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Global Asymptotic Stability of Delayed Bidirectional Associative Memory Neural Networks with Fuzzy Logic

Dianbo Ren
(Associate Professor, School of Automotive Engineering, Harbin Institute of Technology at Weihai, China)
Corresponding Author: Dianbo Ren

Abstract: *In this paper, without assuming the boundedness, monotonicity and differentiability of the activation functions, we present new conditions ensuring existence, uniqueness, and global asymptotical stability of the equilibrium point of bidirectional associative memory neural networks with fuzzy logic and time delays. The results are applicable to both symmetric and nonsymmetric interconnection matrices, and all continuous non-monotonic neuron activation functions. Since the criterion is independent of the delays and simplifies the calculation, it is easy to test the conditions of the criterion in practice. An example is given to demonstrate the feasibility of the criterion.*

Keywords - *bidirectional associative memory, neural networks, global asymptotic stability, fuzzy logic, time delays*

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I. Introduction

Bidirectional associative memory (BAM) neural networks known as an extension of the unidirectional autoassociator of Hopfield [1] was first introduced by Kosto[2]. It is composed of neurons arranged in two layers. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnection among neurons in the same layer. Through iterations of forward and backward propagation information flows between the two layers, which performs a two-way associative search for stored bipolar vector pairs and generalize the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched heteroassociative circuits. Due to the BAM neural networks has been used in many fields such as pattern recognition, image processing, and automatic control. Therefore, the BAM neural networks have attracted great attention of many researchers .One can refer to the articles [3-18] for detailed discussion on these aspects.

When a neural network is employed as an associative memory, the existence of many equilibrium points is a necessary feature. However, in applications to parallel computation and signal processing involving solution optimization problems, it is required that there be a well-defined computable solution for all possible initial states. From a mathematical viewpoint, this means that the network should have a unique equilibrium point that is globally asymptotically stable. In hardware implementation, time delays occur due to finite switching speeds of the amplifiers, and the existence of time delays frequently causes oscillation or instability in neural networks, Thus, the study of globally asymptotical stability of BAM neural networks with time delays is practically required. In [12-18], some sufficient conditions have been obtained for globally asymptotic stability of delayed bidirectional associative memory networks.

In this paper, we investigate a kind of delayed BAM neural networks with fuzzy logic which integrates fuzzy logic into the structure of traditional BAM neural networks with time delays. Unlike traditional BAM neural networks structures, the kind of BAM neural networks has fuzzy logic between its template and input besides the operation of sum of product. Studies have been revealed that neural networks with fuzzy logic has inherent connections to mathematical morphology ,which is a cornerstone in image processing and pattern recognition[19]. Some results on stability have been derived for fuzzy neural networks, for example [20-22], but few studies have considered the stability for the BAM neural networks with fuzzy logic. Our objective is to study the existence of unique equilibrium point and its global asymptotic stability for delayed BAM neural networks with fuzzy logic. Without assuming the boundedness, monotonicity and differentiability of activation functions, by using M-matrix theory, Liapunov functions, we present new conditions ensuring existence, uniqueness, and global asymptotical stability of the equilibrium point for the class of BAM networks with fuzzy logic and time delays.

II. Notation and preliminaries

For convenience, we introduce some notations. $x=(x_1,\cdots,x_n)^{\mathrm{T}}\in R^n$ denotes a column vector. $|x|$ denotes the absolute-value vector given by $|x|=(|x_1|,\cdots,|x_n|)^{\mathrm{T}}$, $\|x\|$ denotes a vector norm defined by $\|x\|=(x_1^2+\cdots+x_n^2)^{1/2}$. For matrix $A=(a_{ij})_{n\times n}$, A^{T} denotes the transpose of A, A^{-1} denotes the inverse of A, $[A]^s$ is defined as $[A]^s=(A^{\mathrm{T}}+A)/2$, and $|A|$ denotes absolute-value matrix given by $|A|=(|a_{ij}|)_{n\times n}$, $\|A\|$ denotes a matrix norm defined by $\|A\|=(\max\{\lambda:\lambda$ is an eigenvalue of $A^{\mathrm{T}}A\})^{1/2}$. $\wedge$ and $\vee$ denote the fuzzy AND and fuzzy OR operation, respectively.

The dynamical behavior of bidirectional associative memory neural networks with fuzzy logic and time delays can be described by the following nonlinear differential equations:

$$\dot{u}_i(t)=-\alpha_i u_i(t)+\sum_{j=1}^{m}a_{ij}f_j(v_j(t-\tau_{ij}))+\bigwedge_{j=1}^{m}b_{ij}f_j(v_j(t-\tau_{ij}))+\bigvee_{j=1}^{m}c_{ij}f_j(v_j(t-\tau_{ij}))+I_i,\ i=1,2,\cdots,n, \tag{1a}$$

$$\dot{v}_j(t)=-\beta_j v_j(t)+\sum_{i=1}^{n}d_{ji}g_i(u_i(t-\sigma_{ji}))+\bigwedge_{i=1}^{n}k_{ji}g_i(u_i(t-\sigma_{ji}))+\bigvee_{i=1}^{n}l_{ji}g_i(u_i(t-\sigma_{ji}))+J_j,\ j=1,2,\cdots,m, \tag{1b}$$

where $\alpha_i>0$, $\beta_j>0$ for $i=1,2,\cdots,n$, $j=1,2,\cdots,m$ denote the passive decay rates; a_{ij}, d_{ji} for $i=1,2,\cdots,n$, $j=1,2,\cdots,m$ are the synaptic connection strengths; b_{ij}, k_{ji} and c_{ij}, l_{ji} for $i=1,2,\cdots,n$, $j=1,2,\cdots,m$ are elements of fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively; f_j for $j=1,2,\cdots,m$ and g_i for $i=1,2,\cdots,n$ denote the propagational signal functions ; I_i for $i=1,2,\cdots,n$ and J_j for $j=1,2,...,m$ are the exogenous inputs. The initial conditions associated with (1) are of the form

$$u_i(s)=\phi_i(s),\ s\in[-\sigma,0],\ \sigma=\max_{\substack{1\le i\le n\\1\le j\le m}}\sigma_{ji},\ i=1,2,\cdots,n,$$

$$v_j(s)=\psi_j(s),\ s\in[-\tau,0],\ \tau=\max_{\substack{1\le i\le n\\1\le j\le m}}\tau_{ij},\ j=1,2,\cdots,m,$$

where it is usually assumed that $\phi_i\in C([-\sigma,0],R)$ for $i=1,2,\cdots,n$ and $\psi_j\in C([-\tau,0],R)$ for $j=1,2,\cdots,m$.

Assumption (A) For each $i\in[1,n]$, $j\in[1,m]$, $f_j:R\to R$ and $g_i:R\to R$ are globally Lipschitz with Lipschitz constant $F_j>0$, $G_j>0$, i.e., $|f_j(y_j)-f_j(v_j)|\le F_j|y_j-v_j|$ for all y_j, v_j; $|g_i(x_i)-g_i(u_i)|\le G_i|x_i-u_i|$ for all x_i,u_i.

In the following, we let

$$u=(u_1,...,u_n)^{\mathrm{T}},\ v=(v_1,\cdots,v_m)^{\mathrm{T}},\ \alpha=\mathrm{diag}(\alpha_1,\cdots,\alpha_n),\ \beta=\mathrm{diag}(\beta_1,\cdots,\beta_m),\ A=(a_{ij})_{n\times m},$$

$$B=(b_{ij})_{n\times m},\ C=(c_{ij})_{n\times m},\ D=(d_{ji})_{m\times n},\ K=(k_{ji})_{m\times n},\ L=(l_{ji})_{m\times n},\ f(v)=(f_1(v_1),\cdots,f_m(v_m))^{\mathrm{T}},$$

$$g(u)=(g_1(u_1),\cdots,g_n(u_n))^{\mathrm{T}},\ F=\mathrm{diag}(F_1,F_2,\cdots,F_n),\ G=\mathrm{diag}(G_1,G_2,\cdots,G_m).$$

Lemma 1[23]**.** Let Ω be a $(n+m)\times(n+m)$ matrix with non-positive off-diagonal elements. Then the following statements are equivalent:

(i) Ω is an M-matrix,

(ii) The real parts of all eigenvalues of Ω are positive,

(iii) There exists a vector $\xi>0$, such that $\xi^{\mathrm{T}}\Omega>0$,

(iv) Ω is nonsingular and all elements of Ω^{-1} are nonnegative,

(v) There exists a positive definite $(n+m)\times(n+m)$ diagonal matrix Q such that matrix $\Omega Q+Q\Omega^{\mathrm{T}}$ is positive definite.

Lemma 2[19]**.** Suppose $(x,y)\in R^{n\times m}$ and $(u,v)\in R^{n\times m}$ are two states of system (1), then

$$|\bigwedge_{i=1}^{n} k_{ji}g_i(x_i)-\bigwedge_{i=1}^{n} k_{ji}g_i(u_i)|\le\sum_{i=1}^{n}|k_{ji}||g_i(x_i)-g_i(u_i)|,$$

$$|\bigvee_{i=1}^{n} l_{ji}g_i(x_i)-\bigvee_{i=1}^{n} l_{ji}g_i(u_i)|\le\sum_{i=1}^{n}|l_{ji}||g_i(x_i)-g_i(u_i)|,\ i=1,2,\cdots,n;$$

$$|\bigwedge_{j=1}^{m} b_{ij}f_j(y_j)-\bigwedge_{j=1}^{m} b_{ij}f_j(v_j)|\le\sum_{j=1}^{m}|b_{ij}||f_j(y_j)-f_j(v_j)|,$$

$$|\bigvee_{j=1}^{m} c_{ij}f_j(y_j)-\bigvee_{j=1}^{m} c_{ij}f_j(v_j)|\le\sum_{j=1}^{m}|c_{ij}||f_j(x_j)-f_j(v_j)|,\ j=1,2,\cdots,m.$$

Lemma 3[18]**.** If $H(x)\in C^0$ satisfies the following conditions, then $H(x)$ is a homeomorphism of R^{n+m}.

(1) $H(x)$ is injective on R^{n+m},

(2) $\lim_{\|x\|\to\infty}\|H(x)\|\to\infty$.

III. Existence and uniqueness of the equilibrium

In the section, we study the existence and uniqueness of the equilibrium point of (1). We firstly study the nonlinear map associated with (1) as follows:

$$\begin{cases} H_i(u_i)=-\alpha_i u_i+\sum_{j=1}^{m}a_{ij}f_j(v_j)+\bigwedge_{i=1}^{m}b_{ij}f_j(v_j)+\bigvee_{i=1}^{m}c_{ij}f_j(v_j)+I_i,\ i=1,2,\cdots,n,\\ H_{n+j}(v_j)=-\beta_j v_j+\sum_{i=1}^{n}d_{ji}g_i(u_i)+\bigwedge_{i=1}^{n}k_{ji}g_i(u_i)+\bigvee_{i=1}^{n}l_{ji}g_i(u_i)+J_j,\ j=1,2,\cdots,m.\end{cases} \tag{2}$$

Let $H(u,v)=(H_1(u_1),H_2(u_2),\cdots,H_n(u_n),H_{n+1}(v_1),H_{n+2}(v_2),\cdots,H_{n+m}(v_m))^{\mathrm{T}}$. It is known that if there exists a point (u^*,v^*) such that $H(u^*,v^*)=0$, then, the point (u^*,v^*) is the equilibrium in (1). So in order to investigated the existence and uniqueness of the equilibrium in (1), we firstly investigate the existence and uniqueness of the solution for nonlinear equation (2). If map $H(u,v)$ is a homeomorphism on $R^{n\times m}$, then there exists a unique point (u^*,v^*) such that $H(u^*,v^*)=0$, i.e., systems (1) have a unique equilibrium (u^*,v^*). Based on the Lemma 3, we get the conditions of the existence and uniqueness of the equilibrium for system (1) as follows.

Theorem 1 Suppose f, g satisfy Assumption (A), and Ω is an M-matrix, then, for every pair of input (I,J), systems (1) have a unique equilibrium (u^*,v^*). Ω is defined as

$$\Omega=\begin{bmatrix}\alpha & -[|A|+|B|+|C|]F\\ -[|D|+|K|+|L|]G & \beta\end{bmatrix}.$$

Proof. In order to prove that systems (1) have a unique equilibrium point (u^*,v^*), it is only need to prove that $H(u,v)$ is a homeomorphism on $R^{n\times m}$. In the following, we shall prove that map $H(u,v)$ is a homeomorphism through two steps.

In the first step**,** we prove that $H(u,v)$ is an injective on $R^{n\times m}$. Suppose, for purposes of contradiction, that there exist $(x,y)\in R^{n\times m}$, $(u,v)\in R^{n\times m}$ with $(x,y)\ne(u,v)$ such that $H(x,y)=H(u,v)$. From (2), by Assumption (A) and Lemma 2, we get

$$\begin{bmatrix}|H_i(x_i)-H_i(u_i)|\\ |H_{n+j}(y_j)-H_{n+j}(v_j)|\end{bmatrix}$$

$$=\begin{bmatrix} |-\alpha_i(x_i-u_i)+\sum_{j=1}^{m}a_{ij}[f_j(y_j)-f_j(v_j)]+\bigwedge_{j=1}^{m}b_{ij}f_j(y_j)-\bigwedge_{j=1}^{m}b_{ij}f_j(v_j) \\ +\bigvee_{j=1}^{m}c_{ij}f_j(y_j)-\bigvee_{j=1}^{m}c_{ij}f_j(v_j)| \\ |-\beta_j(y_j-v_j)+\sum_{i=1}^{n}d_{ji}[g_i(x_i)-g_i(u_i)]+\bigwedge_{i=1}^{n}k_{ji}g_i(x_i)-\bigwedge_{i=1}^{n}k_{ji}g_i(u_i) \\ +\bigvee_{i=1}^{n}l_{ji}g_i(x_i)-\bigvee_{i=1}^{n}l_{ji}g_i(u_i)| \end{bmatrix}$$

$$\geq\begin{bmatrix} \alpha_i|x_i-u_i|-\sum_{j=1}^{m}|a_{ij}||f_j(y_j)-f_j(v_j)|-|\bigwedge_{j=1}^{m}b_{ij}f_j(y_j)-\bigwedge_{j=1}^{m}b_{ij}f_j(v_j)| \\ -|\bigvee_{j=1}^{m}c_{ij}f_j(y_j)-\bigvee_{j=1}^{m}f_j(v_j)| \\ \beta_j|y_j-v_j|-\sum_{i=1}^{n}|d_{ji}||g_i(x_i)-g_i(u_i)|-|\bigwedge_{i=1}^{n}k_{ji}g_i(x_i)-\bigwedge_{i=1}^{n}k_{ji}g_i(u_i)| \\ -|\bigvee_{i=1}^{n}l_{ji}g_i(x_i)-\bigvee_{i=1}^{n}l_{ji}g_i(u_i)| \end{bmatrix}$$

$$\geq\begin{bmatrix} \alpha_i|x_i-u_i|-\sum_{j=1}^{m}|a_{ij}||f_j(y_j)-f_j(v_j)|-\sum_{j=1}^{m}|b_{ij}||f_j(y_j)-f_j(v_j)|-\sum_{j=1}^{m}|c_{ij}||f_j(y_j)-f_j(v_j)| \\ \beta_j|y_j-v_j|-\sum_{i=1}^{n}|d_{ji}||g_i(x_i)-g_i(u_i)|-\sum_{i=1}^{n}|k_{ji}||g_i(x_i)-g_i(u_i)|-\sum_{i=1}^{n}|l_{ji}||g_i(x_i)-g_i(u_i)| \end{bmatrix}$$

$$\geq\begin{bmatrix} \alpha_i|x_i-u_i|-\sum_{j=1}^{m}|a_{ij}|F_j|y_j-v_j|-\sum_{j=1}^{m}|b_{ij}|F_j|y_j-v_j|-\sum_{j=1}^{m}|c_{ij}|F_j|y_j-v_j| \\ \beta_j|y_j-v_j|-\sum_{i=1}^{n}|d_{ji}|G_i|x_i-u_i|-\sum_{i=1}^{n}|k_{ji}|G_i|x_i-u_i|-\sum_{i=1}^{n}|l_{ji}|G_i|x_i-u_i| \end{bmatrix}$$

$$=\begin{bmatrix} \alpha_i & -\sum_{j=1}^{m}(|a_{ij}|+|b_{ij}|+|c_{ij}|)F_j \\ -\sum_{i=1}^{n}(|d_{ji}|+|k_{ji}|+|l_{ji}|)G_i & \beta_j \end{bmatrix}\begin{bmatrix}|x_i-u_i| \\ |y_j-v_j|\end{bmatrix}, \tag{3}$$

$$i=1,2,\cdots,n,\ j=1,2,\cdots,m.$$

From (3), we get

$$|H(x,y)-H(u,v)|\geq\begin{bmatrix}\alpha & -(|A|+|B|+|C|)F \\ -(|D|+|K|+|L|)G & \beta\end{bmatrix}\begin{bmatrix}|x-u| \\ |y-v|\end{bmatrix}=\Omega\begin{bmatrix}|x-u| \\ |y-v|\end{bmatrix}.$$

By the supposition that $H(x,y)=H(u,v)$, we obtain $\Omega\begin{bmatrix}|x-u| \\ |y-v|\end{bmatrix}\leq 0$. Since Ω is an M-matrix, from Lemma 1, we know that all elements of Ω^{-1} are non-negative. Hence, $|x-u|=0$, $|y-v|=0$ i.e., $x=u$, $y=v$, which is a contradiction. So map $H(u,v)$ is injective.

In the second step, we prove that $\lim_{\|(u,v)\|\to\infty}\|H(u,v)\|\to\infty$. If $f(v)$ and $g(u)$ are bounded, it is easy to verify that when $\|(u,v)\|\to+\infty$, $\|H(u,v)\|\to+\infty$. In the following, we will discuss the case that $f(v)$ or $g(u)$ is unbounded. Let $\overline{H}(u,v)=H(u,v)-H(0,0)$ .To prove that $H(u,v)$
is a homeomorphism, it suffices to show that $\overline{H}(u,v)$ is a homeomorphism. Because of Ω is an M-matrix, from Lemma 1, there exists a positive define diagonal matrix $Q=\text{diag}(q_1,q_{2,},\cdots,q_{n+m})$, such that

$$[Q\Omega]^s \le -\varepsilon E_{n+m} < 0 \tag{4}$$

for sufficiently small ε >0. E_{n+m} is the identity matrix. By Assumption (A), Lemma 2 and (4), we get

$$
\begin{aligned}
&[u^{\mathrm{T}}, v^{\mathrm{T}}]Q\overline{H}(u,v)\\
&=\sum_{i=1}^{n} u_i q_i \{-\alpha_i u_i + \sum_{j=1}^{m} a_{ij}[f_j(v_j)-f_j(0)] + \bigwedge_{j=1}^{m} b_{ij} f_j(v_j) - \bigwedge_{j=1}^{m} b_{ij} f_j(0) + \bigvee_{j=1}^{m} c_{ij} f_j(v_j) - \bigvee_{j=1}^{m} c_{ij} f_j(0)\}\\
&+\sum_{j=1}^{m} v_j q_{n+j}\{-\beta_j v_j + \sum_{i=1}^{n} d_{ji}[g_i(u_i)-g_i(0)]\\
&+\bigwedge_{i=1}^{n} k_{ji} g_i(u_i) - \bigwedge_{i=1}^{n} k_{ji} g_i(0) + \bigvee_{i=1}^{n} l_{ji} g_i(u_i) - \bigvee_{i=1}^{n} l_{ji} g_i(0)\}\\
&\le \sum_{i=1}^{n} q_i\{-\alpha_i u_i^2 + |u_i|[\sum_{j=1}^{m} |a_{ij}||f_j(v_j)-f_j(0)| + |\bigwedge_{j=1}^{m} b_{ij} f_j(v_j) - \bigwedge_{j=1}^{m} b_{ij} f_j(0)|\\
&+|\bigvee_{j=1}^{m} c_{ij} f_j(v_j) - \bigvee_{j=1}^{m} c_{ij} f_j(0)|]\} + \sum_{j=1}^{m} q_{n+j}\{-\beta_j v_j^2 + |v_j|[\sum_{i=1}^{n} |d_{ji}||g_i(u_i)-g_i(0)|\\
&+|\bigwedge_{i=1}^{n} k_{ji} g_i(u_i) - \bigwedge_{i=1}^{n} k_{ji} g_i(0)| + |\bigvee_{i=1}^{n} l_{ji} g_i(u_i) - \bigvee_{i=1}^{n} l_{ji} g_i(0)|]\}\\
&\le \sum_{i=1}^{n} q_i\{-\alpha_i u_i^2 + |u_i|[\sum_{j=1}^{m} |a_{ij}||f_j(v_j)-f_j(0)| + \sum_{j=1}^{m} |b_{ij}||f_j(v_j)-f_j(0)|\\
&+\sum_{j=1}^{m} |c_{ij}||f_j(v_j)-f_j(0)]\} + \sum_{j=1}^{m} q_{n+j}\{-\beta_j v_j^2 + |v_j|[\sum_{i=1}^{n} |d_{ji}||g_i(u_i)-g_i(0)|\\
&+\sum_{i=1}^{n} |k_{ji}||g_i(u_i)-g_i(0)| + \sum_{i=1}^{n} |l_{ji}||g_i(u_i)-g_i(0)|]\}\\
&\le \sum_{i=1}^{n} q_i\{-\alpha_i u_i^2 + |u_i|[\sum_{j=1}^{m} |a_{ij}|F_j|v_j| + \sum_{j=1}^{m} |b_{ij}|F_j|v_j| + \sum_{j=1}^{m} |c_{ij}|F_j|v_j|]\}\\
&+\sum_{j=1}^{m} q_{n+j}\{-\beta_j v_j^2 + |v_j|[\sum_{i=1}^{n} |d_{ji}|G_i|u_i| + \sum_{i=1}^{n} |k_{ji}|G_i|u_i| + \sum_{i=1}^{n} |l_{ji}|G_i|u_i|]\}\\
&=[|u|^{\mathrm{T}}, |v|^{\mathrm{T}}]C\begin{bmatrix} -\alpha & 0 \\ 0 & -\beta \end{bmatrix}\begin{bmatrix} |u| \\ |v| \end{bmatrix} + [|u|^{\mathrm{T}}, |v|^{\mathrm{T}}]Q\begin{bmatrix} 0 & (|A|+|B|+|C|)F \\ (|D|+|K|+|L|)G & 0 \end{bmatrix}\begin{bmatrix} |u| \\ |v| \end{bmatrix}\\
&=[|u|^{\mathrm{T}}, |v|^{\mathrm{T}}]Q\begin{bmatrix} -\alpha & (|A|+|B|+|C|)F \\ (|D|+|K|+|L|)G & -\beta \end{bmatrix}\begin{bmatrix} |u| \\ |v| \end{bmatrix}\\
&=[|u|^{\mathrm{T}}, |v|^{\mathrm{T}}][Q\Omega]^s\begin{bmatrix} |u| \\ |v| \end{bmatrix}\\
&\qquad \le -\varepsilon\|(u,v)\|^2 .
\end{aligned} \tag{5}
$$

Using Schwartz inequality, from (5), we get

$$\varepsilon\|(u,v)\|^2 \le \|C\|\|(u,v)\|\|\overline{H}(u,v)\|,$$

namely, $\frac{\varepsilon\|(u,v)\|}{\|C\|} \le \|\overline{H}(u,v)\|$. So when $\|(u,v)\| \to +\infty$, $\|\overline{H}(u,v)\| \to +\infty$, i.e., $\|H(u,v)\| \to +\infty$.

From the above two steps, according to Lemma 3, we know that for any pair of input (I, J), map $H(u,v)$ is a homeomorphism on R^{n+m}. Hence system (1) has a unique equilibrium point.The proof is completed.

IV. Global asymptotic stability of the equilibrium point

In the section, we study the global asymptotically stability of the equilibrium point of system (1).

Theorem 2 Suppose Assumption (A) holds. If Ω is an M-matrix, then for any pair of input (I, J), system (1) has a unique equilibrium point (u^*, v^*), which is globally asymptotically stable, independent of the delays.

Proof. Since Ω is an M-matrix, from Theorem 1, system (1) has a unique equilibrium point (u^*, v^*). By means of coordinate translation $x(t) = u(t) - u^*$, $y(t) = v(t) - v^*$, system (1) can be written as

$$\dot{x}_i(t) = -\alpha_i x_i(t) + \sum_{j=1}^{m} a_{ij}[f_j(y_j(t-\tau_{ij}) + v_j^*) - f_j(v_j^*)] + \bigwedge_{j=1}^{m} b_{ij} f_j(y_j(t-\tau_{ij}) + v_j^*)$$
$$- \bigwedge_{j=1}^{m} b_{ij} f_j(v_j^*) + \bigvee_{j=1}^{m} c_{ij} f_j(y_j(t-\tau_{ij}) + v_j^*) - \bigvee_{j=1}^{m} c_{ij} f_j(v_j^*),\ i = 1,2,\cdots,n, \quad (6a)$$

$$\dot{y}_j(t) = -\beta_j y_j(t) + \sum_{i=1}^{n} d_{ji}[g_i(x_i(t-\sigma_{ji}) + u_i^*) - g_i(u_i^*)] + \bigwedge_{i=1}^{n} k_{ji} g_i(x_i(t-\sigma_{ji}) + u_i^*)$$
$$- \bigwedge_{i=1}^{n} k_{ji} g_i(u_i^*) + \bigvee_{i=1}^{n} l_{ji} g_i(x_i(t-\sigma_{ji}) + u_i^*) - \bigvee_{i=1}^{n} l_{ji} g_i(u_i^*),\ j = 1,2,\cdots,m. \quad (6b)$$

System (6) has a unique equilibrium at $(x, y) = (0,0)$. Clearly, (u^*, v^*) is globally asymptotically stable for (1) if and only if the trivial solution of (6) is global asymptotically stable.

Due to $\Omega = \begin{bmatrix} \alpha & -[|A|+|B|+|C|]F \\ -[|D|+|K|+|L|]G & \beta \end{bmatrix}$ being an M-matrix, from Lemma 1, there exist $\xi_i > 0 \quad (i = 1,..., n)$, $\eta_j > 0$ $(j = 1,..., m)$ such that

$$\xi_i \alpha_i - \sum_{j=1}^{m} \eta_j (|d_{ji}| + |k_{ji}| + |l_{ji}|) G_i > 0,\ (i = 1,2,..., n), \quad (7a)$$

$$\eta_j \beta_j - \sum_{i=1}^{n} \xi_i (|a_{ij}| + |b_{ij}| + |c_{ij}|) F_j > 0,\ (j = 1,2,..., m). \quad (7b)$$

Consider a Liapunov functional $V(t) = V(x, y)(t)$ defined by

$$V(x, y)(t) = \sum_{i=1}^{n} \xi_i \{|x_i| + \sum_{j=1}^{m} F_j (|a_{ij}| + |b_{ij}| + |c_{ij}|) \int_{t-\tau_{ij}}^{t} |y_j(s)| ds\}$$
$$+ \sum_{j=1}^{m} \eta_j \{|y_j| + \sum_{i=1}^{n} G_i (|d_{ji}| + |k_{ji}| + |l_{ji}|) \int_{t-\sigma_{ji}}^{t} |x_i(s)| ds\}. \quad (8)$$

Calculating the upper right derivative D^+V of V along the solutions of (6). By using Assumption (A) and Lemma 2, we get

$$D^+V(x, y)(t) = \sum_{i=1}^{n} \xi_i \{\mathrm{sgn}\ x_i \frac{dx_i}{dt} + \sum_{j=1}^{m} F_j (|a_{ij}| + |b_{ij}| + |c_{ij}|)(|y_j(t)| - |y_j(t-\tau_{ij})|)\}$$
$$+ \sum_{j=1}^{m} \eta_j \{\mathrm{sgn}\ y_j \frac{dy_j}{dt} + \sum_{i=1}^{n} G_i (|b_{ji}| + |k_{ji}| + |l_{ji}|)(|x_i(t)| - |x_i(t-\sigma_{ji})|)\}$$
$$= \sum_{i=1}^{n} \xi_i \{\mathrm{sgn}\ x_i [-\alpha_i x_i(t) + \sum_{j=1}^{m} a_{ij} (f_j(y_j(t-\tau_{ij}) + v_j^*) - f_j(v_j^*))$$
$$+ \bigwedge_{j=1}^{m} b_{ij} f_j(y_j(t-\tau_{ij}) + v_j^*) - \bigwedge_{j=1}^{m} b_{ij} f_j(v_j^*) + \bigvee_{j=1}^{m} c_{ij} f_j(y_j(t-\tau_{ij}) + v_j^*)$$
$$- \bigvee_{j=1}^{m} c_{ij} f_j(v_j^*)] + \sum_{j=1}^{m} F_j (|a_{ij}| + |b_{ij}| + |c_{ij}|)(|y_j(t)| - |y_j(t-\tau_{ij})|)\}$$

$$+\sum_{j=1}^{m}\eta_j\{\operatorname{sgn}\ y_j[-\beta_j y_j(t)+\sum_{i=1}^{n}d_{ji}(g_i(x_i(t-\sigma_{ji})+u_i^*)-g_i(u_i^*))$$

$$+\bigwedge_{i=1}^{n}k_{ji}g_i(x_i(t-\sigma_{ji})+u_i^*)-\bigwedge_{i=1}^{n}k_{ji}g_i(u_i^*)+\bigvee_{i=1}^{n}l_{ji}g_i(x_i(t-\sigma_{ji})+u_i^*)$$

$$-\bigvee_{i=1}^{n}l_{ji}g_i(u_i^*)]+\sum_{i=1}^{n}G_i(|b_{ji}|+|k_{ji}|+|l_{ji}|)(|x_i(t)|-|x_i(t-\sigma_{ji})|)\}$$

$$\le\sum_{i=1}^{n}\xi_i\{-\alpha_i|x_i(t)|+\sum_{j=1}^{m}|a_{ij}||f_j(y_j(t-\tau_{ij})+v_j^*)-f_j(v_j^*)|$$

$$+|\bigwedge_{j=1}^{m}b_{ij}f_j(y_j(t-\tau_{ij})+v_j^*)-\bigwedge_{j=1}^{m}b_{ij}f_j(v_j^*)|+|\bigvee_{j=1}^{m}c_{ij}f_j(y_j(t-\tau_{ij})+v_j^*)$$

$$-\bigvee_{j=1}^{m}c_{ij}f_j(v_j^*)|+\sum_{j=1}^{m}F_j(|a_{ij}|+|b_{ij}|+|c_{ij}|)(|y_j(t)|-|y_j(t-\tau_{ij})|)\}$$

$$+\sum_{j=1}^{m}\eta_j\{-\beta_j|y_j(t)|+\sum_{i=1}^{n}|d_{ji}||g_i(x_i(t-\sigma_{ji})+u_i^*)-g_i(u_i^*)|$$

$$+|\bigwedge_{i=1}^{n}k_{ji}g_i(x_i(t-\sigma_{ji})+u_i^*)-\bigwedge_{i=1}^{n}k_{ji}g_i(u_i^*)|+|\bigvee_{i=1}^{n}l_{ji}g_i(x_i(t-\sigma_{ji})+u_i^*)$$

$$-\bigvee_{i=1}^{n}l_{ji}g_i(u_i^*)|+\sum_{i=1}^{n}G_i(|b_{ji}|+|k_{ji}|+|l_{ji}|)(|x_i(t)|-|x_i(t-\sigma_{ji})|)\}$$

$$\le\sum_{i=1}^{n}\xi_i\{-\alpha_i|x_i(t)|+\sum_{j=1}^{m}(|a_{ij}|+|b_{ij}|+|c_{ij}|)|f_j(y_j(t-\tau_{ij})+v_j^*)-f_j(v_j^*)|$$

$$+\sum_{j=1}^{m}F_j(|a_{ij}|+|b_{ij}|+|c_{ij}|)(|y_j(t)|-|y_j(t-\tau_{ij})|)\}$$

$$+\sum_{j=1}^{m}\eta_j\{-\beta_j|y_j(t)|+\sum_{i=1}^{n}(|d_{ji}|+|k_{ji}|+|l_{ji}|)|g_i(x_i(t-\sigma_{ji})+u_i^*)-g_i(u_i^*)|$$

$$+\sum_{i=1}^{n}G_i(|b_{ji}|+|k_{ji}|+|l_{ji}|)(|x_i(t)|-|x_i(t-\sigma_{ji})|)\}$$

$$\le\sum_{i=1}^{n}\xi_i\{-\alpha_i|x_i(t)|+\sum_{j=1}^{m}(|a_{ij}|+|b_{ij}|+|c_{ij}|)F_j|y_j(t-\tau_{ij})|$$

$$+\sum_{j=1}^{m}F_j(|a_{ij}|+|b_{ij}|+|c_{ij}|)(|y_j(t)|-|y_j(t-\tau_{ij})|)\}$$

$$+\sum_{j=1}^{m}\eta_j\{-\beta_j|y_j(t)|+\sum_{i=1}^{n}(|d_{ji}|+|k_{ji}|+|l_{ji}|)G_i|x_i(t-\sigma_{ji})|$$

$$+\sum_{i=1}^{n}G_i(|b_{ji}|+|k_{ji}|+|l_{ji}|)(|x_i(t)|-|x_i(t-\sigma_{ji})|)\}$$

$$=\sum_{i=1}^{n}\xi_i\{-\alpha_i|x_i(t)|+\sum_{j=1}^{m}(|a_{ij}|+|b_{ij}|+|c_{ij}|)F_j|y_j(t)|\}$$

$$+\sum_{j=1}^{m}\eta_j\{-\beta_j|y_j(t)|+\sum_{i=1}^{n}(|d_{ji}|+|k_{ji}|+|l_{ji}|)G_i|x_i(t)|\}$$

$$=\sum_{i=1}^{n}\{-\xi_i\alpha_i+\sum_{j=1}^{m}\eta_j(|d_{ji}|+|k_{ji}|+|l_{ji}|)G_i\}|x_i(t)|$$
$$+\sum_{j=1}^{m}\{-\eta_j\beta_j+\sum_{i=1}^{n}\xi_i(|a_{ij}|+|b_{ij}|+|c_{ij}|)F_j\}|y_j(t)|$$
$$\le-\mu\sum_{i=1}^{n}|x_i(t)|-\lambda\sum_{j=1}^{m}|y_j(t)|, \quad (9)$$

where,

$$\mu=\min_{1\le i\le n}\{\xi_i\alpha_i-\sum_{j=1}^{m}\eta_j(|d_{ji}|+|k_{ji}|+|l_{ji}|)G_i\}>0,\ \lambda=\min_{1\le j\le m}\{\eta_j\beta_j-\sum_{i=1}^{n}\xi_i(|a_{ij}|+|b_{ij}|+|c_{ij}|)F_j\}>0.$$

By the standard Liapunov-type Theorem in functional differential equations[24], the trivial solution of (6) is global asymptotically stable, and therefore, (u^*,v^*) is global asymptotically stable for (1). The proof is completed.

V. A illustrative example

Consider the following BAM neural networks with fuzzy logic:

$$\dot u_i=-\alpha_i u_i+\sum_{j=1}^{2}a_{ij}f_j(v_j(t-0.2))+\bigwedge_{j=1}^{2}b_{ij}f_j(v_j(t-0.2))+\bigvee_{j=1}^{2}c_{ij}f_j(v_j(t-0.2))+2,\ i=1,2, \quad (10a)$$
$$\dot v_j=-\beta_j v_j+\sum_{i=1}^{2}d_{ji}g_i(u_i(t-0.5))+\bigwedge_{i=1}^{2}k_{ji}g_i(u_i(t-0.5))+\bigvee_{i=1}^{2}l_{ji}g_i(u_i(t-0.5))+3,\ j=1,2, \quad (10b)$$

where

$$\alpha=\mathrm{diag}(\alpha_1,\alpha_2)=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ \beta=\mathrm{diag}(\beta_1,\beta_2)=\begin{bmatrix}1.5&0\\0&1.5\end{bmatrix},$$
$$A=(a_{ij})_{2\times2}=\begin{bmatrix}0.1&-0.2\\0.2&0.1\end{bmatrix},B=(b_{ij})_{2\times2}=\begin{bmatrix}0.2&0.2\\0.2&-0.3\end{bmatrix}\ C=(c_{ij})_{2\times2}=\begin{bmatrix}-0.2&-0.1\\-0.2&0.1\end{bmatrix},$$
$$D=(d_{ji})_{2\times2}=\begin{bmatrix}0.1&-0.1\\0.2&0.1\end{bmatrix},K=(k_{ji})_{2\times2}=\begin{bmatrix}0.2&0.2\\0.3&0.2\end{bmatrix},L=(l_{ji})_{2\times2}=\begin{bmatrix}0.3&-0.3\\0.2&0.2\end{bmatrix},\ f_1(u)=\sin((\sqrt{2}/2)u)+u,$$
$$f_2(u)=\sin(u),\ g_1(u)=g_2(u)=(e^u-e^{-u})/(e^u+e^{-u}).$$

It is easy to verify that $f_1(u)$, $f_2(u)$ and $g_1(u)$, $g_2(u)$ satisfy Assumption (A) with $F_1=1+\sqrt{2}/2$, $F_2=1$ and $G_1=G_2=1$, therefore

$$F=\mathrm{diag}(F_1,F_2)=\begin{bmatrix}1.707&0\\0&1\end{bmatrix},\ G=\mathrm{diag}(G_1,G_2)=\begin{bmatrix}1&0\\0&1\end{bmatrix}.$$

Thus, we obtain

$$\Omega=\begin{bmatrix}\alpha&-[|A|+|B|+|C|]F\\-[|D|+|K|+|L|]G&\beta\end{bmatrix}=\begin{bmatrix}1&0&-0.854&-0.5\\0&1&-0.6&-0.5\\-0.6&-0.6&1.5&0\\-0.7&-0.5&0&1.5\end{bmatrix}$$

is a M-matrix, so by Theorem 2, neural network (10) is globally exponential stable.

VI. Conclusion

In the paper, the stability for a class of bidirectional associative memory neural networks with fuzzy logic and time delays has been presented. The sufficient condition related to the existence of unique equilibrium point and its global asymptotic stability for the class of bidirectional associative memory neural networks are derived by using M-matrix theory and Liapunov functions. Furthermore, the obtained sufficient condition is delay-dependent, it is easy to test the conditions of the criterion in practice.

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