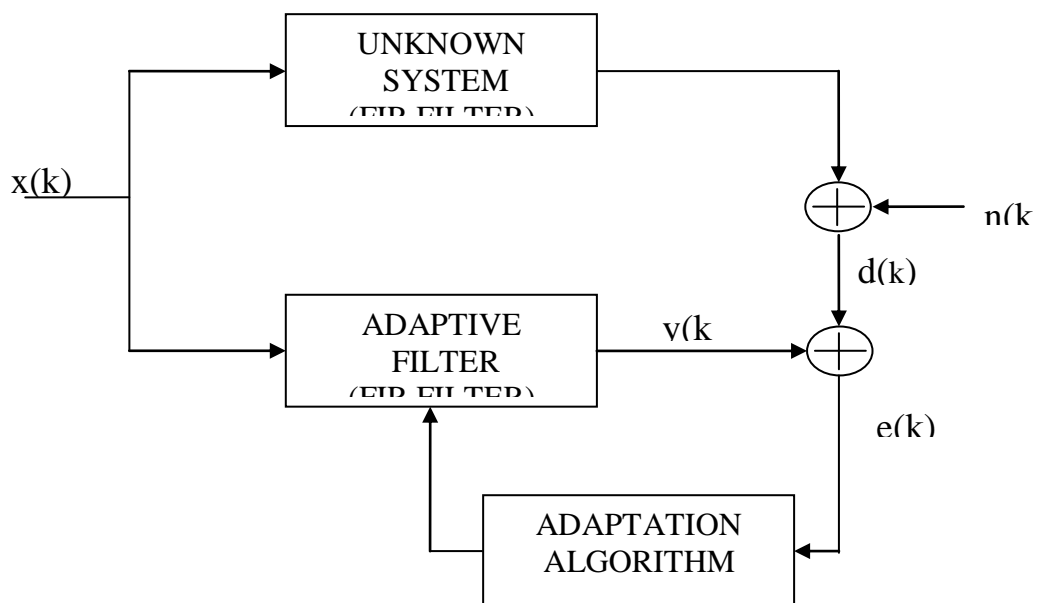
ABSTRACT— A new framework for designing robust adaptive filters is introduced. It is based on the optimization of a certain cost function subject to a time-dependent constraint on the norm of the filter update. Particularly, we present a robust variable step-size NLMS algorithm which optimizes the square of the a posteriori error. We also show the link between the proposed algorithm and another one derived using a robust statistics approach. In addition, a theoretical model for predicting the transient and steady-state behavior and a proof of almost sure filter convergence are provided. The algorithm is then tested in different environments for system identification and acoustic echo cancelation applications.

INDEX TERMS—Acoustic echo cancelation, adaptive filtering, impulsive noise, normalized least-mean-square (NLMS) algorithm, robust filtering.

I. INTRODUCTION

A least mean squares (LMS) filter is an adaptive filter that adjusts its transfer function according to an optimizing algorithm. You provide the filter with an example of the desired output together with the input signal. The filter then calculates the filter weights, or coefficients, that produce the least mean squares of the error between the output signal and the desired signal. To improve the convergence performance of the LMS algorithm, the normalized variant (NLMS) uses an adaptive step size based on the signal power. As the input signal power changes, the algorithm calculates the input power and adjusts the step size to maintain an appropriate value. Thus the step size changes with time. As a result, the normalized algorithm converges more quickly with fewer samples in many cases. For input signals that change slowly over time, the normalized LMS can represent a more efficient LMS approach.

II. GENERALIZED BLOCK DIAGRAM



Block Diagram for System Identification

III. LEAST MEAN SQUARE ALGORITHM

LMS algorithm uses the estimates of the gradient vector from the available data. LMS incorporates an iterative procedure that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. Compared to other algorithms LMS algorithm is relatively simple; it does not require correlation function calculation nor does it require matrix inversions. LMS algorithms have a step size that determines the amount of correction to apply as the filter adapts from one iteration to the next. Choosing the appropriate step size requires experience in adaptive filter design.

Filter convergence is the process where the error signal (the difference between the output signal and the desired signal) approaches an equilibrium state over time

The LMS algorithm initiated with some arbitrary value for the weight vector is seen to converge and stay stable for

$$0 < \mu < 1/\lambda_{\max}$$

Where λ_{\max} is the largest eigenvalue of the correlation matrix R. The convergence of the algorithm is inversely proportional to the eigenvalue spread of the correlation matrix R. When the eigenvalues of R are widespread, convergence may be slow. The eigenvalue spread of the correlation matrix is estimated by computing the ratio of the largest eigenvalue to the smallest eigenvalue of the matrix. If μ is chosen to be very small then the algorithm converges very slowly. A large value of μ may lead to a faster convergence but may be less stable around the minimum value. One of the literatures [will provide reference number here] also provides an upper bound for μ based on several approximations as $\mu \leq 1/(3\text{trace}(R))$.

The LMS algorithm can be summarized by following equations

$$\begin{aligned} y(n) &= \mathbf{w}^h \mathbf{x}(n) \\ e(n) &= \mathbf{d}^*(n) - y(n) \\ \mathbf{w}(n+1) &= \mathbf{w}(n) + \mu \mathbf{x}(n) e^*(n) \end{aligned}$$

IV. NORMALIZED LEAST MEAN SQUARE ALGORITHM

As indicated by Equation LMS's step size is restricted by its region of stability which is determined by the energy in the excitation signal. For signals that have time-varying short-time energy, like speech, a constant stepsize means the speed of convergence will vary with the short-time energy. NLMS overcomes this problem by normalizing the step size every update with the squared Euclidian norm of the excitation vector, $\mathbf{x}(n)$. NLMS can be derived by considering a sample-by-sample cost function that minimizes the size of the coefficient update under the constraint that the *a posteriori error* (the error after the coefficient update) for that sample period is zero. Thus,

$$J_n = \delta \mathbf{r}^T(n) \mathbf{r}(n) + \square^2(n)$$

where, $\mathbf{r}(n)$ is the coefficient update vector at sample period n , $\square(n)$ is the a posteriori error, and δ is a weighting factor between the size of the update and the a posteriori error. The a posteriori error can be expressed as

$$\begin{aligned} \square(n) &= s(n) - \mathbf{x}^T(n) \mathbf{h}(n) \\ &= s(n) - \mathbf{x}^T(n) [\mathbf{h}(n-1) + \mathbf{r}(n)] \\ &= e(n) - \mathbf{x}^T(n) \mathbf{r}(n) \end{aligned}$$

Thus, the cost function can be written as,

$$J_n = \delta \mathbf{r}^T(n) \mathbf{r}(n) + [e(n) - \mathbf{x}^T(n) \mathbf{r}(n)]^2$$

Also we have

$$\xi = \frac{\mu_{\text{NLMS}} \square \mathbf{x}(n) \square}{\square \mathbf{x}(n) \square^2 + \delta}$$

NLMS algorithm can be written in the two steps of its usual implementation form as,

$$\begin{aligned} e(n) &= s(n) - \mathbf{x}^T(n) \mathbf{h}(n-1) \\ \mathbf{h}(n) &= \mathbf{h}(n-1) + \mu_{\text{NLMS}} [\mathbf{x}^T(n) \mathbf{x}(n) + \delta]^{-1} \mathbf{x}(n) e(n) \end{aligned}$$

where, the NLMS step-size parameter, μ_{NLMS} has been added as a *relaxation* factor and the stability range of μ_{NLMS} for NLMS is $0 < \mu_{\text{NLMS}} < 1$. The parameter δ in the NLMS coefficient update is also known as the regularization parameter. It is seen that when δ is non-zero (it is always non-negative) the coefficient update is prevented from becoming unstable when $\mathbf{x}^T(n) \mathbf{x}(n) = 0$.

For sufficiently large L , $\square \mathbf{x}(n) \square^2$ may be approximated as $L \sigma_x^2$

$$\xi = \frac{\mu_{\text{NLMS}} L \sigma}{L \sigma^2 + \delta}$$

This ξ is known as the noise amplification factor. Both μ_{NLMS} and δ help control the size of the noise amplification factor. When the excitation signal's energy, σ_x^2 is very small, ξ is limited in its growth by the regularization factor δ in its denominator. A high regularization value results in slower convergence and very low regularization leads to a larger noise amplification factors. Choosing the correct regularization value is important for optimizing the performance of the adaptive filter. In general, compared to LMS, NLMS with regularization is faster and more stable for all kinds of excitation signals (white noise, colored noise and speech).

V.AFFINE PROJECTION ALGORITHMS

APA is a generalization of NLMS. Where the coefficient update NLMS can be viewed as a rank-1 affine projection, a rank- N projection with N^3+1 is made in APA. As the projection rank increases, the convergence speed of the adaptive filter increases as well, unfortunately so does the computational complexity. The N -order affine projection algorithm, in a relaxed and regularized form, can be defined as,

$$e(n) = s(n) - X^T(n) h(n-1)$$

$$\square(n) = [X^T(n) X(n) + \delta I]^{-1} e(n)$$

$$h(n) = h(n-1) + \mu_{APA} X^T(n) \square(n)$$

When the projection order of APA is 1, it is equivalent to NLMS. However, the convergence of APA gets better with the increased in the projection order and APA demonstrates very good convergence properties with colored excitation signals.

VI.RESULTS OBTAINED

Here the input signal generated is of random form. As it is noticed that if the algorithm works properly for random input it obviously works for any type of input signal given to the system. The figure below on the left hand side shows the input signal given and on the right hand side shows the LMS algorithm output which is similar to that of the desired input signal

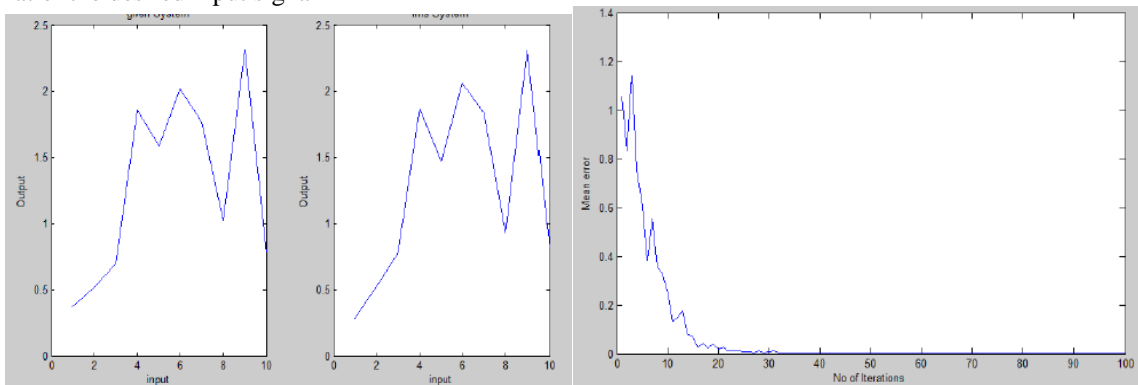


fig 1

fig 2

Here for the above graph we have taken number of inputs as 10 and number of iterations as 100. The system has used LMS adaptive algorithm. At every iteration it compares the desired output and output of the LMS system and calculates the error signal. This error signal is used to generate the weight matrix which adds the correction on the input side so as to get desired signal at the output. If the graph of mean error versus number of iteration is calculated it is as shown above in fig2

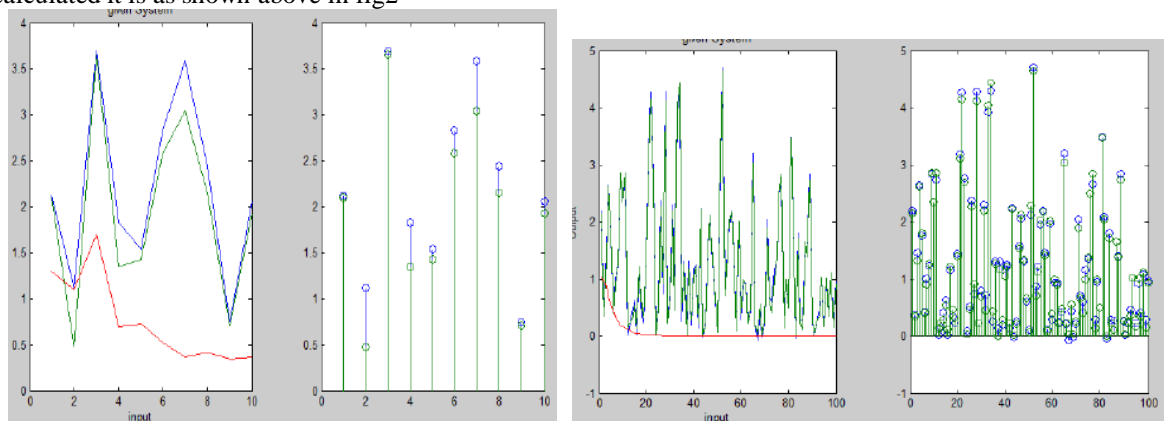


fig 3

fig 4

When the graphs in fig 1 and fig 2 are plotted in the one figure window the output is as shown in fig 3. The Blue line shows the desired output, Green line shows the output of the LMS algorithm and the Red line shows the error signal. It is observed that the LMS output and desired signal are almost the same. The error signal shows that the algorithm converges to the desired output in almost 30 iterations and the error is almost negligible thereafter. The right hand side fig 3 shows the plot of coefficients of input signal to the output signal

Here fig 4 shows the LMS output along with the desired input signal and the error signal when the number of inputs is taken as 100. It is observed that though the number of inputs increases there is no alteration in the output of the algorithm. The convergence rate also remains almost the same as above graphs

Variation of Step Size

The variation of step size plays an very important role in the performance of LMS algorithm. It is the step size which decides the correction to be added to the input signal at every iteration so as to get the desired output. If the system with sine wave as desired signal is considered it is observed that if value of step size is very less then the algorithm takes more number of iterations to get to the required signal. So we can conclude that if the step size is very small then the system convergence requires more number of iterations. If the step size of the system is kept very high then the convergence rate is faster but the system becomes unstable which is not expected so the step size is chosen such that it is neither very less nor very high.

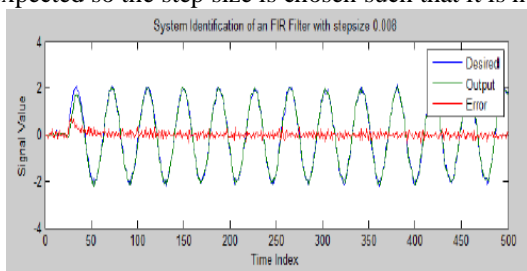


fig 5

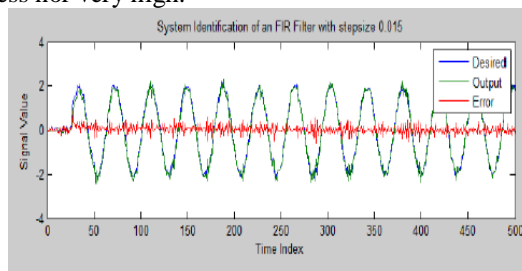


fig 6

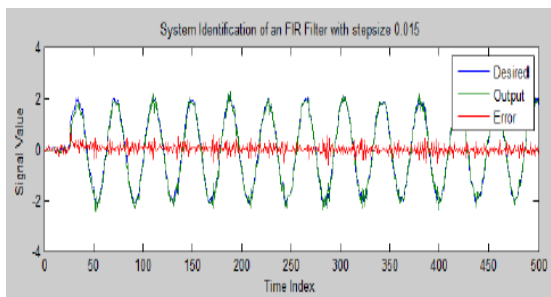


fig 7

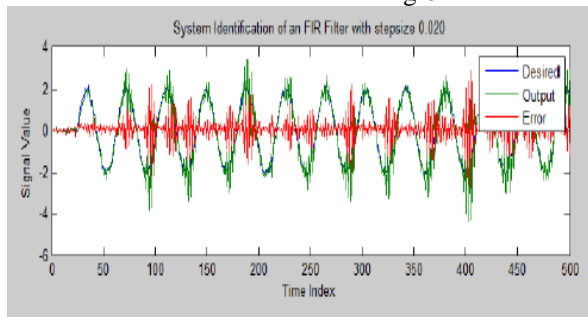


fig 8

The value step size is not constant if the input signal changes obviously the step size should be changed. The brief study of an adaptive filter design may give the idea to select the step size. Here the graph below shows the output when step size is taken as 0.08. It is observed that system converges faster and also it is a stable system. The increase in the step size causes more error in the output signal and there is probability that system becomes unstable.

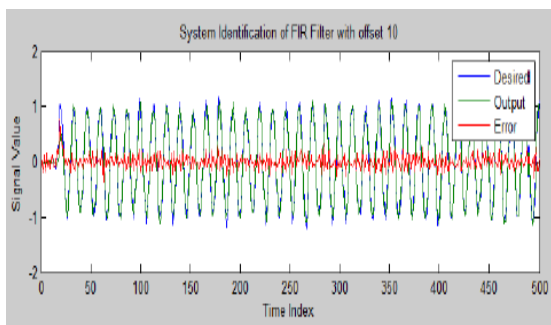


fig 9

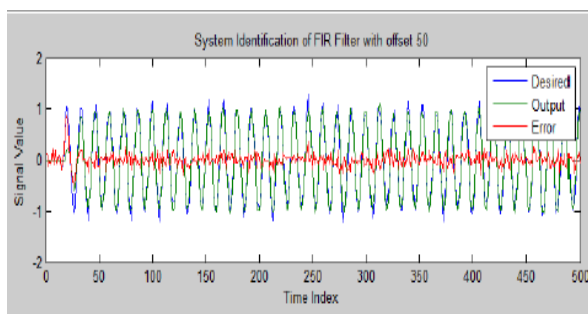


fig 10

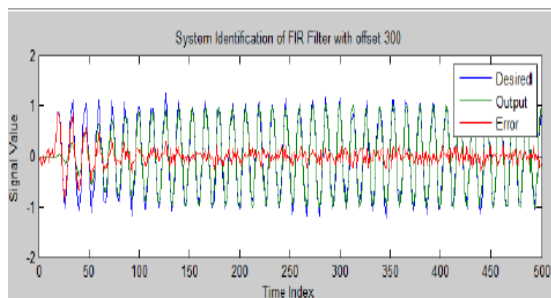


fig 11

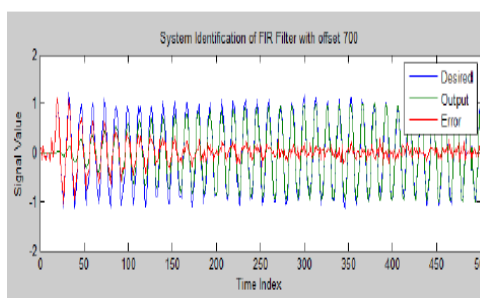
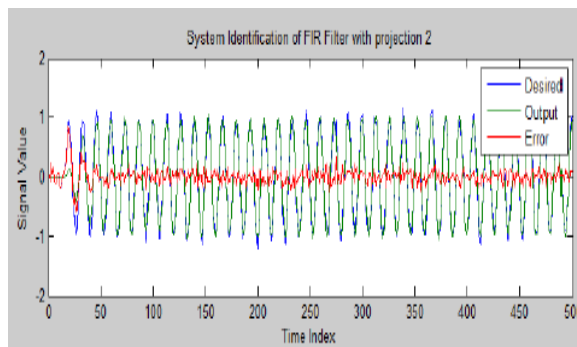
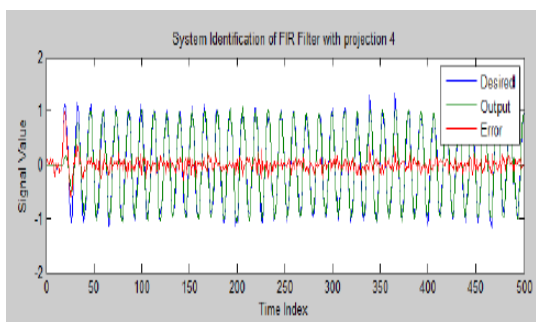


fig 12

If NLMS algorithm is considered with same signal as an input it is observed as the input signal varies the stepsize also varies according to the regularization parameter or we may call it as offset value. If the value of offset is increased the system converges faster but it may become unstable that is why the offset value should not be either too high or too small. The graph in fig 9, fig 10, fig 11 and fig 12 shows that a better result is obtained for an offset value of 50. It is observed that the performance of NLMS is better than that of LMS output signal with a slight increase in computational complexity. In APA, if the projection order is increased, the computational complexity goes on increasing but the convergence rate is much faster than that of LMS and NLMS.



Thus it is observed that the convergence rate of the APA algorithm is faster than NLMS and LMS algorithms but the complexity of the algorithm goes on increasing from LMS to NLMS and finally to APA algorithm as the number of input values L increases. Table 1 gives a brief idea about the complexity of the algorithm, as APA is a generalization of

Equation	Multiplications
Complexity of LMS algorithm	
$e(n) = d^*(n) - w^h(n)x(n)$	L
$w(n+1) = w(n) + \mu_{LMS} x(n)e^*(n)$	L
Total complexity	$2L$
Complexity of NLMS algorithm	
$e(n) = s(n) - x^T(n)h(n-1)$	L
$\square(n) = [x^T(n) \ x(n) + \delta I]^{-1} e(n)$	O_{NLMS}
$h(n) = h(n-1) + \mu_{NLMS} x(n) \square(n)$	L
Total complexity	$2L + O_{NLMS}$
Complexity of APA algorithm	
$e(n) = s(n) - X^T(n)h(n-1)$	NL
$\square(n) = [X^T(n) \ X(n) + \delta I]^{-1} e(n)$	$O_{APA}N^2$
$h(n) = h(n-1) + \mu_{APA} X^T(n) \square(n)$	NL
Total complexity	$2NL + O_{APA}N^2$

Table 1

VII.CONCLUSION

It is seen that the complexity of the algorithm is low in the LMS algorithm but the rate of convergence is high. In Normalized LMS the convergence is faster than LMS with slightly increase in complexity. In APA algorithm further the convergence rate is faster than the NLMS but the complexity is more as compared to LMS and NLMS adaptive algorithms

REFERENCES

- 1) *A learning method for system identification* Nagumo, J.; Noda, A. Automatic Control, IEEE Transactions on Volume: 12 , Issue: 3 Digital Object Identifier: 0.1109/TAC.1967.1098599 Publication Year: 1967 , Page(s): 282 - 287
- 2) *Efficient Implementation of the Affine Projection Sign Algorithm*, Jingen Ni; Feng Li Signal Processing Letters, IEEE Volume: 19 , Issue: 1 Digital Object Identifier: 0.1109/LSP.2011.2174784 Publication Year: 2012 , Page(s): 24 – 26
- 3) *Low-Complexity Implementation of the Affine Projection Algorithm* Zakharov, Y.V.Signal Processing Letters, IEEE Volume: 15 Digital Object Identifier: 10.1109/LSP.2008.2001111 Publication Year: 2008 , Page(s): 557 – 560
- 4) *A new sign algorithm for interference suppression in DS-CDMA systems* Liping Sun; Guangrui Hu Communications Letters, IEEE Volume: 7 , Issue: 5 Digital Object Identifier: 10.1109/LCOMM.2003.811194 Publication Year: 2003 , Page(s): 233 – 235
- 5) *A robust mixed-norm adaptive filter algorithm* Chambers, J.; Avlonitis, A.Signal Processing Letters, IEEE Volume: 4 , Issue: 2 Digital Object Identifier: 10.1109/97.554469 Publication Year: 1997 , Page(s): 46 – 48
- 6) *An Affine Projection Sign Algorithm Robust Against Impulsive Interferences* Tiange Shao; Zheng, Y.R.; Benesty, J. Signal Processing Letters, IEEE Volume: 17 , Issue: 4 Digital Object Identifier: 10.1109/LSP.2010.2040203 Publication Year: 2010 , Page(s): 327 – 330
- 7) *Proportionate Affine Projection Sign Algorithms for Network Echo Cancellation* Zengli Yang; Zheng, Y.R.; Grant, S.L. Audio, Speech, and Language Processing, IEEE Transactions on Volume: 19 , Issue: 8 Digital Object Identifier: 10.1109/TASL.2011.2125955 Publication Year: 2011 , Page(s): 2273 – 2284
- 8) *Low-complexity constrained affine-projection algorithms* Werner, S.; Apolinario, J.A., Jr.; de Campos, M.L.R.; Diniz, P.S.R. Signal Processing, IEEE Transactions on Volume: 53 , Issue: 12 Digital Object Identifier: 10.1109/TSP.2005.859348 Publication Year: 2005 , Page(s): 4545 – 4555
- 9) *A New Robust Variable Step-Size NLMS Algorithm* Vega, L.R.; Rey, H.; Benesty, J.; Tressens, S. Signal Processing, IEEE Transactions on Volume: 56 Issue: 5 Digital Object Identifier: 10.1109/TSP.2007.913142 Publication Year: 2008 , Page(s): 1878 – 1893