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Regions of reversed and non-reversed flows through unsteady laminar incompressible viscous stagnation point flow under the effect of oscillating motion

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Abstract
The classical Hiemenz solution describes incompressible two-dimensional stagnation point flow at a solid wall. Blyth & Hall consider an unsteady version of this problem, examining particularly the response close to the wall when the solution at infinity is modulated in time by a periodic factor of specified amplitude and frequency. they find that there exists a threshold frequency above which the flow is regular and periodic, Our current study deals with the stagnation point flow in the presence of an oscillating motion at infinity, affected by the change in the oscillation frequency and the parameter of unsteadiness, in addition to an oscillatory function describing the suction and blowing at the wall, which changes by the effect of amplitude and frequency of oscillation. The set of influencing parameters was called the flow parameters set (FPS), as the study found a threshold separating the parameters that make the flow reversible from the parameters that make the flow non-reversible.

Key words. *Stagnation Flow, Unsteadiness, Oscillation, Blowing*

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I. Introduction.

The two-dimensional unsteady motion of incompressible viscous fluid in the vicinity of a forward stagnation point was studied by (Rott, 1956), (Yang,1958), and (Williams, 1968).

Hiemenz 's earlier research tackled a stagnation point flow adjoining the vertical solid wall, while Batchelor's addition was incorporation of oscillatory motion at infinity the resulting solution can be used to describe the local dynamics around a stagnation point on an oscillating body. Equally, the surface may be considered as fixed, with the flow at infinity changing periodically in time.

(Riley and Vasantha, 1989) focused on the phenomenon, where the free-stream in the far field is totally in oscillation state and found the solution numerically.

The inclusion of a mean flow component at far field flow was interested by (Grosch and Salwen, 1982), where the mean component is large compared to the oscillatory component and oscillations frequency is high. As mentioned before, a Stokes-layer occurs in the region closed to the surface of body, which have to match to a layer that has a thickness on the order of the square root of the dimensionless oscillation frequency parameter, multiply by the Stokes layer thickness.

(Merchant and Davis, 1989) concluded the study of (Grosch and Salwen, 1982), developing it to address the problem when the dimensionless frequency parameter value is high enough and the oscillatory component is much larger than the mean component. When the amplitude parameter is at a high critical value, the flow becomes reversal during a specific interval of the time. (Hall and Papageorgiou's, 1999) problem was the non-stationary incompressible stagnation point flow induced in an infinite channel.

(Stuart, 1959) examined the flow consisting a time-independent orthogonal stagnation point flow, a shear flow with constant vorticity and a uniform stream. the flow characterized in terms of a stream function. A similarity solution is created because this stream function in the potential flow does not in matching to the velocity on the wall.

After that, (Drazin and Riley 2006) generalized the lasted work, to contain a free parameter at free-stream, which relates to the strength of the uniform stream in the potential flow. (Stuart, 2012) considered the viscous flow in the zone adjacent to a stagnation point when the external flow has uniform vorticity.

Upon increasing the free parameter, the shear velocity profile creates a region of reversed flow in the zone adjacent to the wall. They summarized their analysis with a discussion of the gradient of the resembled stream-line closed to the vertical surface, with the same results to that of (Dorrepaal, 1986). Particularly, it was shown that the ratio of the resembling stream-line rate of change near the surfacel, to that of the rate of change in the potential flow field is not a function of the flow vorticity.

(Blyth & Hall, 2003) concerned with the behavior in the vicinity of the stagnation point, where the body surface may be considered to be locally flat. In this context they allow for fluctuations of arbitrary amplitude and frequency. The problem concluded that above the critical relative amplitude equations blow-up at a finite time singularity.

In current study, generalization of Blyth & Hall problem had done, in addition, assume viscous flow adjacent to stagnation point. we assume a specific unsteady function at the far-field flow and an unsteady blowing/suction at the surface. It was interesting in analysis finding that the solutions below a critical value of oscillation parameter break down at a finite time. According to our assumptions, for each determined value of the unsteadiness parameter, the critical value of oscillation parameter approximately constant with changing of the blowing parameter. So we could consider a border line separates to regions of flow parameters, the region above the line refers to the flow parameters leading to a non-reversed flow motion, while the region below the line refers to the flow parameters leading to reversed flow motion where the flow break down at a finite time.

II. Methodology

The system of equations describing unsteady stagnation point flow problem is Navier Stoke's equations which are PDE, similarity solution is appropriate to solve this system. So, the following steps were achieved:

1- Similarity transformation of Navier Stoke's equations.
2- Assuming problem assumptions.
3- Numerical solution, A fully implicit finite difference based PDE solver is used (Implicit Euler).

Similarity transformations:

The problem under consideration is that of two-dimensional version of flow approaching a vertical flat plate. Referring to a set of Cartesian axes (x, z), the flat plate occupies $-\infty < x < \infty,\ z = 0$.

The velocity components are expressed as $u(x,z,t),\ w(x,z,t)$ in the x, z directions, respectively, governing equations which describe the fluid motion in this case are the two dimensional unsteady Navier-Stokes equations.

A stagnation point flow develops and the streamline is perpendicular to the surface of the rigid body. The flow in the vicinity of this stagnation point is characterized by Navier-Stokes equations. By introducing coordinate variable transformation, the number of independent variables is reduced by one or more.

The governing equations can be simplified to the non-linear ordinary differential equations and are analytic solvable

Navier-Stokes Equations

The full Navier-Stokes equations are difficult or impossible to obtain an exact solution in almost every real situation because of the analytic difficulties associated with the

nonlinearity due to convective acceleration. The existence of exact solutions are fundamental not only in their own right as solutions of particular flows, but also are agreeable in accuracy checks for numerical solutions.

The Navier-Stokes equations are a system of non-linear, coupled partial differential equations (PDEs) which are derived from the principles of mass and momentum conservation.

The equation of mass conservation, or continuity equation, can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

The equations of momentum conservation for a fluid are obtained from the application of the force-momentum principle, and can be written:

x- momentum :

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}P_x + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right] \qquad (2)$$

z- momentum :

$$w_t\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right] \qquad (3)$$

with the parameters, kinematic viscosity μ , pressure P and density ρ.

The boundary conditions are taken as:

$$u = 0, \quad w = w_0(t) \qquad at \qquad z = 0$$
$$u \to U_{free}(t) \qquad at \qquad z \to \infty \qquad (4)$$

Where $u, \& w$ the velocity components of the flow through the boundary layer are, $w_0(t)$ is the velocity of blowing\suction through the wall, $U_{free}(t)$ is unsteady potential velocity component.

near the surface, because of the no slip condition not being satisfied, a similarity solution is employed. We defined dimensionless similarity variables as ξ, η since:

$$\xi = Gx \qquad (5)$$

$$\eta = \sqrt{\frac{G}{\mu}} * z \qquad (6)$$

Where, G is a constant related with the body geometry, where $G > 0$.

The velocity components $u \,\&\, w$ of the boundary-layer flow are assumed to have solutions of the following form:

$$u = \xi * f_\eta(\eta, t) \qquad (7)$$

Then, the z-direction velocity component w of the potential flow is immediately determined from the continuity equation (3.1), by substituting the foregoing velocity components u and w.

$$w = -\sqrt{\mu G}\, f(\eta, t) \qquad (8)$$

Consider the unsteady periodic motion of an incompressible viscous fluid in the vicinity of the stagnation point at $x = z = 0$ on a blunt body. The potential flow approaches the body in the negative z-direction, impinges on the surface normally at the stagnation point flows away radially in all directions along the surface, and is assumed to have unsteady velocity components:

$$U_{free} = \xi r(t) \qquad (9)$$

Where $a(t)$ is an arbitrary time-dependent function, as a case study it was chosen $r(t)$ as:

$$a(t) = \frac{1}{1 + \frac{\alpha}{\omega}\sin \omega t}, \qquad (10)$$

Where, a is a constant related to free stream acceleration and Ω is the potential flow frequency.

When $a = 0$, the problem reduces to the steady case, that means $U_{free} \to Gx$.

Corresponding to $a \,\&\, G$, we can define the unsteadiness parameter S , $S = \frac{a}{G}$

The equations of motion [1 to 3] for the two-dimensional unsteady flow of incompressible viscous fluid in the vicinity of a forward stagnation point are reduced to two partial differential equations for a potential flow field chosen to vary periodically as a function of time. Finally, using similarity transformation, we get the final similarity equation describes the stream-wise flow in stagnation point flow boundary layer:

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{\partial f}{\partial \eta} * \frac{\partial f}{\partial \eta} - f * \frac{\partial^2 f}{\partial \eta^2} = \left(\frac{1}{G}\right)\left(\frac{\partial^2 f}{\partial \eta \partial t} + r_t(t)\right) + [r(t)]^2 \qquad (11)$$

Initial and boundary conditions:

Boundary conditions for the differential equation are expressed as follows:

$$\frac{\partial f(0,\eta)}{\partial \eta} = \frac{u_0(t)}{Gx}, \qquad f(0,t) = -\sqrt{\frac{1}{G\mu}}\, w_0(t),$$
$$\eta \to \infty, \qquad \frac{\partial f}{\partial \eta} \to r(t) \qquad (12)$$

To satisfy the interspersed wall boundary conditions and to match to the outer potential solution.

A mathematical model was created to generalize the flow boundary conditions, and studying the effect of flow parameters change on the characteristics of boundary layer.

The boundary conditions in eq.(12) in a generalized form and our assumptions were applied to achieve a special case which under the effect of flow parameters' changing.
For that, the boundary conditions are reformed according to our case study as:
$w_0(t)$ was chosen as:

$$w_0(t) = \lambda \cos \Omega t \qquad (13)$$

Where λ , is the amplitude of the oscillating flow due to blowing/suction at the wall, and Ω is the oscillation frequency.
By substituting by eq.(12) in eq.(13), we get:
$f(0,t) = -\sqrt{\frac{1}{G\mu}}\lambda \cos \Omega t$, hence, G, μ, λ are constants, we can introduce, $B = -\sqrt{\frac{1}{G\mu}}\lambda$, and named it as "blowing parameter
In this research the classification of sets of flow parameters based on the reversibility.
The flow velocity is inversely proportional dependent of pressure therefore, if the pressure gradient has a positive sign along flow direction that leads to (reversed flow), and if it has a negative sign, that leads to (non-reversed flow)
The pressure gradient sign is dependent of the value of unsteadiness parameter. Table (1), summarize this relation:

Table (1): regions of reversed & non-reversed flows (ODE solution)

	$\frac{\partial P}{\partial x}$	$\frac{\partial u}{\partial x}$	Flow type
$S<1$	-	+	Non-revered
$S>1$	+	-	Reversed

PDE solution enabled us to assume any unsteady function either at the surface or at the potential flow, in our study as mentioned, our supposed unsteady function as:

$$U_{free} = \frac{Gx}{1 + \frac{a}{\Omega}\sin \Omega t}$$

This supposed oscillation function was applied, the behavior of flow could be observed under the effect of change of $B \& \Omega$, at $S < 1$, and found that, for any arbitrary values of B, Ω , the flow at $\eta = 0$ is always non-reversed all the time, which is the same flow behavior in (ODE solution) case (table 1). But, there is an interesting behavior observed in our results due to our new assumed unsteady function, it was the existing of a non-reversed fluctuations during the interval $(0 \le t \le 10)$ flow even the unsteadiness parameter exceeds a unity.
If (S $>$ 1) at a t higher values of oscillating frequency Ω , It could be found at each ($S \& B$) parameter values, a corresponding value of Ω which we distinguished as Ω^*, where Ω^* is the obtained highest value of oscillation frequency at the border of irreversibility.
If $\Omega < \Omega^*$, it was observed the possibility of existence of reversed flow at definite time. Here was the point of excellence that, our assumptions have created. So we could modify table (2) to take the following form:

Table (2): regions of reversed & non-reversed flows (due to our assumptions)

	Frequency Ω	$\frac{\partial P}{\partial x}$	$\frac{\partial u}{\partial x}$	Flow type
$S<1$	any	-	+	Non-revered
$S>1$	$\Omega > \Omega^*$	-	+	Non-revered
	$\Omega < \Omega^*$	+	-	Revered

As an important benefit of our work was finding a general map shows two sets of flow parameters which defined in study as (FPS). So, it was possible to find a borderline formed by obtained values of Ω^*.
Values which were found under different flow conditions (FPS). The border line separates two regions, where the area above the borderline represents to parameters' values related with flow irreversibility parameters, and the area under represents to the variables values related with flow reversibility parameters. B-values were selected to vary from 5 to 700, and the selected S values were [$S=2,\ S=4,\ S=6$]

The indicator representing the $u-component$ velocity direction at $\eta=0$ is $\frac{\partial^2 f}{\partial \eta^2}(0)$. Where if:

$$\frac{\partial^2 f}{\partial \eta^2}(0)>0 \quad non-reversed\ \ flow\ (\ at\ \ t: o\le t\le 10)$$

The flow under the effect of flow parameters set was observed accuretely, the following values have been got and inserted in the following table.

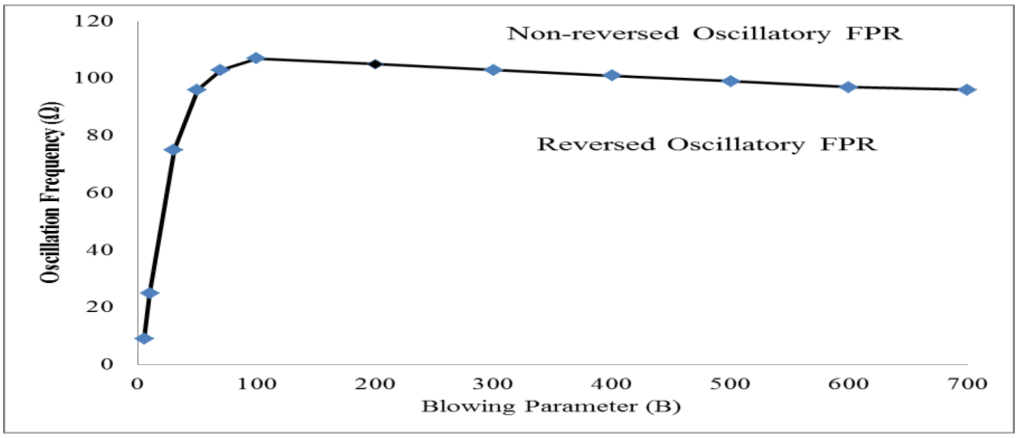
Table (3): values of Ω^* at which the point separates reversed and non-reversed regions of parameters at the wall (reversed flow at the wall exists at definite time)

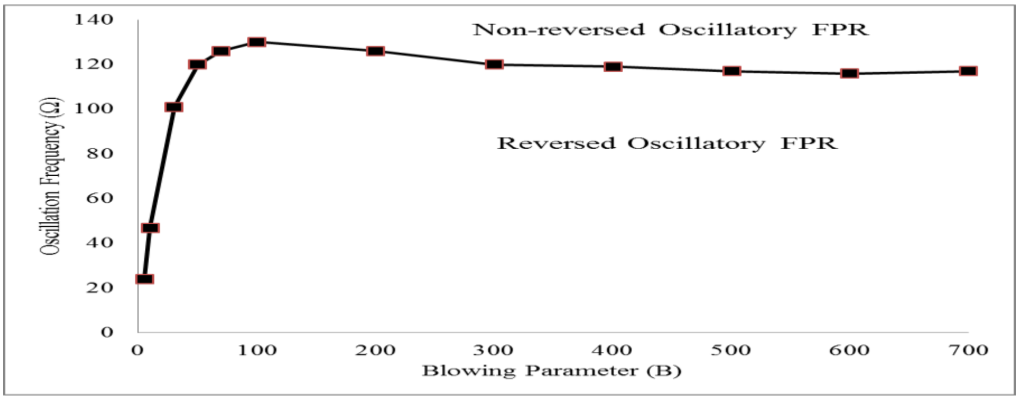
B		5	10	30	50	70	100	200	300	400
Ω	S=2	9	25	75	96	103	107	105	103	101
	S=4	24	47	101	120	126	130	126	120	119
	S=6	38	65	117	131	137	140	139	131	127

According to this table, the following observations can be commented:

- At values of $B<5$, $\frac{\partial^2 f}{\partial \eta^2}\ (\eta=0)\to\infty$
- Above $B\approx 500$, Ω^* has small variations and can be considered as constantly changes with B.
- Reversibility of flow may be exist at any time, so Ω^* values were searched corresponding $\frac{\partial^2 f}{\partial \eta^2}\ (\eta=0)$, where $\frac{\partial^2 f}{\partial \eta^2}(0)^*$ is the value of $\frac{\partial^2 f}{\partial \eta^2}(0)$ when inflected from negative to positive sign.

Finally, from the previous discussion results can be showed in figure (1)





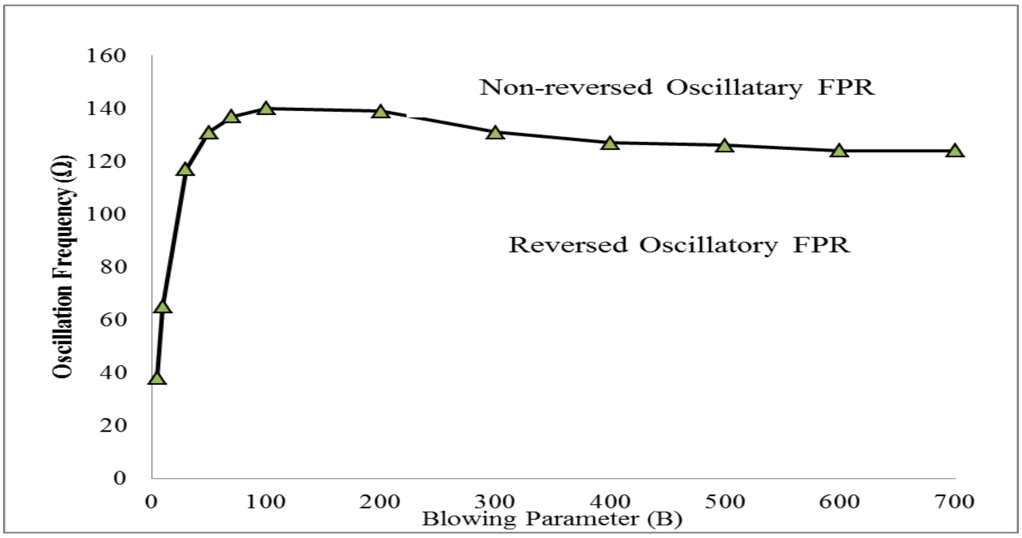


Figure (1): Reversed and non-reversed FPS at the wall (reversed flow at the wall exists at definite time)

III. Conclusion

The study assumed a specific periodic motion at the free stream flow and an periodic motion at the wall. It was interesting in analysis finding that the solutions below a critical value of oscillation parameter break down at a finite time. According to our assumptions, for each different (FPS) of the unsteadiness parameter, the critical value of oscillation parameter approximately constant with changing of the blowing parameter. So we could consider a threshold separates to areas of flow parameters, the region above refers to the flow parameters leading to a non-reversed flow, while the region below the line refers to the flow parameters leading to reversed flow where the flow break down at a finite time.

It was observed that the flow at a specific value of the unsteadiness parameter where:

at $S > 1$ can be non-reversed at the surface all the time if $\Omega > \Omega^*$.

at $S > 1$ & $\Omega < \Omega^*$ the flow becomes reversed at definite period of time.

at $S \leq 1$ the flow at the wall is non-reversed all the time regardless of the value of Ω.

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