

Low Complexity Channel Estimation for MC-CDMA System Based Chaos under Fast Multipath Channel

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Abstract: *Based on interpolation methods, a low complexity channel estimation method for MC-CDMA system based chaos under fast fading multipath channel is proposed. Different interpolation methods are used; Linear interpolation (LI), cubic spline interpolation (CSI), and piecewise cubic Hermite interpolation (PCHI). The multi-carrier scheme is established by using a chaotic generator with different initial conditions, while the pilots are established using another chaotic generator with another different initial conditions. The proposed channel estimator without and with interpolation methods for single user and multiusers is tested under fast frequency selective Rayleigh fading channel. Simulation results demonstrate that the channel estimation of MC-CDMA system based chaos using interpolation methods and when the pilots are chaos signal too, achieves better performance improvement in fast multipath environment compared to the conventional MC-CDMA system when the pilots are considered as Walsh-Hadamard codes. The proposed system is also built and tested using Simulink program.*

Keywords: *chaos based spread spectrum, chaos generator, MC-CDMA system, multicarrier system, interpolation, spreading codes, Walsh-Hadamard codes.*

I. INTRODUCTION

Recently, multi-carrier code division multiple access (MC-CDMA), which uses a number of low rate orthogonal sub-carriers to reduce the intersymbol interference (ISI) resulting from frequency selective channel, has been attracting much attention [1]. In MC-CDMA system, the channel transfer function of radio channel is usually frequency selective and time variant. Therefore, a dynamic estimation of the channel is necessary for demodulation of MC-CDMA signals [1].

In [2], blind channel estimation for MC-CDMA systems using aperiodic spreading codes was studied. The multiuser parameter estimation problem was solved using relatively low complexity. Since the method is a blind estimation method, it possesses the simplicity essential for practical implementation of MC-CDMA technologies on uplink.

The channel estimation of a multi-carrier code-division multiple access (MC-CDMA) system based on least square (LS) algorithm and linear interpolation in frequency domain over multipath fading channels was proposed in [3]. Different detection receivers for MC-CDMA were also introduced; their algorithms include maximum ratio combining (MRC), minimum mean square error (MMSE), parallel interference cancellation (PIC), and interference cancellation based on expectation-maximization (EM) algorithm. It was proved in [3] that MMSE algorithm provides significant performance gain compared to that with MRC algorithm and the system performance can be further improved by using PIC and EM algorithms.

In [4], performance of a MC-CDMA system employing MMSE multiuser detection (MUD) at receiver, was evaluated using two pilot symbol assisted (PSA) channel estimation schemes: the maximum likelihood (ML) estimation and the MMSE estimation. It was shown that MMSE estimator significantly outperforms the ML estimator in non-sample-spaced channels where path delays are closely spaced with respect to the time resolution of the system.

A cyclic pilot assisted channel estimation technique (CPACE) suitable for MC-CDMA using overlap Frequency-domain equalization (FDE) was proposed in [5]. It was proved that FDE based on the MMSE criterion can take advantage of the channel frequency-selectivity and improve the average bit error rate (BER) performance due to frequency diversity gain. Overlap FDE technique has been proposed that requires no GI insertion and the MC-CDMA using overlap FDE can provide almost the same BER performance as the conventional MC-CDMA downlink using the GI insertion.

The objective of this paper is to design a low complexity channel estimation method for MC-CDMA system based chaos under fast fading environment. Both the multicarrier scheme and pilot's insertion are based on chaos signals generated from two different chaos generators. Least square (LS) channel estimator is used with interpolation methods. The interpolation methods are: linear interpolation (LI), piecewise cubic Hermite interpolation (PCHI) and cubic spline interpolation (CSI). In comb type pilot channel estimation [6], the pilots were inserted within a fixed distance between the symbols while in this research; the pilots (chaos based) are

inserted randomly between the symbols using random interleaver. The symbol error rate (SER) performance of the proposed channel estimation method is realized for single user and multi-users in 3-paths fast frequency selective Rayleigh fading channel without and with different interpolation methods. Simulation results demonstrate that the performance for the proposed channel estimation based chaos achieves good performance under fast fading channel with and without interpolation methods compared with pilot channel estimation in [3,6] for conventional MC-CDMA that uses Walsh-Hadamard codes for spreading and pilot sequences. The proposed system is also build and tested using Simulink program.

This paper is organised as follows, section II demonstrate the mathematical concepts of interpolation, and section III presents the system description. In section IV, performance evaluation is illustrated using MATLAB program with the proposed system build and tested using Simulink program. Finally, Conclusion is presented in section V.

II. INTERPOLATION

Interpolation is the process of defining a function that takes on specified values at specified points. Any two points in the plane, (x_1, y_1) and (x_2, y_2) , with $x_1 \neq x_2$, determine a unique first-degree polynomial in x whose graph passes through the two points. There are many different formulas for the polynomial, but they all lead to the same straight line graph [7].

Given n points in the plane, $(x_i; y_i), i = 1, \dots, n$, with distinct x_i 's, there is a unique polynomial in x of degree less than n whose graph passes through the points. This polynomial is called the interpolating polynomial because it exactly reproduces the given data [7];

$$p_n(x_i) = (y_i); i = 1, \dots, n \quad (1)$$

2.1 Linear Interpolation (LI)

To generalize the concept of linear interpolation [8], let $n \geq 1$, and suppose that $x_i, i = 0, 1, \dots, n$, are distinct real numbers (i.e., $x_i \neq x_j$ for $i \neq j$) and $y_i, i = 0, 1, \dots, n$, are real numbers; we wish to find $p_n \in P_n$ such that $p_n(x_i) = y_i, i = 0, 1, \dots, n$.

Lemma 2.1.1: Suppose that $n \geq 1$. There exist polynomials $L_k \in P_n, k = 0, 1, \dots, n$, such that [8];

$$L_k(x_i) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases} \quad (2)$$

for all $i, k = 0, 1, \dots, n$. Moreover;

$$p_n(x) = \sum_{k=0}^n L_k(x)y_k \quad (3)$$

satisfies the above interpolation conditions; in other words, $p_n \in P_n$ such that $p_n(x_i) = y_i, i = 0, 1, \dots, n$.

Proof: For each fixed $k, 0 \leq k \leq n$, L_k is required to have n zeros $x_i, i = 0, 1, \dots, n, i \neq k$ thus, $L_k(x)$ is of the form;

$$L_k(x) = C_k \prod_{\substack{i=0 \\ i \neq k}}^n (x - x_i) \quad (4)$$

Where, $C_k \in R$ is a constant to be determined. It is easy to find the value of C_k by recalling that $L_k(x_k) = 1$; using this in (4) yields;

$$C_k = \prod_{\substack{i=0 \\ i \neq k}}^n \frac{1}{x_k - x_i} \quad (5)$$

Inserting (5) into (4), yield;

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^n \frac{x - x_i}{x_k - x_i} \quad (6)$$

As the function p_n defined by (3) is a linear combination of the polynomials $L_k \in P_n, k = 0, 1, \dots, n$, also $p_n \in P_n$. Finally, $p_n(x_i) = y_i, i = 0, 1, \dots, n$ is a trivial consequence of using (2) in (3). The interpolating polynomial is easily described once the form of $L_k(x)$ is known. This polynomial, called the Lagrange interpolating polynomial in which the linear interpolation is based on [8].

2.2 Piecewise Cubic Hermite Interpolation

Many of the most effective interpolation techniques are based on piecewise cubic polynomials. Let h_k denote the length of the k th subinterval [7];

$$h_k = x_{k+1} - x_k \tag{7}$$

Then the first divided difference, δ_k , is given by;

$$\delta_k = \frac{y_{k+1} - y_k}{h_k} \tag{8}$$

Let d_k denote the slope of the interpolant at x_k ;

$$d_k = P'(x_k) \tag{9}$$

For the piecewise linear interpolant, $d_k = \delta_{k-1}$ or δ_k , but this is not necessarily true for higher order interpolants.

Consider the following function on the interval $x_k < x < x_{k+1}$ expressed in terms of local variables $s = x - x_k$ and $h = h_k$ [7];

$$P(x) = \frac{3hs^2 - 2s^3}{h^3} y_{k+1} + \frac{h^3 - 3hs^2 + 2s^3}{h^3} y_k + \frac{s^2(s-h)}{h^2} d_{k+1} + \frac{s(s-h)^2}{h^2} d_k \tag{10}$$

This is a cubic polynomial in s , and hence in x , that satisfies four interpolation conditions, two on function values and two on the possibly unknown derivative values;

$$P(x_k) = y_k, P(x_{k+1}) = y_{k+1} \tag{11}$$

$$P'(x_k) = d_k, P'(x_{k+1}) = d_{k+1} \tag{12}$$

Functions that satisfy interpolation conditions on derivatives are known as Hermite interpolants.

If both function values and first derivative values at a set of data points are known, then piecewise cubic Hermite interpolation can reproduce those data. But if the derivative values were not given, then the idea is to determine the slopes d_k so that the function values do not overshoot the data values, at least locally [7].

If δ_k and δ_{k-1} have opposite signs or if either of them is zero, then x_k is a discrete local minimum or maximum, so the value of $d_k = 0$.

If δ_k and δ_{k-1} have the same sign and the two intervals have the same length, then d_k is taken to be the harmonic mean of the two discrete slopes;

$$\frac{1}{d_k} = \frac{1}{2} \left(\frac{1}{\delta_{k-1}} + \frac{1}{\delta_k} \right) \tag{13}$$

In other words, at the breakpoint, the reciprocal slope of the Hermite interpolant is the average of the reciprocal slopes of the piecewise linear interpolant on either side.

If δ_k and δ_{k-1} have the same sign, but the two intervals have different lengths, then d_k is a weighted harmonic mean, with weights determined by the lengths of the two intervals;

$$\frac{w_1 + w_2}{d_k} = \left(\frac{w_1}{\delta_{k-1}} + \frac{w_2}{\delta_k} \right) \tag{14}$$

Where, $w_1 = 2h_k + h_{k-1}$ and $w_2 = h_k + 2h_{k-1}$

2.3 Cubic spline interpolation

In fact, spline interpolants are also piecewise cubic Hermite interpolating polynomials, but with different slopes [7]. The mathematical spline must have a continuous second derivative and satisfy the same interpolation constraints. The breakpoints of a spline are also referred to as its *knots*.

The first derivative $P'(x)$ of our piecewise cubic function is defined by different formulas on either side of a knot x_k . Both formulas yield the same value d_k at the knots, so $P'(x)$ is continuous [7]. On the k th subinterval, the second derivative is a linear function of $s = x - x_k$ [7];

$$p''(x) = \frac{(6h - 12s)\delta_k + (6s - 2h)d_{k+1} + (6s - 4h)d_k}{h^2} \tag{15}$$

If $x = x_k, s = 0$, and

$$p''(x_k +) = \frac{6\delta_k - 2d_{k+1} - 4d_k}{h_k} \tag{16}$$

The plus sign in $x_k +$ indicates that this is a one-sided derivative. If $x = x_{k+1}, s = h_k$ and

$$p''(x_k -) = \frac{-6\delta_k + 4d_{k+1} + 2d_k}{h_k} \tag{17}$$

On the $(k - 1)$ st interval, $P''(x)$ is given by a similar formula involving δ_{k-1}, d_k and d_{k-1} . At the knot x_k ;

$$p''(x_{k+1} -) = \frac{-6\delta_{k-1} + 4d_k + 2d_{k-1}}{h_{k-1}} \tag{18}$$

Requiring $P''(x)$ to be continuous at $x = x_k$ means that $p''(x_k +) = p''(x_k -)$. This leads to the condition [7];

$$h_k d_{k-1} + 2(h_{k-1} + h_k) d_k + h_{k-1} d_{k+1} = 3(h_k \delta_{k-1} + h_{k-1} \delta_k) \quad (19)$$

If the knots are equally spaced, so that h_k does not depend on k , this becomes

$$d_{k-1} + 4d_k + d_{k+1} = 3(\delta_{k-1} + \delta_k) \quad (20)$$

The slopes d_k of a spline are closely related to the differences δ_k . In the spline case, they are a kind of running average of the δ_k 's [7].

III. System Description

Figure (1) illustrates the MC-CDMA with the proposed channel estimation. The proposed method is based on LS estimator and interpolation methods. The interpolation methods used are linear interpolation (LI), piecewise cubic Hermite interpolation (PCHI) and cubic spline interpolation (CSI).

The information data is modulated using QPSK modulator. The data modulated symbols are represented by:

$$D(m) = [d_1, d_2, d_3, \dots, d_M], \quad (21)$$

The user specific scrambling code c_v is generated from a chaotic generator by changing its initial conditions and is considered as spreading in frequency. [9];

$$C(v) = [c_1, c_2, c_3, \dots, c_v] \quad (22)$$

where,

$$c_n = 4(c_{n-1}^2 - 3c_{n-1}), n = 1, 2, \dots, v \quad (23)$$

The spreaded data symbols are converted from serial-to-parallel (s/p) vector X_s of length $N_d = m \times v$. The pilot vector X_p of length N_p is represented by;

$$X_p(N_p) = [x_1, x_2, \dots, x_{N_p}] \quad (24)$$

Where ,

$$x_n = 4x_{n-1}(1 - x_{n-1}), n = 1, 2, \dots, N_p \quad (25)$$

X_p is generated by varying the initial conditions using the chaotic generator given in (25), then this vector is concatenated with the vector X_s to form a concatenated vector X_{sp} of length N_c ;

$$X_{sp} = [X_s X_p] \quad (26)$$

$N_c = N_p + N_d$ is the number of sub-carriers of inverse fast Fourier transform (IFFT). X_{sp} is input to a random interleaver to form an interleaved vector X_{spI} of length N_c . IFFT block is used to transform X_{spI} to a time domain signal. A guard interval (GI), which is chosen to be larger than the expected delay spread of the channel, is inserted to X_{spI} to prevent inter-symbol interference. After parallel-to-serial (p/s) convertor, the transmitted signal will pass through a 3-paths frequency selective Rayleigh fading channel with additive white Gaussian noise (AWGN). The channel is modeled using Jake's model [10] where its discrete impulse response, h_n is defined as:

$$h_n = \sum_{l=1}^L h_l \text{sinc} \left\{ \frac{\tau_l}{T_s} - n \right\} \quad (27)$$

where, T_s is the input sample period to the channel, τ_l is the set of path delays, L is the total number of propagation paths, and h_l is the l^{th} path gain.

At the receiver, after removing GI, (s/p) conversion, the received signal is sent to FFT block which transforms it into a frequency domain vector $Q(k)$ of length N_c .

$$Q(k) = X_{spI}(k)H(k) + W(k), \quad k = 1, 2, \dots, N_c \quad (28)$$

Let $X(k) = X_{spI}$ for simplicity, $H(k) = FFT\{h(n)\}$ and $W(k) = FFT\{AWGN\}$. Now $Q(k)$ is deinterleaved, the received pilot signals $P(k)$ of length N_p are extracted and used to estimate the channel impulse response for the data sub-channels in the channel estimation block. LS channel estimator is used [7, 8, 9]:

$$H_{LS} = \frac{P(k)}{X_p} \quad (29)$$

H_{LS} is interpolated using the interpolation methods which were described in section II to obtain the interpolated channel estimation vector H_{est_i} . The received data from the deinterleaver, $QQ(k)$ of length N_d is now multiplied with H_{est_i} to obtain the estimated data vectors. As shown in Fig.1, the estimated data vectors each of length N_d are summed, p/s converted and applied to a despreader to obtain the estimated output data.

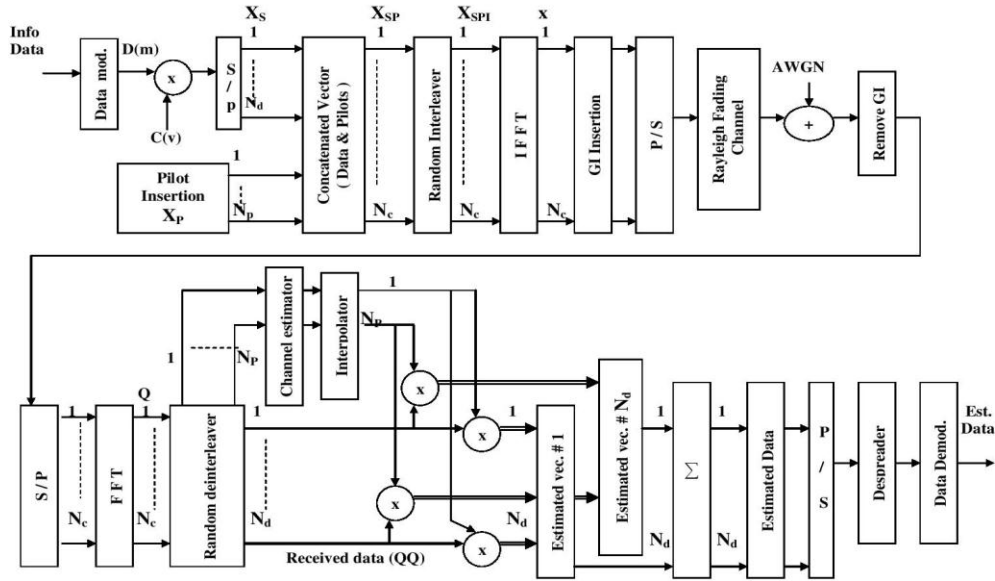


Fig. 1 MC-CDMA system based chaos with proposed channel estimation method

IV. Performance Evaluation

The performances of single user and multi-users (8 users) of MC-CDMA system based chaos under fast fading environment with the proposed channel estimation method are evaluated using MATLAB simulation program as shown in Fig.2, Fig. 3, and Fig. 4. The proposed system is also build and tested using Simulink program as shown in Fig. 5. Table 1, list the simulation parameters that are used in this work.

Table.1 Simulation Parameters

Number of transmitted bits	100000
Type of modulation	QPSK
Spreading factor	8
Interleaver type	Random interleaver
Size of FFT and IFFT	$N = 512$
Length of Guard insertion (GI)	64
Pilot type	Chaos sequence , Walsh – Hadamard sequence
Pilot numbers	16, 32
Rayleigh Fading channel paths	3
Rayleigh Fading channel gains	[0 -3 -8] dB
Doppler Frequency	40 Hz
AWGN	(0 – 30) dB
Number of users	1 , 8
Interpolation types	Linear, nearest, pchip, and spline
Channel estimation	LS estimator

Figure 2 and Fig.3 illustrate the SER performances of MC-CDMA system based chaos in single user and eight users with proposed channel estimation without interpolation and with three types of interpolation methods. It can be noticed from both figures that SER performance using Pchip interpolation method is nearly the same as spline interpolation method but spline interpolation method is better for single and eight users in frequency selective Rayleigh fading channel under fast fading.

In Fig. 4, SER performance performances of MC-CDMA system based chaos using spline interpolation for eight users is presented. Different types and numbers of pilot signal are used, 16 and 32 as chaos and Walsh-

Hadamard sequences. It can be seen that as the number of pilots is increased, the SER performances using chaos and Walsh-Hadamard sequences becomes closer.

In Fig. 5, the proposed system is build and tested using Simulink program, the input is represent by a random integer generator frame based, Both the spreading code and pilots which are chaos based are built as subprograms using MATLAB functions. The LS channel estimation is also build using MATLAB function while the interpolator is represented by FIR interpolator block. Using the parameters which are in Table 1, the proposed system is tested for SNR= 0dB in AWGN and 3-paths fast Rayleigh fading channel where the SER is recorded as 0.3167.

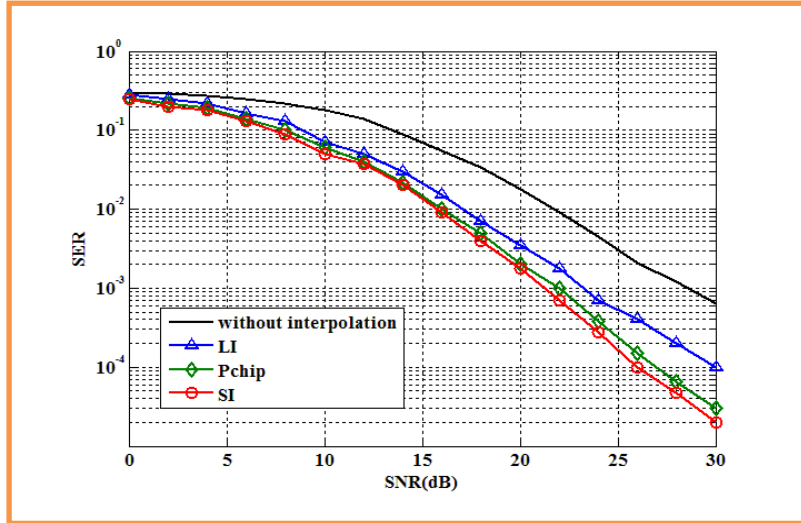


Fig.2 SER performance of MC-CDMA system for one user with and without interpolation

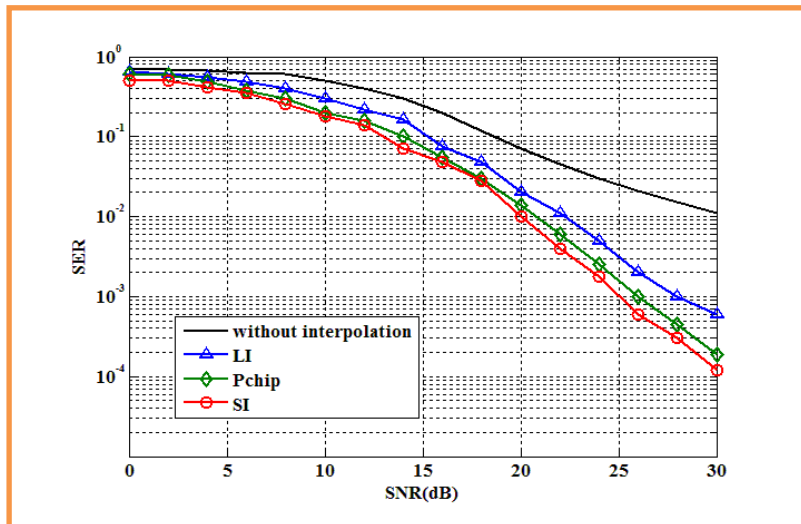


Fig.3 SER performance of MC-CDMA system for eight users with and without interpolation

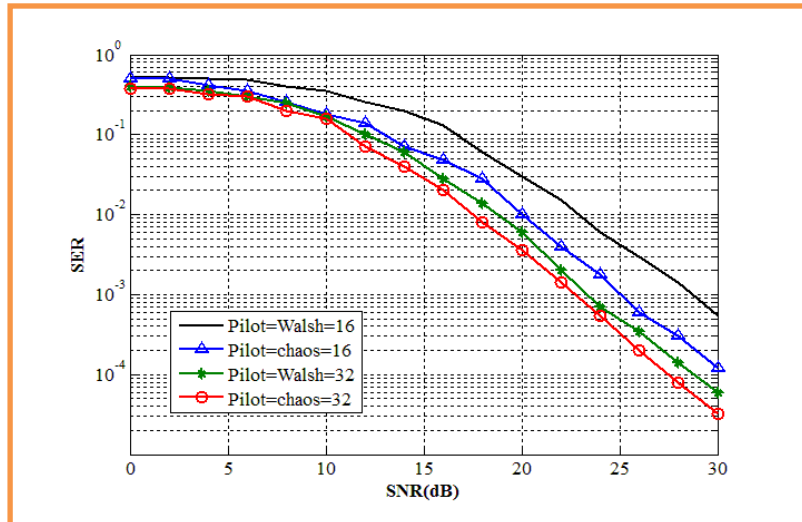


Fig.4 SER performance of MC-CDMA system for eight users with SI and using different pilot types

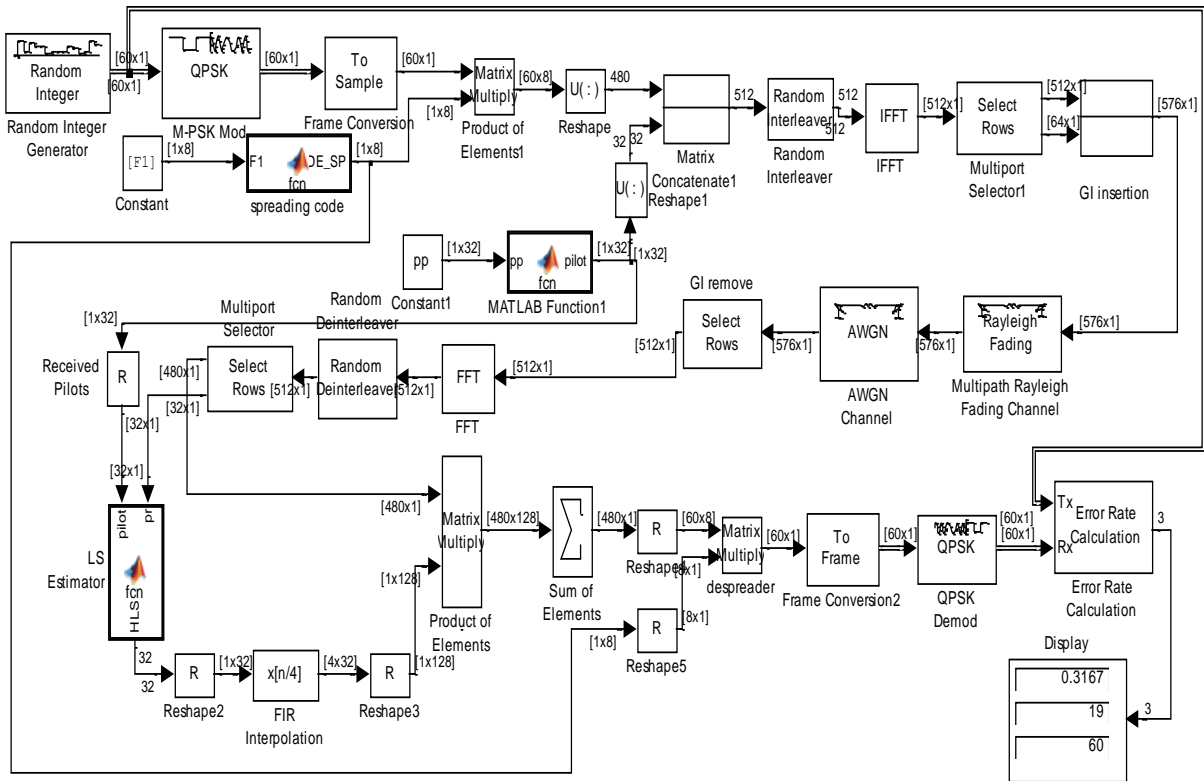


Fig. 5 MC-CDMA system based chaos with proposed channel estimation using Simulink

V. CONCLUSION

A low complexity channel estimation method for MC-CDMA system based chaos was designed for single user and multi – users in fast frequency selective Rayleigh fading channel. Both the multicarrier scheme and pilot's insertion were based on chaos signals generated from two different chaos generators. Random interleaver was used and the proposed method was based on LS channel estimator with interpolator. Three interpolation methods were used. Simulation results demonstrated that the channel estimation of MC-CDMA system using interpolation methods and when the pilots are chaos signal achieves better performance

improvement compared to the same proposed system when the pilots are considered as Walsh-Hadamard codes (conventional MC-CDMA), but the performance becomes closer as the number of pilots is increased. The proposed system was also built and tested using Simulink program.

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