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Pythagorean Fuzzy Ideals: Primary And Semiprimary Structures

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Abstract
The theory of Pythagorean fuzzy sets provides important advantages for modeling vagueness and complex uncertainty. Pythagorean fuzzy information captures the ambiguity inherent in subjective judgments and allows for a more flexible assessment of fuzziness and imprecision. In this paper, we define certain properties of Pythagorean fuzzy primary ideals and Pythagorean fuzzy semiprimary ideals, and we present several results derived from these properties.
Keywords: *Fuzzy sets, Intuitionistic fuzzy sets, pythagorean fuzzy sets, intuitionistic fuzzy primary ideal, intuitionistic fuzzy semiprimary ideal*

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I. Introduction

Recognizing the inherent imprecision in decision-making, Zadeh [20] introduced the concept of fuzzy sets, characterized by a membership function ρ. This function assigns to each element of the universe of discourse a value from the unit interval $[0,1]$, representing the degree of membership in the corresponding set. In a fuzzy set, ρ quantifies the extent to which an element belongs to a particular class: $\rho = 0$ indicates no membership, $\rho = 1$ denotes full membership, and intermediate values capture partial membership.

Although foundational, the classical theory of fuzzy sets has certain limitations, most notably the absence of a non-membership function and the lack of formal treatment for hesitation. To overcome these limitations, Atanassov [11] proposed the concept of intuitionistic fuzzy sets (IFSs), defined by a membership function ρ, a non-membership function σ, and a hesitation margin π (representing indeterminacy). These satisfy the constraints $\rho + \sigma \leq 1$ and $\rho + \sigma + \pi = 1$. This extension provides a richer framework for modeling uncertainty and vagueness.

However, situations also arise where $\rho + \sigma \geq 1$, which cannot be adequately represented by IFSs. To address this issue, Yager [18, 19] introduced Pythagorean fuzzy sets (PFSs), where the membership and non-membership degrees satisfy $\rho^2 + \sigma^2 \leq 1$, and consequently $\rho^2 + \sigma^2 + \pi^2 = 1$, with π denoting the Pythagorean fuzzy index. This formulation allows for greater flexibility in capturing vagueness and uncertainty. Further studies and applications of PFSs can be found in [2, 10, 12, 16].

Algebraic structures play an essential role across disciplines such as theoretical physics, information science, computer science, and control engineering. This broad relevance has motivated researchers to extend classical algebraic concepts within the framework of fuzzy set theory. For instance, Kim and Jun [13] explored the intuitionistic fuzzification of semigroup ideals, while Kumbhojkar and Bapat [14] established a correspondence theorem for fuzzy ideals. Palanivelrajan and Nandakumar [15] further advanced the field by defining and analyzing operations on intuitionistic fuzzy primary and semiprimary ideals.

In this paper, we focus on algebraic structures within the setting of Fermatean fuzzy sets. Specifically, we introduce the notions of Fermatean fuzzy primary and Fermatean fuzzy semiprimary ideals and establish several important results concerning their properties. The paper is organized as follows: Section 2 reviews the necessary preliminaries and definitions, including fuzzy sets, fuzzy primary ideals, intuitionistic fuzzy sets, Pythagorean fuzzy sets etc. Section 3 introduces Pythagorean fuzzy primary and semiprimary ideals and investigates their fundamental properties. Finally, Section 4 concludes the paper with a summary of the key results.

II. Preliminaries And Definition

We will review the related concepts of fuzzy sets, intuitionistic fuzzy sets, and pythagorean fuzzy sets in this section.

Definition 2.1 *A fuzzy set F in a universal set X is defined as*

$$F = \{\langle \xi, \rho_F(\xi) \rangle : \xi \in X\},$$

where $\rho_F : X \to [0,1]$ is a mapping that is known as the fuzzy set's membership function.
The complement of ρ is defined by $\bar{\rho}(\xi) = 1 - \rho(\xi)$ for all $\xi \in X$ and denoted by $\bar{\rho}$.

Definition 2.2 *Let X be a fixed set. An intuitionistic fuzzy set (IFS) A in X is an expression having the form*

$$A = \{\langle \xi, \rho_A(\xi), \sigma_A(\xi) \rangle : \xi \in X\},$$

where the $\rho_A(\xi)$ is tha memebership grade and $\sigma_A(\xi)$ is the non-membership grade of the element $\xi \in X$ respectively.
Also $\rho_A : X \to [0,1], \sigma_A : X \to [0,1]$ and satisfy the condition

$$0 \leq \rho_A(\xi) + \sigma_A(\xi) \leq 1,$$

for all $\xi \in X$.
The degree of indeterminacy $h_A(\xi) = 1 - \rho_A(\xi) - \sigma_A(\xi)$.

Definition 2.3 *A Pythagorean fuzzy set P in a finite universe of discourse X is given by*

$$P = \{\langle \xi, \rho_P(\xi), \sigma_P(\xi) \rangle | \xi \in X\},$$

where $\rho_P(\xi) : X \to [0,1]$ denotes the degree of membership and $\sigma_P(\xi) : X \to [0,1]$ represents the degree to which the element $\xi \in X$ is not a member of the set P, with the condition that

$$0 \leq (\rho_P(\xi))^2 + (\sigma_P(\xi))^2 \leq 1,$$

for all $\xi \in X$.
The degree of indeterminacy $h_P(\xi) = \sqrt{1 - (\rho_P(\xi))^2 - (\sigma_P(\xi))^2}$.

Definition 2.4 *A fuzzy ideal ρ of a ring R is called fuzzy primary ideal, if for all $a, b \in R$ either $\rho(ab) = \rho(a)$ or else $\rho(ab) \leq (b^m)$ for some $m \in Z^+$.*
Definition 2.5 *A fuzzy ideal u of a ring R is called fuzzy semiprimary ideal, if for all $a, b \in R$ either $\rho(ab) \leq \rho(a^n)$, for some $n \in Z^+$, or else $\rho(ab) \leq (b^m)$ for some $m \in Z^+$*
Definition 2.6 *A fuzzy ideal A of a ring R is called Pythagorean fuzzy semiprimary ideal if for all a, $b \in R$, either $\rho(ab) \leq \rho_(a^m)$ and $\sigma_A(ab) \geq \sigma_A(a^m)$ for some $n \in Z^+$ or else $\rho(ab) \leq \rho A(b^m)$ and $v_A(ab) \geq \sigma_A a(b^m)$ for some $m \in Z^+$.*

III. Main Results

Theorem 3.1 *If $\mathcal{P}$ and Q are Pythagorean fuzzy primary ideal of a ring R then $\mathcal{P} \times Q$ is also an Pythagorean fuzzy primary ideal of R.*
Proof: Let $(\xi, \zeta) \in \mathcal{P} \times Q$ where $\xi_1, \xi_2 \in \mathcal{P}$ and $\zeta_1, \zeta_2 \in Q$.
Consider

$$\begin{aligned} \rho_{\mathcal{P} \times Q}((\xi_1, \zeta_1), (\xi_2, \zeta_2)) &= \rho_{\mathcal{P} \times Q}(\xi_1 \xi_2, \zeta_1 \zeta_2) \\ &= \min(\rho_{\mathcal{P}}(\xi_1, \xi_2), \rho_Q(\zeta_1, \zeta_2)) \\ &= \min(\rho_{\mathcal{P}}(\xi_1), \rho_Q(\zeta_1)) \end{aligned}$$

Therefore, $\rho_{\mathcal{P} \times Q}(((\xi_1, \zeta_1).(\xi_2, \zeta_2)) = \rho_{\mathcal{P} \times Q}(\xi_1, \zeta_1)$.
Again,

$$\begin{aligned} \sigma_{\mathcal{P} \times Q}((\xi_1, \zeta_1), (\xi_2, \zeta_2)) &= \sigma_{\mathcal{P} \times Q}(\xi_1 \xi_2, \zeta_1 \zeta_2) \\ &= \max(\rho_A(\xi_1, \xi_2), \sigma_Q(\zeta_1, \zeta_2)) \\ &= \max(\sigma_A(\xi_1), \sigma_Q(\zeta_1)). \end{aligned}$$

Therefore, $\sigma_{\mathcal{P} \times Q}(((\xi_1, \zeta_1).(\xi_2, \zeta_2)) = \sigma_{\mathcal{P} \times Q}(\xi_1, \zeta_1)$.
Therefore, $\mathcal{P} \times Q$ is an Pythagorean fuzzy primary ideal of R.

Theorem 3.2 *If $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ are Pythagorean fuzzy primary ideal of a ring R then $\mathcal{P}_1 \cup (\mathcal{P}_2 \cap \mathcal{P}_3) = (\mathcal{P}_1 \cup \mathcal{P}_2) \cap (\mathcal{P}_1 \cup \mathcal{P}_3)$ is also an Pythagorean fuzzy primary ideal of R.*
Proof: Consider

$$\begin{aligned} \rho_{\mathcal{P}_1 \cup (\mathcal{P}_2 \cap \mathcal{P}_3)}(\xi\zeta) &= \max(\rho_{\mathcal{P}_1}(\xi\zeta), \rho_{\mathcal{P}_2 \cap \mathcal{P}_3}(\xi\zeta)) \\ &= \max(\rho_{\mathcal{P}_1}(\xi), \rho_{\mathcal{P}_2 \cap \mathcal{P}_3}(\xi)) \\ &= \max(\rho_{\mathcal{P}_1}(\xi), \min(\rho_{\mathcal{P}_2}(\xi), \rho_{\mathcal{P}_3}(\xi))) \\ &= \min(\max(\rho_{\mathcal{P}_1}(\xi), \rho_{\mathcal{P}_2}(\xi)), \max(\rho_{\mathcal{P}_1}(\xi), \rho_{\mathcal{P}_3}(\xi))) \\ &= \min(\rho_{\mathcal{P}_1 \cup \mathcal{P}_2}(\xi), \rho_{\mathcal{P}_1 \cup \mathcal{P}_3}(\xi)) \end{aligned}$$

Also,

$$\sigma_{\mathcal{P}_1 \cup (\mathcal{P}_2 \cap \mathcal{P}_3)}(\xi\zeta) = \min(\sigma_{\mathcal{P}_1}(\xi\zeta), \sigma_{\mathcal{P}_2 \cap \mathcal{P}_3}(\xi\zeta))$$

$$= \min(\sigma_{\mathcal{P}_1}(\xi), \sigma_{\mathcal{P}_2 \cap \mathcal{P}_3}(\xi))$$
$$= \min(\sigma_{\mathcal{P}_1}(\xi), \max(\sigma_{\mathcal{P}_2}(\xi), \sigma_{\mathcal{P}_3}(\xi)))$$
$$= \max(\min(\sigma_{\mathcal{P}_1}(\xi), \sigma_{\mathcal{P}_2}(\xi)), \min(\sigma_{\mathcal{P}_1}(\xi), \sigma_{\mathcal{P}_3}(\xi)))$$
$$= \max(\sigma_{\mathcal{P}_1 \cup \mathcal{P}_2}(\xi), \sigma_{\mathcal{P}_1 \cup \mathcal{P}_3}(\xi))$$

Therefore, $\mathcal{P}_1 \cup (\mathcal{P}_2 \cap \mathcal{P}_3) = (\mathcal{P}_1 \cup \mathcal{P}_2) \cap (\mathcal{P}_1 \cup \mathcal{P}_3)$ is an Pythagorean fuzzy primary ideal of R.

Theorem 3.3 *If $\mathcal{P}$ and $\mathcal{Q}$ are Pythagorean fuzzy primary ideal of a ring R then $\mathcal{P} \cup (\mathcal{P} \cap \mathcal{Q}) = \mathcal{P}$ is also an Pythagorean fuzzy primary ideal of R.*

Proof: Consider,

$$\rho_{\mathcal{P} \cup (\mathcal{P} \cap \mathcal{Q})}(\xi\zeta) = \max\{\rho_{\mathcal{P}}(\xi\zeta), \rho_{\mathcal{P} \cap \mathcal{Q}}(\xi\zeta)\}$$
$$= \max\{\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{P} \cap \mathcal{Q}}(\xi)\}$$
$$= \max\{\rho_{\mathcal{P}}(\xi), \min(\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{Q}}(\xi))\}$$
$$= \max\{\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{Q}}(\xi)\}.$$

Therefore, $\rho_{\mathcal{P} \cup (\mathcal{P} \cap \mathcal{Q})(\xi\zeta)} = \rho_{\mathcal{P}}(\xi)$.

Again,

$$\sigma_{\mathcal{P} \cup (\mathcal{P} \cap \mathcal{Q})}(\xi\zeta) = \min\{\sigma_{\mathcal{P}}(\xi\zeta), \sigma_{\mathcal{P} \cap \mathcal{Q}}(\xi\zeta)\}$$
$$= \min\{\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{P} \cap \mathcal{Q}}(\xi)\}$$
$$= \min\{\sigma_{\mathcal{P}}(\xi), \max(\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{Q}}(\xi))\}$$
$$= \min\{\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{Q}}(\xi)\}$$

Therefore, $\sigma_{\mathcal{P} \cup (\mathcal{P} \cap \mathcal{Q})(\xi\zeta)} = \sigma_{\mathcal{P}}(\xi)$.

Therefore, $\mathcal{P} \cup (\mathcal{P} \cap \mathcal{Q}) = \mathcal{P}$ is an Pythagorean fuzzy primary ideal of R.

Theorem 3.4 *If $\mathcal{P}$ and $\mathcal{Q}$ are Pythagorean fuzzy primary ideal of a ring R then $\mathcal{P} \cap (\mathcal{P} \cup \mathcal{Q}) = \mathcal{P}$ is also an Pythagorean fuzzy primary ideal of R.*

Proof: Consider

$$\rho_{\mathcal{P} \cap (\mathcal{P} \cup \mathcal{Q})}(\xi\zeta) = \min(\rho_{\mathcal{P}}(\xi\zeta), \rho_{\mathcal{P} \cup \mathcal{Q}}(\xi\zeta))$$
$$= \min(\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{P} \cup \mathcal{Q}}(\xi))$$
$$= \min(\rho_{\mathcal{P}}(\xi), \max(\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{Q}}(\xi)))$$
$$= \min(\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{Q}}(\xi)).$$

Therefore, $\rho_{\mathcal{P} \cap (\mathcal{P} \cup \mathcal{Q})}(\xi\zeta) = \rho_{\mathcal{P}}(\xi)$.

Again,

$$\sigma_{\mathcal{P} \cap (\mathcal{P} \cup \mathcal{Q})}(\xi\zeta) = \max(\sigma_{\mathcal{P}}(\xi\zeta), \sigma_{\mathcal{P} \cup \mathcal{Q}}(\xi\zeta))$$
$$= \max(\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{P} \cup \mathcal{Q}}(\xi))$$
$$= \max(\sigma_{\mathcal{P}}(\xi), \min(\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{Q}}(\xi)))$$
$$= \max(\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{Q}}(\xi)).$$

Therefore, $\sigma_{\mathcal{P} \cap (\mathcal{P} \cup \mathcal{Q})}(\xi\zeta) = \sigma_{\mathcal{P}}(\xi)$.

Hence, $\mathcal{P} \cap (\mathcal{P} \cup \mathcal{Q}) = \mathcal{P}$ is an Pythagorean fuzzy primary ideal of R.

Theorem 3.5 *If $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{S}$ are Pythagorean fuzzy primary ideal of a ring R then $\mathcal{P} \cap (\mathcal{Q} \cup \mathcal{S}) = (\mathcal{P} \cap \mathcal{Q})(\mathcal{P} \cap \mathcal{S})$ is also an Pythagorean fuzzy primary ideal of R.*

Proof: Consider

$$\rho_{\mathcal{P} \cap (\mathcal{Q} \cup \mathcal{S})}(\xi\zeta) = \min(\rho_{\mathcal{P}}(\xi\zeta), \rho_{\mathcal{Q} \cup \mathcal{S}}(\xi\zeta))$$
$$= \min(\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{Q} \cup \mathcal{S}}(\xi))$$
$$= \min(\rho_{\mathcal{P}}(\xi), \max(\rho_{\mathcal{Q}}(\xi), \rho_{\mathcal{S}}(\xi)))$$
$$= \max((\min\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{Q}}(\xi)), (\min\rho_{\mathcal{P}}(\xi), \rho_{\mathcal{Q}}(\xi)))$$
$$= \max(\rho_{\mathcal{P} \cap \mathcal{Q}}(\xi), \rho_{\mathcal{P} \cap \mathcal{S}}(\xi))$$

Therefore, $\rho_{\mathcal{P} \cap (\mathcal{Q} \cup \mathcal{S})}(\xi\zeta) = \rho_{\mathcal{P} \cap \mathcal{Q}}(\xi), \rho_{\mathcal{P} \cap \mathcal{S}}(\xi)$

Again,

$$\sigma_{\mathcal{P} \cap (\mathcal{Q} \cup \mathcal{S})}(\xi\zeta) = \max(\sigma_{\mathcal{P}}(\xi\zeta), \sigma_{\mathcal{Q} \cup \mathcal{S}}(\xi\zeta))$$
$$= \max(\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{Q} \cup \mathcal{S}}(\xi))$$
$$= \max(\sigma_{\mathcal{P}}(\xi), \min(\sigma_{\mathcal{Q}}(\xi), \sigma_{\mathcal{S}}(\xi)))$$
$$= \min((\max\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{Q}}(\xi)), (\max\sigma_{\mathcal{P}}(\xi), \sigma_{\mathcal{Q}}(\xi)))$$
$$= \min(\sigma_{\mathcal{P} \cap \mathcal{Q}}(\xi), \sigma_{\mathcal{P} \cap \mathcal{S}}(\xi))$$

Therefore, $\sigma_{\mathcal{P} \cap (\mathcal{Q} \cup \mathcal{S})}(\xi\zeta) = \sigma_{\mathcal{P} \cap \mathcal{Q}}(\xi), \rho_{\mathcal{P} \cap \mathcal{S}}(\xi)$.

Hence, $\mathcal{P} \cap (\mathcal{Q} \cup \mathcal{S}) = (\mathcal{P} \cap \mathcal{Q})(\mathcal{P} \cap \mathcal{S})$ is also an Pythagorean fuzzy primary ideal of R.

Theorem 3.6 *If $\mathcal{P}$ is an Pythagorean fuzzy primary ideal of a ring R then $\bar{\bar{\mathcal{P}}}$ is also an Pythagorean fuzzy*

primary ideal of R.
Proof: Consider $\rho_{\bar{\bar{\mathcal{P}}}}(\xi\zeta) = \sigma_{\bar{\mathcal{P}}}(\xi\zeta) = \rho_A(\xi\zeta)$.
Therefore, $\rho_{\bar{\bar{\mathcal{P}}}}(\xi\zeta) = \rho_{\mathcal{P}}(\xi)$.
Similarly, $\sigma_{\bar{\bar{\mathcal{P}}}}(\xi\zeta) = \rho_{\bar{\mathcal{P}}}(\xi\zeta) = \sigma_{\mathcal{P}}(\xi\zeta)$.
Therefore, $\sigma_{\bar{\bar{\mathcal{P}}}}(\xi\zeta) = \sigma_{\mathcal{P}}(\xi)$.
Hence, $\bar{\bar{\mathcal{P}}} = \mathcal{P}$ is an Pythagorean fuzzy primary ideal of a ring R.
Theorem 3.7 *Let be f a homomorphism from a ring R onto a ring R'. Let $\mathcal{P}$ and $\mathcal{P}'$ are Pythagorean fuzzy primary ideals of R and R' respectively then the following statements are true.*
(i) $f(\mathcal{P})$ is a fuzzy primary ideal of R if $\mathcal{P}$ is f-invariant.
(ii) $f^{-1}(\mathcal{P}')$ is an Pythagorean fuzzy primary ideal of R.
Proof: (i) Let a', $b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$.
Now if $(f_{\rho_{\mathcal{P}}})('^{b'}) > (f(\rho_{\mathcal{P}}))((a')^n)$ for all $n \in Z^+$, then $(f(\rho_{\mathcal{P}}))(f(^n)) > (f(\rho_{\mathcal{P}}))(f(a^n))$.
Which implies that

$$f^{-1}(f(\rho_{\mathcal{P}}))(ab) = \rho_(ab) > \rho_{\mathcal{P}} > \rho_{\mathcal{P}}(^n).$$

i.e., $\rho_{\mathcal{P}}(ab) \le \rho_{\mathcal{P}}(b^m)$ for some $m \in Z_+$, as $\mathcal{P}$ is an Pythagorean fuzzy primary ideal.
Therefore $(f(\rho_{\mathcal{P}}))(a'b') \le (f(\rho))((b')^m)$.
Similarly, a', $b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$.
Now if $(f_{\sigma_{\mathcal{P}}})('^{b'}) < (f(\sigma_{\mathcal{P}}))((a')^n)$ for all $n \in Z^+$, then $(f(\sigma_{\mathcal{P}}))(f(^n)) < (f(\sigma_{\mathcal{P}}))(f(a^n))$. Which implies that

$$f^{-1}(f(\sigma_{\mathcal{P}}))(ab) = \sigma_{\mathcal{P}}(ab) < \sigma_{\mathcal{P}}(a^n).$$

i.e., $\sigma_{\mathcal{P}}(b^m) \le \sigma_{\mathcal{P}}(ab) < \sigma_{\mathcal{P}}(a^n)$ for some $m \in Z^+$.
That is $\sigma_{\mathcal{P}}(ab) \ge \sigma_{\mathcal{P}}(b^m)$.
Therefore $(f(\sigma_{\mathcal{P}}))(a'b') \ge (f(\sigma_{\mathcal{P}}))((b')^m)$.
Which implies that $f(\mathcal{P})$ is an Pythagorean fuzzy primary ideal of R'.
(ii) Let $a, b \in R$, $f(a) = a'$ and $f(b) = b'$ then $f^{-1}(\mathcal{P}')$ is an Pythagorean fuzzy primary ideal because $(f^{-1}(\rho_{\mathcal{P}'}))(ab) > (f^{-1}(\rho_{\mathcal{P}'}))(a^n)$, for all $n \in Z^+$.
Which implies that

$$\rho_{\mathcal{P}'}(f(ab)) > \rho_{\mathcal{P}'}(f(a^n)).$$

That is $\rho_{\mathcal{P}'}(f(ab)) > \rho_{\mathcal{P}'}(f(a^n))$.
Therefore $\rho_{\mathcal{P}'}(a'b') \le \rho_{\mathcal{P}'}((b')^m)$ for some $m \in Z^+$, since $\mathcal{P}'$ is an Pythagorean fuzzy primary ideal.
Also, $\rho_{\mathcal{P}'}(a'b') \le \rho_{\mathcal{P}'}(ab)$. Which implies that

$$(f^{-1}(\rho'_{\mathcal{P}}))(ab) \le (f^{-1}(\rho_{\mathcal{P}'}))(b^m).$$

Let $\mathcal{P}$, $b \in R$, $f(a) = a'$ and $f(b) = b'$, $(f^{-1}(\sigma_{\mathcal{P}'}))(ab) < (f^{-1}(\sigma_{\mathcal{P}'}))(^n)$, for all $n \in Z$.
Consider $\sigma_{\mathcal{P}'}(f(b^m)) \le \sigma_{\mathcal{P}'}(f(ab))$.
That is $(f^{-1}(\sigma_{\mathcal{P}'})) < \sigma_{\mathcal{P}'}((a')^n)$.
Which implies that

$$\sigma_{\mathcal{P}'}((b')^m) \le \sigma_{\mathcal{P}'}(a'b').$$

Also, $\sigma_{\mathcal{P}'}(f(b^m)) \le \sigma_{\mathcal{P}'}(f(ab))$.
That is $(f^{-1}(\sigma_{\mathcal{P}'}))(b^m) \le (f^{-1}(\sigma_{\mathcal{P}'}))(ab)$.
Therefore,

$$(f^{-1}(\sigma_{\mathcal{P}'}))(ab) \ge f^{-1}(\sigma_{\mathcal{P}'})(b^m).$$

Hence, $f^{-1}(\mathcal{P}')$ is an Pythagorean fuzzy primary ideal.
Theorem 3.8 *Let f be a homomorphism from a ring R onto a ring R'. Let $\mathcal{P}$ and $\mathcal{P}'$ be an Pythagorean fuzzy semiprimary ideal of R and R' respectively then the following statements are true:*
(i) $f(\mathcal{P})$ is an Pythagorean fuzzy semiprimary ideal of R' provided that $\mathcal{P}$ is f-invariant,
(ii) $f^{-1}(\mathcal{P}')$ is an Pythagorean fuzzy semiprimary ideal of R.
Proof: (i) Let $a', b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$.
Now

$$(f(\rho_{\mathcal{P}}))(a'b') > (f(\rho_{\mathcal{P}}))((a')^n)$$

for all $n \in Z_+$ then $(f(\rho_{\mathcal{P}}))(f(ab)) > (f(\rho_{\mathcal{P}}))(f(a^n)) = f^{-1}(f(\rho_{\mathcal{P}}))(a^n)$.
Which implies that

$$f^{-1}(f(\rho_{\mathcal{P}}))(ab) > \rho_{\mathcal{P}}(ab) > \rho_{\mathcal{P}}(a^n)$$

That is $\rho_{\mathcal{P}}(ab) \le \rho_{\mathcal{P}}(b^m)$ for some $m \in Z^+$ as $\mathcal{P}$ is an intuitionistric fuzzy semiprimary ideal.
Therefore, $(f(\rho_{\mathcal{P}}))(a'b') \le (f(\rho_{\mathcal{P}}))((b')^m)$
Similarly, let $a', b' \in R'$ and let $a, b \in R$ be such that $f(a) = a'$ and $f(b) = b'$.
Now if

$$(f(\sigma_{\mathcal{P}}))(a'b') < (f(\sigma_{\mathcal{P}}))((a')^n)$$

for all $n \in Z^{+}$ then $(f(\sigma_{\mathcal{P}}))(f(ab)) < (f(\rho_{\mathcal{P}}))(f(a^n))$.
Which implies that

$$f^{-1}(f(\sigma_{\mathcal{P}}))(ab) = \sigma_{\mathcal{P}}(ab) < \sigma_{\mathcal{P}}(a^n)$$

That is $\sigma_{\mathcal{P}}(b^m) \leq \sigma_{\mathcal{P}}(ab) < \sigma_{\mathcal{P}}(a^n)$ for some $m \in Z^{+}$.
Also, $\sigma_{\mathcal{P}}(ab) \geq \sigma_{\mathcal{P}}(b^m)$.
Therefore,

$$(f(\sigma_{\mathcal{P}}))(a'b') \geq (f(\sigma_{\mathcal{P}}))((b')^m)$$

Which results $f(\mathcal{P})$ is an intuitionistric fuzzy semiprimary ideal of R
(ii) Let $a, b \in R, f(a) = a'$ and $f(b) = b'$, then $f^{-1}(\mathcal{P}')$ is an intuitionistric fuzzy semiprimary ideal because $(f^{-1}(\rho_{\mathcal{P}'}))(ab) > (f^{-1}(\rho_{\mathcal{P}'}))(a^n)$ for all $n \in Z^{+}$.
Which implies that

$$\rho_{\mathcal{P}'}(f(ab)) > \rho_{\mathcal{P}'}(f(a^n))$$

That is $\rho_{\mathcal{P}'}(a'b') > \rho_{\mathcal{P}'}((a')^n)$.
Therefore, $\rho_{\mathcal{P}'}(a'b') \leq \rho_{\mathcal{P}'}((b')^m)$ for some $m \in Z^{+}$, since $\mathcal{P}'$ is an intuitionistric fuzzy semiprimary ideal.
Also, $\rho_{\mathcal{P}'}(f(ab)) \geq \rho_{\mathcal{P}'}((b)^m)$. Therefore, $(f^{-1}(\rho_{\mathcal{P}'}))(ab) \geq (f^{-1}(\rho_{\mathcal{P}'}))(b^m)$
Let $a, b \in R, f(a) = a'$ and $f(b) = b'$, then $(f^{-1}(\sigma_{\mathcal{P}'}))(ab) < (f^{-1}(\sigma_{\mathcal{P}'}))(a^n)$ for all $n \in Z^{+}$.
Consider $\sigma_{\mathcal{P}'}(f(ab)) < \sigma_{\mathcal{P}'}(f(a^n))$.
That is $\sigma_{\mathcal{P}'}(a'b') < \sigma_{\mathcal{P}'}((a')^n)$.
Which implies that, $\sigma_{\mathcal{P}'}((b')^m) \leq \sigma_{\mathcal{P}'}(a'b')$.
Also, $\sigma_{\mathcal{P}'}(f(b)^m) \geq \sigma_{\mathcal{P}'}(f(ab))$.
That is $(f^{-1}(\sigma_{\mathcal{P}'}))(b^m) \geq (f^{-1}(\sigma_{\mathcal{P}'}))(ab)$.
Therefore

$$f^{-1}(\rho_{\mathcal{P}'}))(ab) \geq (f^{-1}(\sigma_{\mathcal{P}'}))(b^m)$$

for some $m \in Z^{+}$.
Hence, $f^{-1}(\mathcal{P}')$ is an intuitionistric fuzzy semiprimary ideal.

IV. Conclusion

To better address cognitive uncertainty, Pythagorean fuzzy sets provide a powerful extension of intuitionistic fuzzy sets. In this paper, we introduce the notions of Pythagorean fuzzy primary and semiprimary ideals. In addition, we advance the study of Pythagorean fuzzy regular ideals by presenting and analyzing several results concerning Pythagorean fuzzy regular and intraregular ideals within the framework of ordered semigroups.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings.

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