

## New Model of Neural Networks for Solving Unrelated Parallel Machine Scheduling With Common Due Date

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**Abstract:** In this paper the concept of Sequence the jobs on unrelated parallel machine with common due date has been studied, so that the total penalty cost to be minimized. This cost is composed of the total earliness and the total tardiness costs. We designed a new model system of multi layer neural networks base on unit's vector for inputs and out puts. We use Mat lap Software neural networks for training the model to solve a new similar problem. The performance of the new model system has been measured on numerical examples from literature reivews. The results are very encouraging for further investigation.

**Key word:** Unrelated parallel machine, Common due date, Earliness and Tardiness

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### I. Unrelated parallel Machines with Common Due Date

Due to the industrial significance of the just-in-time philosophy, due date problems, i.e., scheduling problems where the due dates are given, have gained increasing attention in recent years. In such problems the jobs due dates are fixed by costumer. In this paper we discuss the case of scheduling  $n$  independent jobs on  $m$  unrelated parallel machines under given common due date. We use the model that earliness and tardiness is cosseted in the same and different rate. In unrelated machine model each job requires a processing time, which are not same for all machines and dependent on the job and machine. Schedules are assigned penalties, which are the sum of the costs related to earliness and tardiness of all jobs. By penalizing both the early and tardy completion of the jobs, costs related to inventory and customer satisfaction are recognized and taken into account. These costs are of a different nature. This is taken into consideration and incorporated into the model by allowing different weights for early and tardy completion.

Let  $n$  be the set of independent jobs  $J_1, J_2, \dots, J_n$  to be processed on  $m$  unrelated parallel machines. The following notation shall be used.

- S Schedule for the  $n$  jobs;
- $P_{ij}$  processing time required by job  $J_i$  on the machine  $j$
- $d$  common due date;
- $C_{ij}$  the completion time of  $J_i$  on machine  $j$ ;
- $E_{ij}$  the earliness of  $J_i$  on machine  $j$ , which is equal to  $\max \{0, d - c_{ij}\}$ ;
- $T_{ij}$  the tardiness of  $J_i$  on machine  $j$ , which is equal to  $\max \{0, c_{ij} - d\}$ ;
- $\alpha, \beta$  the weights associated with earliness and tardiness, respectively.

The problem consider in this paper is to schedule the  $n$  jobs on the  $m$  machines so that the following objective function is minimized.

$$f(S) = \sum_{j=1}^m \sum_{i=1}^n (\alpha E_{ij} + \beta T_{ij}) \quad (1)$$

### II. Neural network Model for Unrelated Parallel Machine

The neural network model for unrelated parallel machine which was proposed for common due date is organized in to three layers of processing units. For example for two unrelated parallel machines, there is an input layer of 20 units in our model, a hidden layer and output layer of two units depends on the number of machine . The number of units in the input and output layers is dictated by the specific representations adopted for this model. In the proposed representation, the input layer contains the information describing the problem in the form of a vector of continuous values. These units' vectors are already applied by (Abdelaziz and Bahrom, 2011). But in this paper we add more four new units, namely unit 8, 9, 10, and 11 so the total units of the model become 20 units, our ideas to generate new model for neural networks to improve the solutions. In the proposed

representation, the input layer contains the information describing the problem in the form of a vector of continuous values. The input for two unrelated machines is defined as follows:

$$unit\ 1 = \frac{P_{i1}}{M_{P_{i1}}}, \tag{2.1}$$

$$unit\ 2 = \frac{P_{i2}}{M_{P_{i2}}} \tag{2.2}$$

$$unit\ 3 = \frac{d}{100}, \tag{2.3}$$

$$unit\ 4 = \frac{SL_{i1}}{M_{SL_{i1}}}, \tag{2.4}$$

$$unit\ 5 = \frac{sl_{i2}}{M_{sl_{i2}}} \tag{2.5}$$

$$unit\ 6 = \frac{P_2}{10}, \tag{2.6}$$

$$unit\ 7 = \frac{P_3}{10} \tag{2.7}$$

$$unit\ 8 = \frac{P_{i1}}{\alpha \times 100} \tag{2.8}$$

$$unit\ 9 = \frac{P_{i2}}{\alpha \times 100} \tag{2.9}$$

$$unit\ 10 = \frac{P_{i1}}{\beta \times 100} \tag{2.10}$$

$$unit\ 11 = \frac{P_{i2}}{\beta \times 100} \tag{2.11}$$

$$unit\ 12 = \frac{\bar{P}_{i1}}{M_{P_{i1}}}, \tag{2.12}$$

$$unit\ 13 = \frac{\bar{P}_{i2}}{M_{P_{i2}}} \tag{2.13}$$

$$unit\ 14 = \frac{\bar{SL}_{i1}}{M_{SL_{i1}}}, \tag{2.14}$$

$$unit\ 15 = \frac{\bar{SL}_{i2}}{M_{sl_{i2}}}, \tag{2.15}$$

$$unit\ 16 = 1 \tag{2.16}$$

$$unit\ 17 = \sqrt{\frac{\sum (P_{i1} - \bar{P}_{i1})^2}{n \times (\bar{P}_{i1})^2}}, \tag{2.17}$$

$$unit\ 18 = \sqrt{\frac{\sum (SL_{i1} - \bar{SL}_{i1})^2}{n \times (\bar{SL}_{i1})^2}}, \tag{2.18}$$

$$unit\ 19 = \sqrt{\frac{\sum (P_{i2} - \bar{P}_{i2})^2}{n \times (\bar{P}_{i2})^2}} \tag{2.19}$$

$$unit\ 20 = \sqrt{\frac{\sum (sl_{i2} - \bar{sl}_{i2})^2}{n \times (\bar{sl}_{i2})^2}} \tag{2.20}$$

Where

$P_{i1}$  : The processing time of  $J_i$  on machine 1;

$P_{i2}$  : The processing time of  $J_i$  on machine 2;

$M_{P_{i1}}$  : Longest processing time among the n jobs on machine 1 =  $\max \{P_{i1}\}$ ;

$M_{P_{i2}}$  : Longest processing time among the n jobs on machine 2 =  $\max \{P_{i2}\}$ ;

$SL_{i1}$  : The different between the due date and processing time of machine 1 =  $d - P_{i1}$ ;

$SL_{i2}$  : The different between the due date and processing time of machine 2 =  $d - P_{i2}$ ;

$\alpha, \beta$  the weights associated with earliness and tardiness, respectively.

$M_{SL_{i1}}$  : Largest slack for the n jobs on machine 1 =  $\max \{SL_{i1}\}, i \in n$ ;

$M_{SL_{i2}}$  : Largest slack for the n jobs on machine 2 =  $\max \{SL_{i2}\}, i \in n$ .

### III. Training of Neural Networks

The neural network is trained by the data Table (1) which was taken from (Abdelaziz and Baharom, 2011) of which the solution is known, so as to compare between the solutions. , the sequence schedule of this training example is as follow  $J_3, J_2, J_6, J_9$ , for machine 1 and  $J_7, J_4, J_8, J_5, J_1, J_{10}$  for machine 2 with total cost for all machines is 1673 units . This example was use for training purpose of our multi-layer neural network model. The 10 jobs are converted first in to their vector representations by using our 20 unit's vectors and the result of this pre- processing stage is illustrated in table (3)

**Table (1) (10 Jobs and 2 machines)**

jobs	$P_{i1}$	$P_{i2}$	d	$\alpha$	$\beta$	$SL_{i1}$	$SL_{i2}$
1	103	44	96	2	3	-7	52
2	29	71	96	2	3	67	25
3	67	106	96	2	3	29	-10
4	50	41	96	2	3	46	55
5	77	36	96	2	3	19	60
6	62	98	96	2	3	34	-2
7	41	40	96	2	3	55	56
8	58	16	96	2	3	38	80
9	95	99	96	2	3	1	-3
10	103	72	96	2	3	-7	24

**Table (2) Input units for training example**

1	0.28	0.65	0.49	0.75	0.6	0.4	0.56	0.92	1
0.415	0.67	1	0.37	0.34	0.93	0.38	0.15	0.93	0.68
0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
-0.1	1	0.43	0.69	0.28	0.51	0.82	0.57	0.02	0.01
0.65	0.31	-0.13	0.69	0.75	-0.03	0.7	1	-0.03	0.3
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67
0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59
0.52	0.15	0.34	0.25	0.39	0.31	0.21	0.29	0.48	0.52
0.22	0.36	0.53	0.21	0.18	0.49	0.2	0.08	0.5	0.36
0.34	0.1	0.22	0.17	0.26	0.21	0.14	0.19	0.32	0.34
0.15	0.24	0.35	0.14	0.12	0.33	0.13	0.05	0.33	0.24
0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42

1	1	1	1	1	1	1	1	1	1
0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47
0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68

**Table (3) Desired output for training example**

0	0.34	0.2	0	0	0.67	0	0	0.9	0
0.76	0	0	0.34	0.62	0	0.2	0.48	0	0.9

**IV. Numerical Examples**

**Example (1)**

In this example we need to sequencing the five jobs of table (4) with common due date equal 14 units of time on two machines so as the objective function (1) be minimized. This example already done by (J.Bank and Fwerner, 2001), they use heuristic methods and they found that the total cost equals 31 units.

**Table (4) (Five Jobs and Two Machines)**

Jobs	J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>	J <sub>5</sub>
P <sub>i1</sub>	4	5	6	6	6
P <sub>i2</sub>	6	3	7	8	9
$\alpha$	1	1	1	1	1
$\beta$	1	1	1	1	1
Sl <sub>i1</sub>	10	9	8	8	8
Sl <sub>i2</sub>	8	11	7	6	5

First we transfer the data of table (4) into the vector unit's bellows by using the 20 unit's vectors (2.1-2.20)

**Table (5) input Units of Example 1**

0.67	0.83	1	1	1
0.67	0.33	0.78	0.89	1
0.14	0.14	0.14	0.14	0.14
1	0.9	0.8	0.8	0.8
0.73	1	0.64	0.55	0.45
0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.1
0.4	0.5	0.6	0.6	0.6
0.6	0.3	0.7	0.8	0.9
0.4	0.5	0.6	0.6	0.6
0.6	0.3	0.7	0.8	0.9
0.87	0.87	0.87	0.87	0.87
0.73	0.73	0.73	0.73	0.73
0.86	0.86	0.86	0.86	0.86
0.67	0.67	0.67	0.67	0.67
1	1	1	1	1
0.16	0.16	0.16	0.16	0.16
0.09	0.09	0.09	0.09	0.09
0.31	0.31	0.31	0.31	0.31
0.28	0.28	0.28	0.28	0.28

After that we use our trained model of multi layer neural network to solve our problem . We find that our scheduling will be as follows , $J_2, J_3, J_5$ , allocated for machine 1 and  $J_1, J_4$  allocated for machine 2 respectively .The total cost is equals 23 which is better than their cost ( See Table 6)

**Table (6) Out put units of Example 1**

The screenshot shows a window titled "Data: Predicted" with a "Value" label. The data is presented as a 2x6 matrix of numerical values:

	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$
$M_1$	0	0.23149	0.4307	0	0.6124
$M_2$	0.24428	0	0	0.46681	0

**Example (2)**

In this example we use the following data (see **Table (7)**) taken from (Coelho and Mand, 2015) in their work they used hybrid Tabu research and truncated branch method, they get cost equals 94 for machine 1 and 186 for machine 2, so their total cost equals 280.

**Table (7) (Data for Example 2)**

Jobs	$M_1$	$M_2$	d	$\alpha$	$\beta$	$SL_{i1}$	$SL_{i2}$
$J_1$	79	45	96	2	3	17	51
$J_2$	51	78	96	2	3	45	18
$J_3$	32	27	96	2	3	64	69
$J_4$	43	90	96	2	3	53	6

We transfer the data of table (7) to table (8) by our 20 unit's vector.

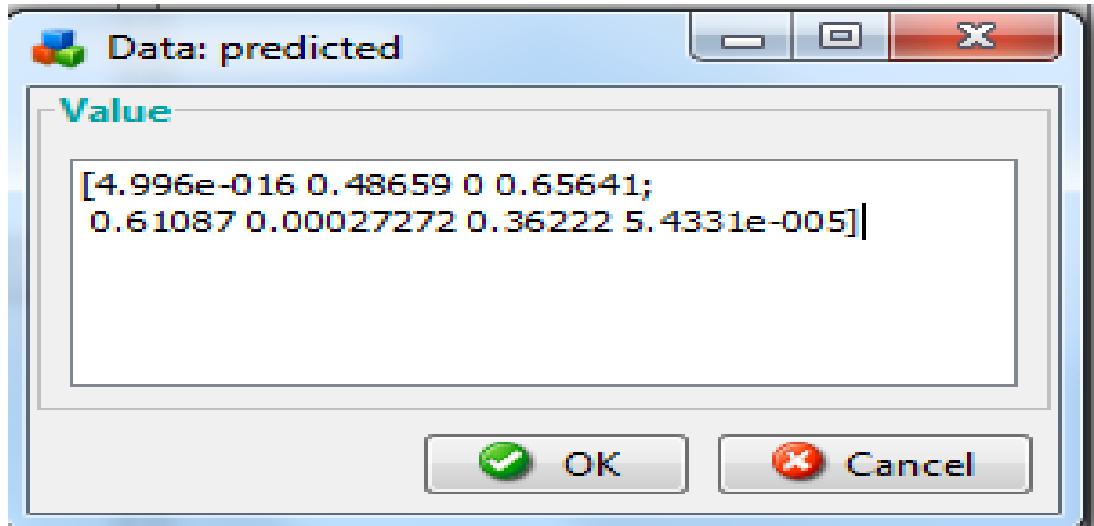
**Table (8) Input units of our example**

1	0.645	0.405	0.544
0.5	0.867	0.3	1
0.96	0.96	0.96	0.96
0.266	0.733	1	0.828
0.739	0.261	1	0.087
0.2	0.2	0.2	0.2
0.3	0.3	0.3	0.3
0.649	0.649	0.649	0.649
0.667	0.667	0.667	0.667
0.699	0.699	0.699	0.699
0.522	0.522	0.522	0.522
1	1	1	1
0.115	0.115	0.115	0.115
0.339	0.339	0.339	0.339
0.151	0.151	0.151	0.151
0.388	0.388	0.388	0.388
0.176	0.176	0.176	0.176
0.42	0.42	0.42	0.42
0.49	0.49	0.49	0.49
0.7	0.7	0.7	0.7

. We find that  $J_2, J_4$  allocated for machine 1 and  $J_3, J_1$  allocated for machine 2 respectively (See Table (9)). We find that the cost of machine 1 equal 94 and for machine 2 equal 150. Then the total cost equal 244, which is better than their cost.

**Table (9) Out puts units**

Machine 1	0	0.48659	0	0.6564
Machine 2	0.61087	0	0.36222	0



**V.Conclusion**

The unrelated parallel machine with common due dates has been studied. The objective was to find an optimal scheduling of jobs on machines, which minimizes a total cost function containing earliness and tardiness costs. We developed a new model of multi layer neural network base on unit's vector variables to solve this problem. It was found that our new model gives us optimal or near optimal solution. Further investigation can be carried out for uniform parallel machines one.

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