

# Analysis of Performance of Fractional Order PID Controller for Continuous Yeast Fermentation Process

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**Abstract:** The PID controllers have found wide acceptance and applications in the industries for the past few decades. In spite of their simple structures, PID controllers are proven to be sufficient for many practical control problems. PID control mechanism, the ubiquitous availability of reliable and cost effective commercial PID modules, and pervasive operator acceptance are among the reasons for the success of PID controllers. An elegant way of enhancing the performance of PID controllers is to use fractional-order controllers where I and D-actions have, non-integer orders. In a  $PI^\lambda D^\delta$  controller, besides the proportional, integral and derivative constants, denoted by  $K_p$ ,  $T_i$  and  $T_d$  respectively, we have two more adjustable parameters: the powers of  $s$  in integral and derivative actions, viz.  $\lambda$  and  $\delta$  respectively. This paper compares the performance of PID and fractional PID controllers used for continuous yeast fermentation process.

**Keywords** –Fractional Calculus, Fractional PID controller, PID controller, yeast fermentation process

## I. INTRODUCTION

The PID controllers have remained, by far; the most commonly used in practically all industrial feedback control applications. PID controllers have been used for several decades in industries for process control applications. The reason for their wide popularity lies in the simplicity of design and good performance including low percentage overshoot and small settling time for slow process plants. The most appreciated feature of the PID controllers is their relative easiness of use, because the three involved parameters have a clear physical meaning. This makes their tuning possible for the operators also by trial-and error and in any case a large number of tuning rules have been developed. Although all the existing techniques for the PID controller parameter tuning perform well, a continuous and an intensive research work is still underway towards system control quality enhancement and performance improvements. On the other hand, in recent years, it is remarkable to note the increasing number of studies related with the application of fractional controllers in many areas of science and engineering. This fact is due to a better understanding of the fractional calculus potentialities. In the field of automatic control, the fractional order controllers which are the generalization of classical integer order controllers would lead to more precise and robust control performances. Although it is reasonably true that the fractional order models require the fractional order controllers to achieve the best performance, in most cases the researchers consider the fractional order controllers applied to regular linear or non-linear dynamics to enhance the system control performances.

This paper is organized as follows: In section 2, we present a brief introduction to fractional calculus. Section 3 deals with fractional order PID controller. Section 4 presents the application of proposed fractional PID controller for yeast fermentation process. Section 5 deals with simulation and results of the system with conclusion.

## II. FUNDAMENTALS OF FRACTIONAL CALCULUS

Fractional calculus is an old mathematical topic since 17th century. Fractional calculus is a subdivision of calculus theory which generalizes the derivative or integral of a function to non-integer order. Fractional calculus helps evaluating  $(d^n y/dt^n)$  n-fold integrals where n is fractional, irrational or complex. For fractional order systems n is considered to be fractional. The number of applications where fractional calculus has been used rapidly grows. These mathematical phenomena allow describing a real object more accurately than the classical "integer-order" methods. The real objects are generally fractional however, for many of them the fractionality is very low. The main reason for using the integer-order models was the absence of solution methods for fractional differential equations. At present there are lots of methods for approximation of fractional derivative and integral and fractional calculus can be easily used in wide areas of applications (e.g.: control theory - new fractional controllers and system models, electrical circuits theory - fractances, capacitor theory, etc.).

The generalized fundamental operator which includes the differentiation and integration is given as:

$$\frac{d^\alpha}{dt^\alpha} \quad R(\alpha) > 0$$

$$\begin{aligned} aD_t^\alpha &= 1 & R(\alpha) &= 0 \\ \int_a^t (d\tau)^\alpha & & R(\alpha) &< 0 \end{aligned} \quad (1)$$

Where

- a – Lower limit of integration
- t – Upper limit of integration
- α – Order of fractional differentiation or integration

Negative α indicates integration and positive α indicates differentiation. [2]

The theory of fractional-order derivative was developed mainly in the 19<sup>th</sup> century. There are several definitions of fractional derivative. Two important and widely applied definitions are:

- Grunwald-Letnikov definition is perhaps the best known one due to its most suitable for the realization of discrete control algorithms. The Grunwald-Letnikov definition is:

$$aD_t^\alpha f(t) = \lim_{h \rightarrow 0} h^{-\alpha} \sum_{j=0}^{\lfloor \frac{t-a}{h} \rfloor} (-1)^j \binom{\alpha}{j} f(t - jh) \quad (2)$$

where  $\lfloor x \rfloor$  means the integer part of  $x$  and  $h$  is time step.

- The Riemann-Liouville definition is given as:

$$f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (3)$$

For a wide class of functions which appear in real physical and engineering applications, the Riemann-Liouville and the Grunwald-Letnikov definitions are equivalent. [2]

### III. FRACTIONAL ORDER PID CONTROLLERS

#### 3.1 Fractional order PID:

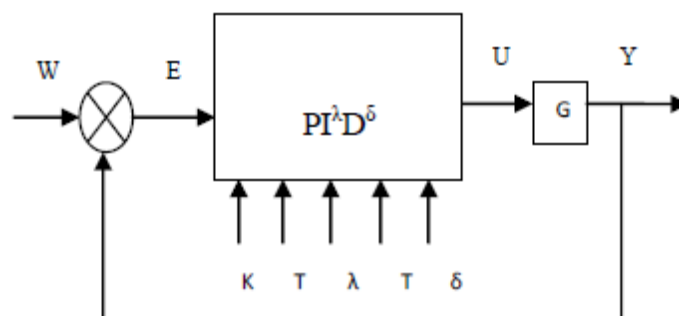
PID controllers belong to dominant industrial controllers and therefore are topics of steady effort for improvements of their quality and robustness. One of the possibilities to improve PID controllers is to use fractional-order controllers with non-integer derivation and integration parts.

A fractional order PID controller is represented as  $PI^\lambda D^\delta$ . It allows us to adjust derivative ( $\lambda$ ) and integral ( $\delta$ ) order in addition to the proportional, integral and derivative constants where the values of  $\lambda$  and  $\delta$  lie between 0 and 1. This gives extra freedom to operator in terms of two extra knobs i.e.

- Order of differentiation
- Order of integration

This also provides more flexibility and opportunity to better adjust the dynamical properties of the control system. The fractional order controller reveals good robustness. The robustness of fractional controller gets more highlighted in presence of a non-linear actuator.

Fig. 1 shows the concept of a fractional PID controller system [5].



**Figure 1. Fractional PID Control System**

The integro-differential equation defining the control action of a fractional order PID controller is given by

$$K_p e(t) + T_i \int e(t) dt + T_d D e(t) = u(t) \quad (4)$$

Applying Laplace transform to this equation with null initial conditions, the transfer function of the controller can be expressed by

$$G_{fc} = K_p + T_i s^{-1} + T_d s \quad (5)$$

Taking  $\lambda=1$  and  $\delta=1$ , we obtain a classical PID controller. If  $\lambda=0$  we obtain a  $PD^\delta$  controller, etc. All these types of controllers are the particular cases of the  $PI^\lambda D^\delta$  controller.

It can be expected that  $PI^\lambda D^\delta$  controller may enhance the system control performance due to more tuning knobs introduced. One of the most important advantages of the fractional order  $PI^\lambda D^\delta$  controller is the possible better control of fractional order dynamical systems. Another advantages lies in the fact that the fractional order  $PI^\lambda D^\delta$  controllers are less sensitive to changes of parameters of a controlled system. This is due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional order control system.

### 3.2 FOPID Controller Tuning

The FOPID controller, has five parameters which can be used to tune the controller, thus a higher flexibility can be achieved, than in the case of a classical PID controller. Due to this reason we expect to obtain with the FOPID controller better closed loop performances than the ones obtained with the PID controllers. Many papers related to tuning methods for FOPID controllers are published in the literature. Although a simple tuning rule, as in the case of PID controllers, does not exist. Barbosa [6] proposed an experimental method for tuning FOPID controllers. The starting point is the parameters determined by using Ziegler-Nichols methods. The parameters of the controller are varied until a satisfactory system response is obtained.

## IV. CONTINUOUS YEAST FERMENTATION PROCESS

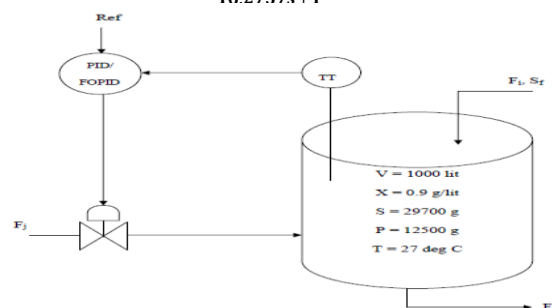
With the presence of living organisms, the control of fermentation process is more complex than conventional chemical reactor. The dynamics of the fermentation process are highly nonlinear and poorly understood. Besides being influenced by external conditions yeasts have their own regulatory mechanism, which mean that the model parameters may not remain unchanged over long time. Therefore we can only change the extracellular environment, which we hope it would affect mechanism rightly. Alcoholic fermentation is one of the most important biochemical processes known to man. The attention directed towards this process has increased during the past two decades, mainly because its product – ethanol – might represent an alternative energy source when used as a partial substitute for gasoline as a fuel. Fig.2 depicts the continuous yeast fermentation process. The fermenter receives a stream, with unknown temperature,  $T_i$  unknown glucose (feed substrate) concentration,  $S_f$ . Temperature of the fermenter,  $T$  is controlled by manipulating the jacket flow rate,  $F_j$ . Few assumptions like Fermenter & Jacket are perfectly mixed, Inlet stream is equal to outlet stream i.e. volume is constant, physical parameters, such as density and heat transfer coefficients are constant. [12].

This reactor is modeled as a continuous-stirred tank with constant substrate feed flow. There is also a constant outlet flow from the reactor that contains the product and substrate, as well as the biomass. The reactor contains three distinct main components:

- The biomass, which is a suspension of yeast and which is fed in batch systems and evacuated continuously;
- The substrate, which is a solution of glucose that feeds the micro-organism and
- The product, ethanol – this is evacuated together with the other components.

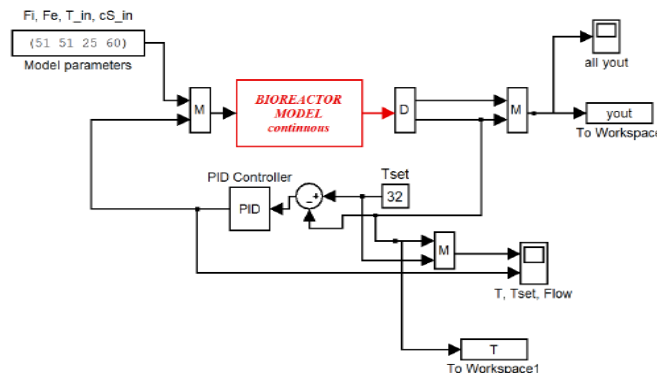
Using the linearized model and the steady state operating values, transfer function is determined. Time is estimated in *hrs*. The FOPDT model is given by [12]

$$G(s) = \frac{-0.146169}{16.2737s+1} e^{-1.8706s} \quad (6)$$



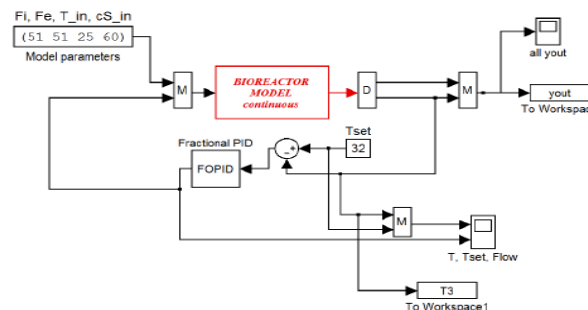
**Figure 2. Continuous yeast fermentation Process**

Fig. 3 shows the Simulink model Simulink model for yeast fermentation bioreactor using PID controller. The model is simulated with effect of variations in flow of jacket,  $F_j$  and inlet temperature,  $T_i$  and output variables are examined. For continuous yeast fermentation process the Fractional order PID controller and PID controller are designed and simulated using simulink model.



**Figure 3. Simulink model for yeast fermentation Bio-reactor using PID controller**

Fig. 4 shows the Simulink model Simulink model for yeast fermentation bioreactor using Fractional Order PID controller.



**Figure 4. Simulink model for yeast fermentation Bio-reactor using FPID controller**

### V. SIMULATION RESULTS

A PID controller is design for the integer order model of **yeast fermentation** bio-reactor using MATLAB PID tuner block. Fig. 3& 4 shows the MATLAB simulink model for bio-reactor control using the conventional PID controller and the Fractional PID controller. The system (Fig. 2) is tuned for the following parameters for the PID controller:

$$K_p = -2.0$$

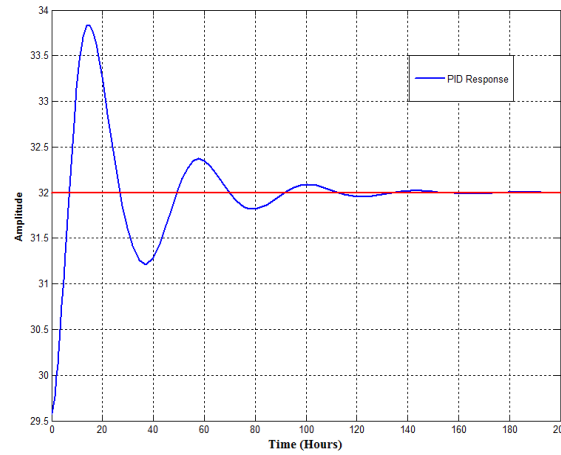
$$K_i = -1.9$$

$$K_d = 1.3$$

Fig. 5 shows the time response characteristics and Table 1 shows its time response specifications.

**Table 1 Time response characteristics of the system**

Parameters	Value
Overshoot	6%
Rise-time	6.5 hours
Settling-time	170 hours



**Figure 5. Time response of fermentation process using PID controller**

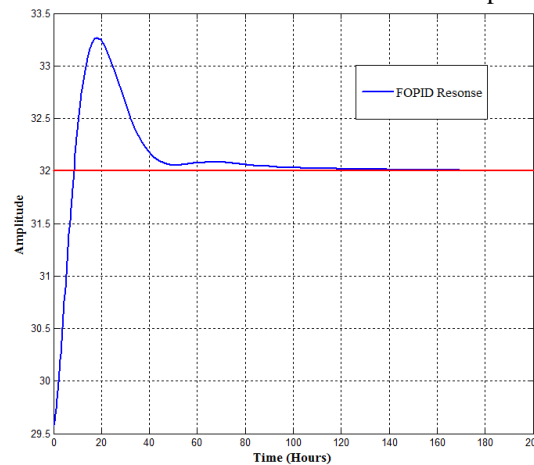
PID controller works well for the system with fixed parameters. However, in the presence of large parameter variations or major external disturbances, the PID controllers usually face trade-off between:

- i. Fast response with significant overshoot.
- ii. Smooth but slow response.

Yeast fermentation bioreactor system is a highly complex, non-linear and uncertain system. For such systems a controller with more number of tuning parameters and which works well for the complex non-linear systems is to be used. Fractional PID controller is one such controller which is to be design for the yeast fermentation bioreactor control. A fractional PID controller is designed for the system by experimental method with following parameters:

$$\begin{aligned}
 K_p &= -2.0 \\
 K_i &= -1.9, \\
 \lambda &= 0.7, \\
 K_d &= 1.3 \\
 \delta &= 0.3
 \end{aligned}$$

Fig.6 shows the time response characteristics and Table 2 shows its time response specifications.



**Figure 6. Time response of fermentation process using FOPID controller**

**Table 2. Performance specifications of the time response for fractional order PID controller**

Parameter	Value
Peak value	4%
Settling time	130 hours
Rise time	7 hours

The PID and fractional order PID controllers are designed for the **yeast fermentation** bio-reactor process. The characteristics are compared in table 3.

**Table 3 Comparison of Time Response Characteristics of PID Controller and Fractional PID Controller**

<b>Characteristics</b>	<b>PID Controller</b>	<b>FOPID Controller</b>
<b>Peak Value</b>	6	4
<b>Settling Time</b>	170	130
<b>Rise Time</b>	6.5	7
<b>Steady State Value</b>	31.9	32

## VI. CONCLUSION

In present work, performance comparison of PID controller with that of fractional order PID controller is presented. Firstly, a simulation model of yeast fermentation process is constructed with the help of MatLab/Simulink module. Then, performance comparison of PID controller with that of fractional order PID controller are simulated and studied. Comparing the step responses with the ones obtained (in simulation) with the PID controller, the better performance of the system with the fractional order PID controller was observed. Fractional order PID controller for integer order plants offer better flexibility in adjusting gain and phase characteristics than the PID controllers, owing to the two extra tuning parameters i.e. order of integration and order of derivative in addition to proportional gain, integral time and derivative time.

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