IOSR Journal of Computer Engineering (IOSR-JCE)
e-ISSN: 2278-0661,p-ISSN: 2278-8727, Volume 25, Issue 6, Ser. 1 (Nov. – Dec. 2023), PP 01-05
www.iosrjournals.org

A Study On Fuzzy Travelling Salesman Problem Using Fuzzy Number

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Abstract:
A fuzzy number be capable to solve numerous real-life problems like Travelling Salesman, Assignment Problems and so on. Travelling Salesman Problem (TSP) solutions are based on the hypothesis that time of travel between nodes is entirely dependent on distance. But in practice, this is not so as road and traffic conditions help to determine time taken to travel between nodes. We initiate fuzzy based TSP solution where distance and road traffic conditions are fuzzy. In this paper, a new method is proposed for solving Travelling Salesman Problems using Transitive Fuzzy Numbers. We use a new Haar algorithm approach to solve a Fuzzy Travelling Salesman Problem (FTSP). The proposed methods are easy to understand and apply to find optimal solution of fuzzy traveling salesman problems occurring in real life situations.
Keywords: *Fuzzy numbers, Transitive Trapezoidal Fuzzy Numbers, Haar Ranking, Modeling, Optimization.*

Date of Submission: 24-10-2023 Date of Acceptance: 04-11-2023

I. INTRODUCTION

Traveling Salesman Problem (TSP) is a particular type of assignment problem where a salesman starting visit from a given city, visiting all other cities only once and finally comes to the same city where he started. The objective of the problem is to find the shortest route of salesman starting from a given city that why minimize total cost.

There are different approaches for solving traveling salesman problems. Linear programming method, heuristic methods like cutting plan algorithms and branch and bound method, Markov chain, simulated annealing and so on. The Hungarian method is one of the classical methods, which is used to solve different types of TSPs. In fact, real life situation, it may not be possible to get the cost or time as a certain quantity. To overcome this, RE Bellman, LA Zadeh, 1970, [1] introduce fuzzy set concepts to deal with imprecision and vagueness. Since then, significant advantages have been made in developing numerous methodologies and their applications to various decision problems. If the cost or time or distance is not crisp values, then it becomes a fuzzy TSP. Amit Kumar et al.2012, [2] solved The Fuzzy TSP with Coefficient as LR parameters. First, Transform the given fuzzy parameters into Haar tuples and then select required nodes (cities) to satisfy the route conditions by considering element-wise addition. R.R. Yager, 1981[3] introduced "A procedure for ordering fuzzy subsets of the unit interval." Since fuzzy TSP is an NP-hard problem, more numbers of algorithms are still proposed to solve the fuzzy TSP. In this paper, the ordering of fuzzy numbers based on Haar wavelet is used to order the fuzzy numbers. The advantage of using this ranking technique is that it converts a given fuzzy number into average and introduced an algorithm for solving a TSP in crisp environment. The uniqueness of the Haar ranking method [4] is that the fuzzification from the defuzzified value is very easy to obtain through up sampling.

Bellman and Zadeh discussed decision making in fuzzy environment in 1970. Lin and Wen [5] solved assignment problem with fuzzy interval cost by a labeling algorithm. They showed that assignment problem can usually be simplified into either a linear fractional programming problem or a bottleneck assignment problem. Wang [6] used graph theory to solve fuzzy assignment problem. Sakwa et al. [7] presented interactive fuzzy programming approach to solve two levels assignment problem. Mukherjee and Kajla [8] used Yager's ranking method for solving fuzzy assignment problem. They transform fuzzy assignment problem into a crisp assignment problem and solve it by simplex method. Nagarajan and Solairaju [9] solved fuzzy assignment problem using same procedure namely robust ranking method of Yager. Ranking of fuzzy numbers are introduced in different forms in the literature [3], [10], [11].

A. Singhal, P. Pandy. 2016 [12] solved travelling salesman problems by dynamic programming algorithm. A. Rehmat, H. Saeed, M S Cheema 2007[13] used fuzzy multi objective linear programming approach for solving travelling salesman problem. Sepideh Fereidouni. 2011[14] was carried out a study travelling salesman problem by using a fuzzy multi-objective linear programming.

In this paper, we consider more realistic TSP with fuzzy cost. This paper presents solution methodology for traveling salesman problem with fuzzy cost. The fuzzy costs are considered as trapezoidal fuzzy numbers. However, decision-making unit always comes in close contact with environment where fuzziness is common in realistic decision-making situation.

Rest of the paper is organized in the following way: Section 2 describes the preliminaries of fuzzy sets required for the paper. Subsections 2.4- 2.13 present the fuzzy arithmetic. Section 3 represents the formulation of FAP. Section 4 is devoted to present the proposed extended Hungarian method to FAP. In Section 5, two problems on fuzzy assignment problems are solved to demonstrate the efficiency of the proposed method. Finally, Section 6 presents the concluding remarks.

II. PRELIMINARIES

Definition 2.1

Fuzzy set: A fuzzy set $\widetilde{A}$ ~~a~~ in a universe of discourse X is defined by $\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x)\rangle \mid x \in X\}$, where $\mu_{\widetilde{A}}(x) : x \in X \to [0,1]$ is called the membership function of $\widetilde{A}$ and $\mu_{\widetilde{A}}(x)$ is the degree of membership to which $x \in \widetilde{A}$.

Fuzzy sets unlike crisp sets are characterized by degree of membership. For example, in a crisp set of tall people, one can either belong to it or not. But in a fuzzy set of tall people the membership is characterized by values which range from 1 for complete membership to zero for non membership. Hence, giants may belong to this to a degree of one while dwarfs belong to this to a degree of zero.

A notation convention for fuzzy sets when the universe of discourse, X, is discrete and finite, is as follows for a fuzzy set

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \cdots$$
$$= \frac{\mu_A(x_i)}{x_i}$$

Definition 2.2

A Fuzzy number $\widetilde{A}$ with membership function satisfies piece wise continuity, convexity and normality.

Definition 2.3 A fuzzy number $\widetilde{A}$ = (a, b, c, d) with membership function defined as

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1 & , b \le x \le c \\ \dfrac{d-x}{d-c} & , c \le x \le d \\ 0 & , otherwise \end{cases}$$

is known as trapezoidal fuzzy number.

Definition 2.4:

The Fuzzy Operations of fuzzy numbers are defined as

Fuzzy Addition:

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

Fuzzy Subtraction:

$$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

Definition 2.5:

If $a_1 \sim a_4$, then the trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$ is called transitive trapezoidal fuzzy number. It is denoted by $A = (a_1, a_4)$, Where a_1 is core (A), a_4 is left width and right width of c.

The parametric form of a transitive trapezoidal fuzzy number is represented by

$$A = [a_1 - a_2(1-r), a_1 + a_3(1-r), a_1 - a_1(1-r)]$$

Definition 2.6:

Consider a Trapezoidal fuzzy number $\tilde{A}$ = (a ,b, c, d) , the Haar tuple of $\tilde{A}$ can be calculated as $H(\tilde{A}) = (\alpha, \beta, \gamma, \delta)$ where $\alpha = {(a+b+c+d)}/{4}$, $\beta = {[(a+b)-(c+d)}/{4}$, $\gamma = {(a-b)}/{2}$ $\delta = {(c-d)}/{2}$ .Here 'a' is the average coefficient and the remaining coefficients are all called as detailed coefficients of the given trapezoidal fuzzy number.

Element wise Addition:

$$(\alpha_1, \beta_1, \gamma_1, \delta_1) + (\alpha_2, \beta_2, \gamma_2, \delta_2) = (\alpha_1 + \alpha_2, \beta_1 + \beta_2, \gamma_1 + \gamma_2, \delta_1 + \delta_2)$$

Element wise Subtraction:

$$(\alpha_1, \beta_1, \gamma_1, \delta_1) - (\alpha_2, \beta_2, \gamma_2, \delta_2) = (\alpha_1 - \alpha_2, \beta_1 - \beta_2, \gamma_1 - \gamma_2, \delta_1 - \delta_2)$$

III. FUZZY TRAVELLING SALESMAN PROBLEM

Suppose a person has to visit n cities. He starts from a particular city, visits each city once and then returns to the starting point. The fuzzy travelling costs from i th city to jth city is given by c_{ij} .The chosen fuzzy travelling salesman problem may be formulated as

Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} c_{ij}$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2, \cdots n$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2, \cdots n$$

IV. PROPOSED ALGORITHM

In this section we used a new search method for solving fuzzy Traveling Salesman Problem to obtain the crisp total optimal distance.

Step 1

Convert the problem into network when the cost is fuzzy numbers.

Step 2

Transform the given fuzzy parameters into Haar tuples. The total cost becomes a crisp number.

Step 3

Find summation of cost along row wise and column wise. After that, select highest Path Finding Indicator (PFI). If a tie occurs then select those PFI along which has minimum cost. If again tie occurs then select PFI arbitrarily and chose minimum cost cell. Then delete related row and column.

Step 4

Continue step-3 up to getting best couple at the last stage which satisfy rout condition and complete the tour.

Step 5

Check the selected paths (cells) and try to create complete tour. If create any sub-tour then chose comparatively next lowest order (next lowest cost) along the selected PFI and connect the city (node) which has next (second) minimum cost cell. If create several sub-tours break it similar way until get a complete tour.

Step 6

According to the selected cell complete the tour.

Numerical Example

Consider the following fuzzy TSP discussed in [7] is given in table 1

Table 1

CITY	A	B	C	D
A	∞	(4,3,3,4)	(1,4,2,1)	(5,13,1,5)
B	(2,6,3,2)	∞	(6,4,2,6)	(2,3,7,2)

C	(5,13,1,5)	(5,6,7,5)	∞	(1,4,2,1)
D	(6,12,1,6)	(1,4,2,1)	(2,6,3,2)	∞

Convert the given fuzzy numbers into Haar tuples. The TSP with Haar tuples as its elements is given in the table 2

Table 2

CITY	A	B	C	D
A	∞	(3.5,0,.5,-.5)	(2,.5,-1.5,.5)	(6,3,-4,-2)
B	(3.25,.75,-2,.5)	∞	(4.5,.5,1,-2)	(3.5,-1,-.5,2.5)
C	(6,3,-4,-2)	(5.75,-.25,-.5,1)	∞	(2,.5,-1.5,.5)
D	(6.25,2.75,-3,-2.5)	2,.5,-1.5,.5)	(3.25,1.5,-2,.5)	∞

Determine sum along row wise elements and column wise elements respectively.

Table 3

CITY	A	B	C	D	Total
A	∞	(3.5,0,.5,-.5)	(2,.5,-1.5,.5)	(6,3,-4,-2)	(11.5,3.5,-5,-2)
B	(3.25,.75,-2,.5)	∞	(4.5,.5,1,-2)	(3.5,-1,-.5,2.5)	(11.25,.25,-1.5,1)
C	(6,3,-4,-2)	(5.75,-.25,-.5,1)	∞	(2,.5,-1.5,.5)	(13.75,3.25,-6,-.5)
D	(6.25,2.75,-3,-2.5)	(2,.5,-1.5,.5)	(3.25,1.5,-2,.5)	∞	(11.5,4.75,-6.5,-1.5)
Total	(15.5,6.5,-9,-4)	(11.25,.25,-1.5,1)	(9.75,2.5,-2.5,-1)	(12,2.5,-6,1)	

Select 4th row & 2nd column. Now delete 4th row & 2nd column. (D →B)

Table 4

CITY	A	B	C	D	Total
A	∞		(2,.5,-1.5,.5)	(6,3,-4,-2)	(8,3.5,-5.5,-1.5)
B	(3.25,.75,-2,.5)		(4.5,.5,1,-2)	(3.5,-1,-.5,2.5)	(11.25,.25,-1.5,1)
C	(6,3,-4,-2)		∞	(2,.5,-1.5,.5)	(8,3.5,-5.5,-1.5)
D					
Total	(3.25,3.75,-6,1.5)		(6.5,1,-.5,-1.5)	(12,2.5,-6,1)	

Select 2nd row & 1st column. Delete 2nd row & 1st column. (B→A)

Table 5

CITY	A	B	C	D	Total
A	∞		(2,.5,-1.5,.5)	(6,3,-4,-2)	(8,3.5,-5.5,-1.5)
B					
C			∞	(2,.5,-1.5,.5)	(8,3.5,-5.5,-1.5)
D					
Total			(2,.5,-1.5,.5)	(8,3.5,-5.5,-1.5)	

Select 3rd row & 4th column and 1st row & 3rd column. (C→D) & (A→C)
The selected paths are D→B; B→A; C→D; A→C.
According to the selected paths constructed tour: D→B→A→C→D.
The fuzzy optimal cost is (5,18,9,5) [(1,4,2,1) + (2,6,3,2) + (1,4,2,1) + (1,4,2,1)]

V. CONCLUSION

We have proposed a fuzzy alternative technique for the solution of fuzzy travelling salesman problems whose decision parameters are trapezoidal fuzzy numbers. By expressing the decision parameters (fuzzy numbers) in the parametric form and applying a new arithmetical operation of trapezoidal fuzzy number we have obtained fuzzy optimal solution which is sharper than the fuzzy optimal solution obtained.

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