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Investigation on Distribution of Nodal Multiplications on T_3 Tree

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Abstract: *The article investigates distribution law of node-multiplications of T3 tree that is an important valuated binary tree. It exhibits the multiplication of two nodes of the T3 tree merely distributes in specific range on specific levels of the tree. By intuitive figures the paper makes it easy to know what range of the multiplication is. Mathematical deductions are showed in detail ,which can enhance the theory of valuated binary tree*

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I. Introduction

In 2016, articles [1] proposed a new approach to study integers by putting odd integers bigger than 1 on a full binary tree from the top to the bottom and from the left to the right and since then, many new properties of the integers have been discovered, such as the symmetric common divisors, the genetic properties, as introduced in articles [2] to [5]. The new approached also exhibited special characteristics in factoring integers, as shown in articles [6] to [9]. Factorization of integers is undoubtedly a topic that has attracted interests of a number of researchers. Knowing about the multiplication of two integers is also undoubtedly a key to help to solve the problem. Article [5] did introduce certain properties of multiplication of two nodes of $\boldsymbol{T}_3$ tree, as stated by the Theorem 6, Corollary 2 to Corollary 5 in the article. However, it can see that, the conclusions are still very rough. In such a point of view, this article follows the study of article [5] that introduced multiplication of two nodes of $\boldsymbol{T}_3$ tree to make a detail study on distribution of the nodal multiplication in the $\boldsymbol{T}_3$ tree. The investigation shows that, multiplication of two nodes of the $\boldsymbol{T}_3$ tree has its own distribution law and might be helpful for designed algorithm of factoring integers.

II. Preliminaries

2.1 Definitions and Notations

Symbol $\boldsymbol{T}_3$ is the $\boldsymbol{T}_3$ tree that was introduced in [1] and [2] and symbol $N_{(k,j)}$ is by default the node at position *j* on level *k* of $\boldsymbol{T}_3$, where $k \ge 0$ and $0 \le j \le 2^k - 1$. Symbol $T_{N_{(k,j)}}$ denotes a subtree whose root is $N_{(k,j)}$ and symbol $N_{(i,\omega)}^{N_{(k,j)}}$ denotes the node at the ω[th] position on the *i*[th] level of $T_{N_{(k,j)}}$. Symbol $N_{(N_{(m,\alpha)} \swarrow \chi)}$ is the node where $N_{(m,\alpha)}$ slides down χ levels along the leftmost path of subtree $T_{N_{(m,\alpha)}}$ and symbol $N_{(N_{(m,\alpha)} \searrow \chi)}$ is that $N_{(m,\alpha)}$ slides down χ levels along the rightmost path of subtree $T_{N_{(m,\alpha)}}$. An odd interval $[a,b]$ is a set of consecutive odd numbers that take *a* as lower bound and *b* as upper bound, for example, $[3,11] = \{3,5,7,9,11\}$. Intervals in this whole article are by default the odd ones unless particularly mentioned. Symbol $A \Rightarrow B$ means conclusion *B* can be derived from condition *A*.

2.2 Lemmas

Lemma 1 (See in [5]). $\boldsymbol{T}_3$ Tree has the following fundamental properties.

(**P0**). Every node is an odd integer and every odd integer bigger than 1 must be on the T_3 tree.

(**P1**). On the k^{th} level with $k = 0,1,...$, there are 2^k nodes starting by $2^{k+1}+1$ and ending by $2^{k+2}-1$, namely, $N_{(k,j)} \in [2^{k+1}+1, 2^{k+2}-1]$ with $j = 0,1,...,2^k - 1$.

(**P2**). $N_{(k,j)}$ is calculated by

$N_{(k,j)} = 2^{k+1} + 1 + 2j,\ j = 0,1,...,2^k - 1$

(**P3**) Nodes $N_{(k+1,2j)}$ and $N_{(k+1,2j+1)}$ on the $(k+1)^{\text{th}}$ level are respectively left son and right son of node $N_{(k,j)}$ on the k^{th} level. The descendants of $N_{(k,j)}$ on the $(k+i)^{\text{th}}$ level are $N_{(k+i,2^i j+\omega)}$ ($0 \le \omega \le 2^i - 1$), which are

$N_{(k+i,2^i j)}, N_{(k+i,2^i j+1)}, N_{(k+i,2^i j+2)}, \ldots, N_{(k+i,2^i j+2^i-1)}$

Particularly,

$$N_{(N_{(m,a)} \square\ \chi)} = 2^{\chi}(N_{(m,\alpha)} - 1) + 1$$

$$N_{(N_{(m,a)} \square\ \chi)} = 2^{\chi}(N_{(m,\alpha)} + 1) - 1$$

(**P4**) The biggest node on left branch of the k^{th} level is $2^{k+1} + 2^k - 1$ whose position is $j = 2^{k-1} - 1$, and the smallest node on the right branch of the k^{th} level is $2^{k+1} + 2^k + 1$ whose position is $j = 2^{k-1}$.

(**P5**) Multiplication of arbitrary two nodes of $\boldsymbol{T}_3$, say $N_{(m,\alpha)}$ and $N_{(n,\beta)}$, is a third node of $\boldsymbol{T}_3$. Let $J = 2^m(1+2\beta)$ $+2^n(1+2\alpha) + 2\alpha\beta + \alpha + \beta$; the multiplication $N_{(m,\alpha)} \times N_{(n,\beta)}$ is given by

$$N_{(m,\alpha)} \times N_{(n,\beta)} = 2^{m+n+2} + 1 + 2J$$

If $J < 2^{m+n+1}$, then $N_{(m,\alpha)} \times N_{(n,\beta)} = N_{(m+n+1,J)}$ lies on level $m+n+1$ of $\boldsymbol{T}_3$; whereas, if $J \ge 2^{m+n+1}$, $N_{(m,\alpha)} \times N_{(n,\beta)} = N_{(m+n+2,\chi)}$ with $\chi = J - 2^{m+n+1}$ lies on level $m+n+2$ of $\boldsymbol{T}_3$. If $0 \le m \le n$, then $N_{(m,\alpha)} \times N_{(n,\beta)}$ can not fall in $T_{N_{(n,\beta)}}$ on level $m+1$ or a higher level.

(**P6**) The i^{th} ($i \ge 0$) level of subtree $T_{N_{(k,j)}}$ ($k \ge 0$) is the $(k+i)^{\text{th}}$ level of T_3 and it contains 2^i nodes. Node at position ω on level i of $T_{N_{(k,j)}}$ is given by

$$N_{(i,\omega)}^{N_{(k,j)}} = 2^i N_{(k,j)} - 2^i + 2\omega + 1$$
$$i = 0,1,2,\ldots; \omega = 0,1,\ldots,2^i - 1$$

III. Results

Theorem 1. Let $N_{(m,\alpha)}$ and $N_{(n,\beta)}$ be two nodes of $\boldsymbol{T}_3$ with $0 \le m \le n$, $0 \le \alpha \le 2^m - 1$ and $0 \le \beta \le 2^n - 1$; then it holds

$$N_{(m,\alpha)} \times N_{(n,\beta)} > \max(N_{(N_{(m,\alpha)} << n+1)}, N_{(N_{(n,\beta)} >> m+1)}) \quad (1)$$

Proof. Direct calculation yields

$$\begin{aligned} N_{(m,\alpha)} \times N_{(n,\beta)} &= (2^{m+1} + 2\alpha + 1) N_{(n,\beta)} \\ &= 2^{m+1} N_{(n,\beta)} + 2\alpha N_{(n,\beta)} + N_{(n,\beta)} \\ &= \underbrace{2^{m+1} N_{(n,\beta)} - 2^{m+1} + 1}_{N_{(N_{(n,\beta)} << m+1)}} + 2^{m+1} - 1 + 2\alpha N_{(n,\beta)} + N_{(n,\beta)} \\ &= N_{(N_{(n,\beta)} << m+1)} + (2\alpha + 1) N_{(n,\beta)} + (2^{m+1} - 1) \end{aligned} \quad (2)$$

$$\begin{aligned} N_{(m,\alpha)} \times N_{(n,\beta)} &= (2^{m+1} + 2\alpha + 1) N_{(n,\beta)} \\ &= 2^{m+1} N_{(n,\beta)} + 2\alpha N_{(n,\beta)} + N_{(n,\beta)} \\ &= \underbrace{2^{m+1} N_{(n,\beta)} + 2^{m+1} - 1}_{N_{(N_{(n,\beta)} >> m+1)}} - 2^{m+1} + 1 + 2\alpha N_{(n,\beta)} + N_{(n,\beta)} \\ &= N_{(N_{(n,\beta)} >> m+1)} + (2\alpha + 1) N_{(n,\beta)} - (2^{m+1} - 1) \end{aligned} \quad (3)$$

$$\begin{aligned} N_{(m,\alpha)} \times N_{(n,\beta)} &= (2^{n+1} + 2\beta + 1) N_{(m,\alpha)} \\ &= 2^{n+1} N_{(m,\alpha)} + 2\beta N_{(m,\alpha)} + N_{(m,\alpha)} \\ &= \underbrace{2^{n+1} N_{(m,\alpha)} - 2^{n+1} + 1}_{N_{(N_{(m,\alpha)} << n+1)}} + 2^{n+1} - 1 + 2\beta N_{(m,\alpha)} + N_{(m,\alpha)} \\ &= N_{(N_{(m,\alpha)} << n+1)} + (2\beta + 1) N_{(m,\alpha)} + (2^{n+1} - 1) \end{aligned} \quad (4)$$

$$
\begin{aligned}
&N_{(m,\alpha)} \times N_{(n,\beta)} = (2^{n+1} + 2\beta + 1)N_{(m,\alpha)} \\
&= 2^{n+1}N_{(m,\alpha)} + 2\beta N_{(m,\alpha)} + N_{(m,\alpha)} \\
&= \underbrace{2^{n+1}N_{(m,\alpha)} + 2^{n+1} - 1}_{N_{(N_{(m,\alpha)}>>n+1)}} - 2^{n+1} + 1 + 2\beta N_{(m,\alpha)} + N_{(m,\alpha)} \qquad (5) \\
&= N_{(N_{(m,\alpha)}>>n+1)} + (2\beta+1)N_{(m,\alpha)} - (2^{n+1} - 1)
\end{aligned}
$$

Since $0 \le \alpha \le 2^m - 1$, (2) and (3) yield

$$N_{(N_{(n,\beta)}<<m+1)} + N_{(n,\beta)} + 2^{m+1} - 1 \le N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(n,\beta)}<<m+1)} + \underbrace{2^{m+1}(N_{(n,\beta)} + 1) - 1}_{N_{(N_{(n,\beta)}>>m+1)}} - N_{(n,\beta)}$$

$$N_{(N_{(n,\beta)}>>m+1)} + N_{(n,\beta)} - (2^{m+1} - 1) \le N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(n,\beta)}>>m+1)} + \underbrace{2^{m+1}(N_{(n,\beta)} - 1) + 1}_{N_{(n,\beta)}<<m+1} - N_{(n,\beta)}$$

Namely,

$$N_{(N_{(n,\beta)}<<m+1)} + N_{(n,\beta)} + 2^{m+1} - 1 \le N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(n,\beta)}<<m+1)} + N_{(N_{(m,\alpha)}>>m+1)} - N_{(n,\beta)}$$

$$N_{(N_{(n,\beta)}>>m+1)} + N_{(n,\beta)} - (2^{m+1} - 1) \le N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(n,\beta)}<<m+1)} + N_{(N_{(n,\beta)}>>m+1)} - N_{(n,\beta)}$$

Likewise, by $0 \le \beta \le 2^n - 1$ (4) and (5) yield

$$N_{(N_{(m,\alpha)}<<n+1)} + N_{(m,\alpha)} + 2^{n+1} - 1 \le N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(m,\alpha)}<<n+1)} + N_{(N_{(m,\alpha)}>>n+1)} - N_{(m,\alpha)}$$

$$N_{(N_{(m,\alpha)}>>n+1)} + N_{(m,\alpha)} - (2^{n+1} - 1) \le N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(m,\alpha)}<<n+1)} + N_{(N_{(m,\alpha)}>>n+1)} - N_{(m,\alpha)}$$

Note that, $N_{(N_{(m,\alpha)}<<n+1)}$ and $N_{(N_{(n,\beta)}>>m+1)}$ are on the same level of ***T***$_3$, and because of $N_{(n,\beta)} = 2^{n+1} + 2\beta + 1 > (2^{m+1} - 1)$, it obtains by (6) and (7)

$$N_{(m,\alpha)} \times N_{(n,\beta)} > \max(N_{(N_{(m,\alpha)}<<n+1)}, N_{(N_{(n,\beta)}>>m+1)})$$

□ In order for readers to know Theorem 1 intuitively, the following figures 1, 2 and 3 demonstrate several cases in details.

Case 1. $m = n$ with $\alpha < \beta$. This time, $N_{(m,\beta)}$ and $N_{(m,\alpha)}$ are on the same level and $N_{(m,\alpha)}$ is left to $N_{(m,\beta)}$, as illustrated in figure 1. Hence it holds $N_{(N_{(m,\alpha)}<<m+1)} < N_{(N_{(m,\beta)}>>m+1)}$ and

$$N_{(m,\alpha)} \times N_{(m,\beta)} > N_{(N_{(m,\beta)}>>m+1)} > N_{(N_{(m,\alpha)}<<m+1)}$$

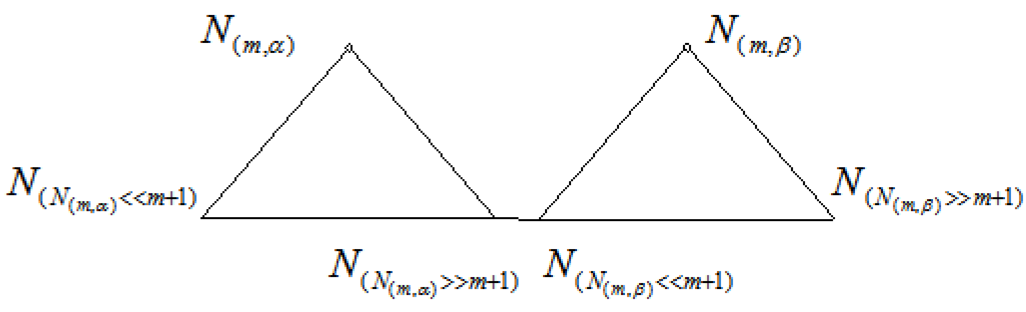

Figure 1. $m = n$ with $\alpha < \beta$

Case 2. $N_{(n,\beta)}$ is a descendant of $N_{(m,\alpha)}$. This time, it holds $N_{(N_{(m,\alpha)}<<n+1)} < N_{(N_{(n,\beta)}>>m+1)}$, as illustrated in figure 2; hence

$$N_{(m,\alpha)} \times N_{(n,\beta)} > N_{(N_{(n,\beta)}>>m+1)} > N_{(N_{(m,\alpha)}<<n+1)}$$

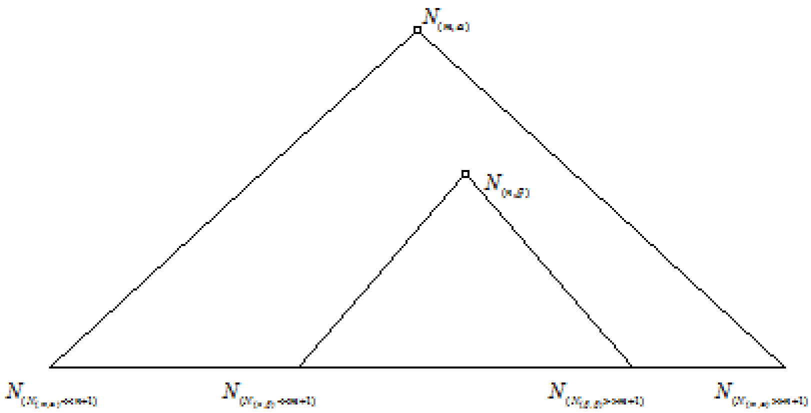

Figure 2. $N_{(n,\beta)}$ Is a descendant of? $N_{(m,\alpha)}$

Case 3. $N_{(n,\beta)}$ is left to $N_{(m,\alpha)}$. This time, it holds $N_{(N_{(m,\alpha)}<<n+1)} > N_{(N_{(n,\beta)}>>m+1)}$, as illustrated in figure 3; hence $N_{(m,\alpha)} \times N_{(n,\beta)} > N_{(N_{(m,\alpha)}<<n+1)} > N_{(N_{(n,\beta)}>>m+1)}$

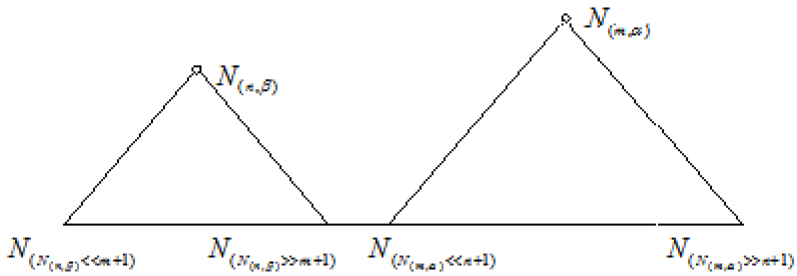

Figure 3. $N_{(n,\beta)}$ is left to $N_{(m,\alpha)}$

Case 4. $N_{(n,\beta)}$ is right to $N_{(m,\alpha)}$. This time, it holds $N_{(N_{(m,\alpha)}<<n+1)} < N_{(N_{(n,\beta)}>>m+1)}$, as illustrated in figure 4; hence $N_{(m,\alpha)} \times N_{(n,\beta)} > N_{(N_{(n,\beta)}>>m+1)} > N_{(N_{(m,\alpha)}<<n+1)}$

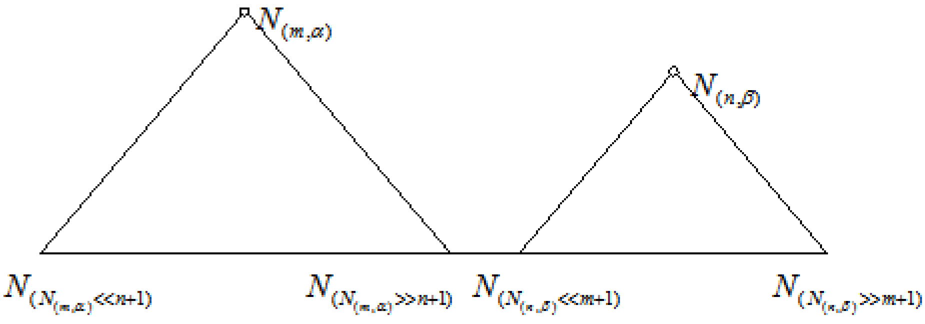

Figure 4. $N_{(n,\beta)}$ is right to $N_{(m,\alpha)}$

Corollary 1. Let $N_{(m,\alpha)}$ and $N_{(n,\beta)}$ be two nodes of $\boldsymbol{T}_3$ with $m \le n$, $0 \le \alpha \le 2^m - 1$ and $0 \le \beta \le 2^n - 1$; if $N_{(m,\alpha)} \times N_{(n,\beta)}$ lies on level $m+n+1$ of $\boldsymbol{T}_3$, then it must be on the right of $T_{N_{(n,\beta)}}$ and it is possibly in $T_{N_{(m,\alpha)}}$ or on the right of $T_{N_{(m,\alpha)}}$.

Proof. By Lemma 1(P5), both $N_{(N_{(m,\alpha)}<<n+1)}$ and $N_{(N_{(n,\beta)}>>m+1)}$ are on level $m+n+1$ of $\boldsymbol{T}_3$. Referring to (1) it knows Corollary 1 holds.

□

Theorem 2. Let $N_{(m,\alpha)}$ and $N_{(n,\beta)}$ be two nodes of $\boldsymbol{T}_3$ with $0 \le m \le n$, $0 \le \alpha \le 2^m - 1$ and $0 \le \beta \le 2^n - 1$; then it holds

$$N_{(m,\alpha)} \times N_{(n,\beta)} < \min(N_{(N_{(m,\alpha)}>>n+2)}, N_{(N_{(n,\beta)}>>m+2)})$$

Proof. Direct calculation shows

$$\begin{aligned}
N_{(m,\alpha)} \times N_{(n,\beta)} &= N_{(N_{(n,\beta)}<<m+1)} + (2\alpha+1)N_{(n,\beta)} + (2^{m+1}-1) \\
&= N_{(N_{(n,\beta)}<<m+2)} + 1 - N_{(N_{(n,\beta)}<<m+1)} + (2\alpha+1)N_{(n,\beta)} + (2^{m+1}-1) \\
&= N_{(N_{(n,\beta)}<<m+2)} + 1 - (2^{m+1}N_{(n,\beta)} - 2^{m+1} + 1) + (2\alpha+1)N_{(n,\beta)} + (2^{m+1}-1) \\
&= N_{(N_{(n,\beta)}<<m+2)} + (2\alpha+1-2^{m+1})N_{(n,\beta)} + (2^{m+2}-1)
\end{aligned} \tag{10}$$

$$\begin{aligned}
N_{(m,\alpha)} \times N_{(n,\beta)} &= N_{(N_{(n,\beta)}>>m+1)} + (2\alpha+1)N_{(n,\beta)} - (2^{m+1}-1) \\
&= N_{(N_{(n,\beta)}>>m+2)} - 1 - N_{(N_{(n,\beta)}>>m+1)} + (2\alpha+1)N_{(n,\beta)} - (2^{m+1}-1) \\
&= N_{(N_{(n,\beta)}>>m+2)} + (2\alpha+1-2^{m+1})N_{(n,\beta)} - (2^{m+2}-1)
\end{aligned} \tag{11}$$

$$\begin{aligned}
N_{(m,\alpha)} \times N_{(n,\beta)} &= N_{(N_{(m,\alpha)}<<n+1)} + (2\beta+1)N_{(m,\alpha)} + (2^{n+1}-1) \\
&= N_{(N_{(m,\alpha)}<<n+2)} + 1 - N_{(N_{(m,\alpha)}<<n+1)} + (2\beta+1)N_{(m,\alpha)} + (2^{n+1}-1) \\
&= N_{(N_{(m,\alpha)}<<n+2)} + (2\beta+1-2^{n+1})N_{(m,\alpha)} + (2^{n+2}-1)
\end{aligned} \tag{12}$$

$$\begin{aligned}
N_{(m,\alpha)} \times N_{(n,\beta)} &= N_{(N_{(m,\alpha)}>>n+1)} + (2\beta+1)N_{(m,\alpha)} - (2^{n+1}-1) \\
&= N_{(N_{(m,\alpha)}>>n+2)} - 1 - N_{(N_{(m,\alpha)}>>n+1)} + (2\beta+1)N_{(m,\alpha)} - (2^{n+1}-1) \\
&= N_{(N_{(m,\alpha)}>>n+2)} + (2\beta+1-2^{n+1})N_{(m,\alpha)} - (2^{n+2}-1)
\end{aligned} \tag{13}$$

Note that, $0 \le \alpha \le 2^m - 1 \Rightarrow 2\alpha+1-2^{m+1} \le -1$ and $0 \le \beta \le 2^n - 1 \Rightarrow 2\beta+1-2^{n+1} \le -1$; it yields

$$(2\alpha+1-2^{m+1})N_{(n,\beta)} + (2^{m+2}-1) \le (2^{m+2}-1) - N_{(n,\beta)} = 2^{m+2} - 2^{n+1} - 2(\beta+1) \tag{14}$$

$$(2\alpha+1-2^{m+1})N_{(n,\beta)} - (2^{m+2}-1) \le -2^{m+2} - 2^{n+1} - 2\beta \tag{15}$$

$$(2\beta+1-2^{n+1})N_{(m,\alpha)} + (2^{n+2}-1) \le 2^{n+2} - 2^{m+1} - 2(\alpha+1) \tag{16}$$

$$(2\beta+1-2^{n+1})N_{(m,\alpha)} - (2^{n+2}-1) \le -2^{n+2} - 2^{m+1} - 2\alpha \tag{17}$$

Then by (14) and (10) it holds

$$N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(n,\beta)}<<m+2)} + 2^{m+2} - 2^{n+1} - 2(\beta+1) \tag{18}$$

By (15) and (11) it holds

$$N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(n,\beta)}>>m+2)} - 2^{n+1} - 2^{m+2} - 2\beta \tag{19}$$

By (16) and (12) it holds

$$N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(m,\alpha)}<<n+2)} + 2^{n+2} - 2^{m+1} - 2(\alpha+1) \tag{20}$$

By (17) and (13) it holds

$$N_{(m,\alpha)} \times N_{(n,\beta)} \le N_{(N_{(m,\alpha)}>>n+2)} - 2^{n+2} - 2^{m+1} - 2\alpha \tag{21}$$

Obviously, (19) and (21) result in

$$N_{(m,\alpha)} \times N_{(n,\beta)} < \min(N_{(N_{(m,\alpha)}>>n+2)}, N_{(N_{(n,\beta)}>>m+2)})$$

□

IV. Conclusion

Multiplication of nodes on the T3 tree can help people to know more about the integers. The research results of this paper shows, the multiplication of two nodes will lie in specific place. This might be helpful for integer factorization because one can use this result to limit the searching area when he/she searches a divisor-node of a composite node. Hope this can be realized in the future.

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