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Simple Complex Operation Utilizing Single Quantum Gate

Daqing Liu[1], Shuyue Chen[1], Yu Xue[1], Xinhua Li[2]
[1]*School of Mathematics and Physics, Changzhou University, Changzhou, 213164, China*
[2]*School of Physics and energy, Normal University of Fujian, Fuzhou, 350117, China*
Corresponding Author: Daqing Liu

Abstract: *In the paper we encoded each qubit state in 2 two-level system into a point in the extended complex plane. According to the one-to-one encoding, an algorithm of realizing complex operation utilizing a single quantum gate was proposed. In some case the algorithm is decorrelation free. We hope the preliminarily discussed results are useful in the quantum simulations.*

Keywords: *complex operation, single quantum gate, extended complex plane*.

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I. Introduction

The proposition of quantum computer dates back to 1980s [1] and it was shown that quantum computers are powerful than classical computers on various specialized problems. For example, the Deutsch-Jozsa algorithm [2], Shor's quantum algorithm for factoring integers [3], Grover's quantum search algorithm [4], algorithms for Hamiltonian simulation of quantum systems [5] and quantum algorithm for linear systems of equations [6,7,8,9].

Here we take an alternative notation on traditional quantum computing. We first notice that an important difference between quantum computer and classical computer is that qubits are usually superpositions of the computational basis. If one encodes each qubit into point in the extended complex plane (ECP), quantum gates can be regarded as complex functions. Thus, we can perform a complex operation using quantum gates. Apparently, complex operations are hardly realized by classical computer and the realizing by quantum computer will exhibit another power of quantum computing. We show how to realize simple complex operation using quantum gates in the manuscript.

II. Representation of qubits on an extended complex plane

In this section we propose how to represent a qubit in an ECP.As we know, qubits can be represented by points on a Bloch sphere (BS) [10,11], using the spherical coordinate system. This representation is based on the fact that any qubit can be represented as

$$|\psi>=\cos\frac{\theta}{2}|0>+\sin\frac{\theta}{2}e^{i\varphi}|1>, \qquad (1)$$

as shown in Fig. 1. In the above expression $\theta\in[0,\pi]$ is the polar angle, phase $\varphi\in[0,2\pi)$ is the azimuth angle. In this representation North and South poles correspond to the basis states, $|0>$ and $|1>$, respectively. Notice that the BS representation can be generalized to represent, for example, entangle states [12] or N-level systems [13].

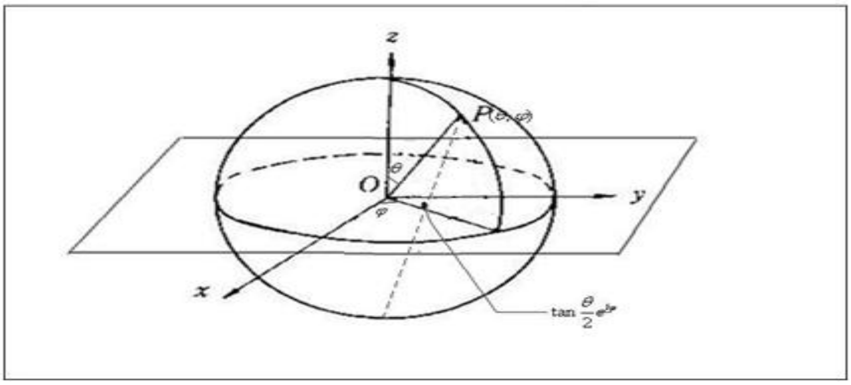

Figure 1 Encoding a point on Bloch sphere into the one in the extended complex plane.

To code points on the BS into the one in the ECP, we firstly set the BS as a unit sphere, the center of which locates at the origin of coordinates. Secondly, we set x-y plane (plus infinite point) as ECP. Apparently, the line, connecting South pole and a point on BS, and ECP intersect at a point. We thus establish the one-to-one correspondence from a qubit, represented by a point on Bloch sphere, to a point in ECP. From Fig. 1 one can verify that a point in BS, the spherical coordinate of which is (θ,φ), corresponds to a point in ECP,

$$z=\tan\frac{\theta}{2}e^{i\varphi}. \tag{2}$$

Notice that since $0\le\theta\le\pi$, $\tan\frac{\theta}{2}\ge 0$, which means that $\tan\frac{\theta}{2}$ is the module of z, $|z|$. Here are some special cases: |0> and |1> correspond to $z=0$ and $z=\infty$ and set of points $(\frac{\pi}{2},\varphi)$ corresponds to a unit circle, $z=e^{i\varphi}$. This representation of qubits by points in ECP provides an isomorphism between qubit and point in ECP.

A quantum gate corresponds to a unitary transformation

$$G\mapsto\begin{pmatrix}\cos\alpha & \sin\alpha e^{i\varphi_2}\\ \sin\alpha e^{i\varphi_3} & -\cos\alpha e^{i(\varphi_2+\varphi_3)}\end{pmatrix}. \tag{3}$$

If qubits are symbolized as points in ECP, one can represent the quantum gate as a map from ECP to ECP, *i.e.*, a complex function, $z'=f(z)$, where both z and z' are points in ECP.

$$G\mapsto z'=e^{i\varphi_3}\frac{\sin\alpha-z\cos\alpha e^{i\varphi_2}}{\cos\alpha+z\sin\alpha e^{i\varphi_2}} \tag{4}$$

Thus, we list complex representations of some quantum gates in the Tab. 1.

Gate	matrix	Function
Hadamard	$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$	$\frac{1-z}{1+z}$
X	$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	z^{-1}
Y	$\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$	$-z^{-1}$
Z	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	-z
Phase	$\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$	iz
$\frac{\pi}{8}$	$\begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}$	$e^{i\pi/4}z$
Rotation	$\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$	$\frac{\sin\alpha+z\cos\alpha}{\sin\alpha-z\cos\alpha}$

Table 1. Matrix representations and complex function ones of usual quantum gates.

The inner product of $z_1=r_1e^{i\varphi_1}$ and $z_2=r_2e^{i\varphi_2}$ is defined as $(z_1,z_2)\equiv\frac{1+r_1r_2e^{i(\varphi_2-\varphi_1)}}{\sqrt{(1+r_1^2)(1+r_2^2)}}$, with the identity $(z_1,z_2)=(z_2,z_1)^*$. After the measurement, the possibility of the state, represented by $z=re^{i\varphi}$, locating in |0> is $|(z,0)|^2 = \frac{1}{1+r^2}$, whereas in |1>, $|(z,\infty)|^2 = \frac{r^2}{1+r^2}$, or in other words, $r = \frac{|(z,\infty)|}{|(z,0)|}$.

III. Simple Complex Operation Using Quantum Gates

In this section we exemplify how to perform a complex function computing using quantum gate. For example, we want to compute $f(z) = \frac{1-z}{1+z}$. In classical computing we face two difficulties: 1) one can hardly realize the operation *f*(*z*) in a small number of steps; 2) if $z \to \pm 1$, the result is overflowing.
However, these difficulties do not occur in the quantum computing. We show the circuit in Fig. 2. To do the complex operation we use a Hadamard gate. To perform the operation $z' = \frac{1-z}{1+z}$ (or $z' = iz$), where $z = re^{i\varphi}$ and $z' = r'e^{i\varphi'}$, we first input an initial state according to $|\psi> = \frac{1}{\sqrt{1+r^2}}|0> + \frac{re^{i\varphi}}{\sqrt{1+r^2}}|1>$. Then, we perform the operation utilizing a Hadamard gate (or phase gate) and get the finial state as $|\psi'> = \frac{1}{\sqrt{1+r'^2}}|0> + \frac{r'e^{i\varphi'}}{\sqrt{1+r'^2}}|1>$. At last, from the final state we obtain the result as $z' = r'e^{i\varphi'}$.

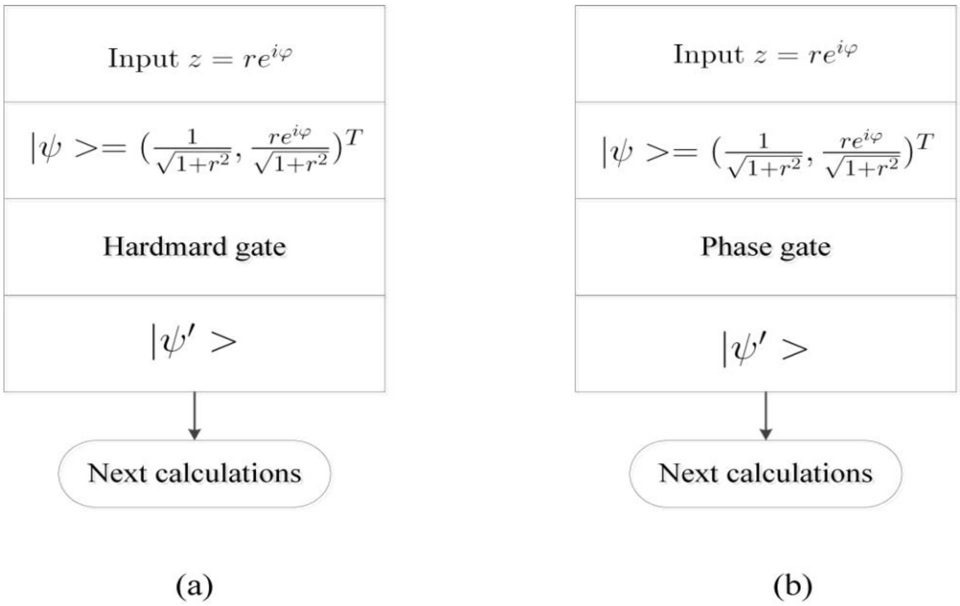

Figure 2. Schedule to perform complex operation $z' = \frac{1-z}{1+z}$ using a Hadamard gate (a) and operation $z' = iz$ using phase gate (b).

Sometime we only focus on $|z'|^2 = r'^2$. From Eq. (2), we need only to obtain $P(0) = |(0,z')|^2 = \frac{1}{1+r'^2}$. In this case the phase of the final state is irrelevant and our schedule is decorrelation free. To clarify the computing proceed we also list the program flow chat of quantum computing simulation of complex operation $z' = \frac{1-z}{1+z}$ in Fig. 3.

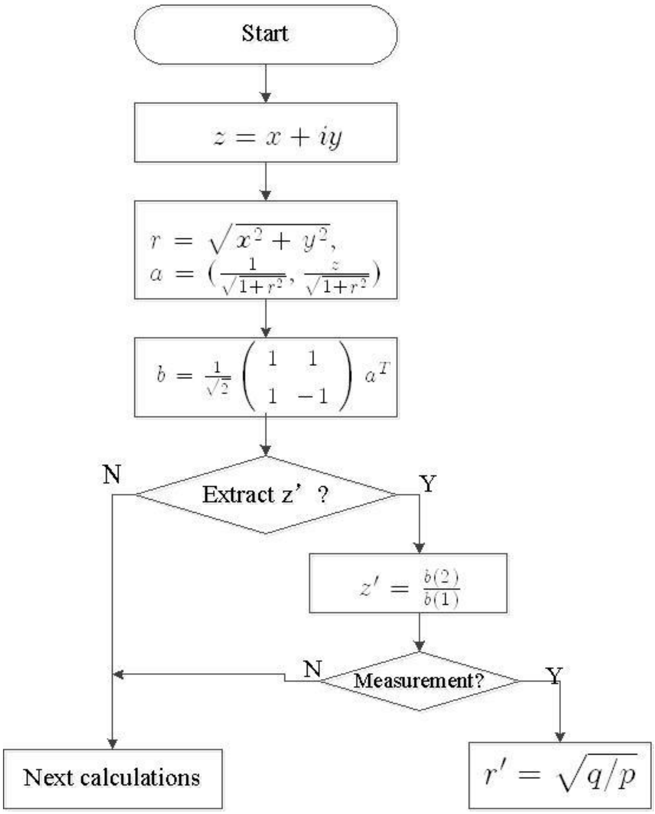

Figure 3. Program flow chat of quantum computing simulation of complex operator $z' = \frac{1-z}{1+z}$. In the figure p is the measurement possibility of state |0>, and q, state |1>.

According to Fig. 3, we list quantum computing simulation results of $z = 1+i$, $z = 3+4i$ and $z = 0.5+0.5i$ in Tab. 2. From the table, one finds that we indeed obtain results of complex operation $z' = \frac{1-z}{1+z}$, as long as we perform enough (simulative) measurements.

z	z'	r'_{40}	r'_{100}	r'_{th}
$z = 1+i$	-0.2-0.4i	0.42	0.44	0.45
$z = 3+4i$	-0.75-0.25i	0.82	0.80	0.79
$z = 0.5+0.5i$	0.2-0.4i	0.38	0.48	0.45

Table 2. Quantum computing simulation results of $z' = \frac{1-z}{1+z}$ with $z = 1+i$, $z = 3+4i$ and $z = 0.5+0.5i$. r'_{40} and r'_{100} are values performed by 40 and 100 times (simulative) measurements respectively, whereas $r'_{th} = |z'|$ are theoretical values (it is in fact given by the expression $r'_{th} = \frac{|b_2|}{|b_1|}$).

IV. Summary

In this letter we find that there is a one-to-one correspondence between one-qubit state in a two-level energy system and a point in the extended complex plane. Taking advantage of the correspondence, one can consider a quantum gate as a complex operation. Thus, we preliminarily discussed a simple schedule to perform complex operations using a single quantum gate. If we are unconcerned with the phase of the final result, such schedule is decorrelation free, as shown in the section 3.

However, it is worthy to note that how to representation of multi-qubit into point in extended complex plane, as authors did in Refs. [12,13], is an open question.

In the area of quantum simulation, for instance, Refs. [14,15,16], one always deals with complex operations directly. We hope that the discussed results are useful in the quantum simulation.

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