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Optimal Seeding And Self-Reproduction From A Mathematical Point of View.

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Abstract: *P. Kabamba developed generation theory as a tool for studying self-reproducing systems. We provide an alternative definition of a generation system and give a complete solution to the problem of finding optimal seeds for a finite self-replicating system. We also exhibit examples illustrating a connection between self-replication and fixed-point theory.*
Keywords: *Algorithm, directed labeled graph, fixed point, generation theory, optimization, seed, self-replication, space robotics.*

I. Introduction

Recent research for space exploration, such as [3], considers creating robot colonies on extraterrestrial planets for mining and manufacturing. However, sending such colonies through space is expensive [14], so it would be efficient to transport just one colony capable of self-replication. Once such a colony is designed, a further effort should be made to identify its optimal seeds, where a seed is a subset of the colony capable of manufacturing the whole colony. The optimization parameters might include the number of robots, the mass of robots, the cost of building robots on an extraterrestrial planet versus shipping them there, the time needed to manufacture robots at the destination, etc. Menezes and Kabamba gave a background and several algorithms for determining optimal seeds for some special cases of such colonies in [7], [8], and [9]. They also gave a lengthy discussion of the merits of the problem and an extensive reference list in [9]. In this paper, which originated in [5], we provide an alternative definition of a generation system, show how this definition simplifies and allows one to solve the problem of finding optimal seeds for a finite self-replicating system, and exhibit a connection between self-replication and fixed-point theory.

The study of self-reproducing systems was initiated by John von Neumann, who discussed cellular automata capable of building other cellular automata in [10]. The subject proved to be a fruitful ground for research. A detailed review of this field with many additional references is contained in [13], [11], [9], and [4]. Generation theory was introduced by Kabamba in [6] to analyze self-reproducing systems. In the framework of generation theory, the entities that can potentially reproduce are called *machines*, regardless of their physical nature (e.g., robots, microbes, or lines of computer code). Reproduction is achieved by the action of machines on available resources, producing an outcome that may or may not be a machine itself.

The remainder of this paper is as follows. We present an alternate definition of a generation system, formulate and solve the problem of finding optimal seeds for a finite self-replicating system, and discuss examples demonstrating connections between self-replication and fixed-point theory.

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Background in Generation Systems.

Menezes and Kabamba in [9] define a generation system as a quadruple (U, M, R, G), where

- U is a universal set that contains machines, resources, and outcomes of attempts at reproduction,
- $M \subseteq U$ is a set of machines.
- $R \subseteq U$ is a set of resources that can be used for reproduction, each resource is an ordered list.
- $G : M \times R \to U$ is a generation function that maps a machine and a resource ordered list into an element in the universal set.

In this paper we would like to use an alternative definition of a generation system. First, note that an outcome of generation might not be a machine, however, we are interested only in outcomes which are machines, so we do not need to introduce the set U. If the outcome of generation is not a machine, we can define it to be the empty set. Hence a generation system becomes a triple (M, R, G). Also there is no need for a resource to be an ordered list. We consider a more general generation function, which maps a subset of the set of machines and a subset of the set of resources into an outcome. We consider only outcomes which are subsets of the set of machines. An outcome might be the empty set. This generalization of generation function is motivated by the observation that several robots might have to work simultaneously in order to produce a new robot, and a family of robots working together might produce more than one robot. Hence a generation function is a function of the form

$G : 2^M \times 2^R \to 2^M$, where 2^S denotes the set of all subsets of the set S.

We allow cannibalization, i.e. certain machines after fulfilling their function can be taken apart for parts or materials, so they become resources. Hence it is possible that $M \cap R \neq \emptyset$. However, we would like to avoid the empty generation situation when a resource becomes a machine without any transformation, i.e. if a machine m_0 is generated by a set of machines $M_0 \subset M$ acting on a set of resources $R_0 \subset R$, so $m_0 \in G(M_0, R_0)$, we require that $m_0 \notin R_0$. Menezes and Kabamba in [9] call such generation systems *weakly regular*.

A set of machines $M_0 \subseteq M$ is called self-replicating if there exists a set of resources $R_0 \subseteq R$ such that $G(M_0, R_0) = M_0$.

Formulation of the Optimal Seeding Problem

The general problem of seeding of a space colony can be put in the following simple form, where it becomes a transportation problem. We need to deliver a set M of machines to a certain destination. We assume that the set M is self-replicating. The set M exists at the base, so one solution of the problem is to load M at the base on some carrier and to transport it to the destination. However, transportation is expensive, and we have a limited budget, so we would like to transport as little as possible. Note that if we are interested in a mining colony, all of the resources R for self-replication of M should be available at the destination except, possibly, for some machines in M which become resources after fulfilling their tasks. For all practical purposes we can assume that both sets M and R are finite. Also assume that for any subsets $M' \subseteq M$ and $R' \subseteq R$ we are given the generation function $G(M', R') = M'' \subseteq M$. In addition, we are given a cost function $T(G(M', R'))$, which might be vector-valued. $T(G)$ provides the cost of running the generation function. The components of the function $T(G)$ might be monetary cost of the manufacturing process, time required for the process, the energy consumption, etc.

First, we would like to find all seeds of M, i.e. subsets of M capable of generating M. Seeds exist, because we assumed that M is self-replicating, so M is a seed of itself. In general, M might be the only seed of itself, however, we hope to find a number of different seeds. Moreover, we would like to find a seed optimal with respect to $T(G)$, and transport it to the destination. If there are several optimal seeds, we can introduce additional components to the function $T(G)$ to narrow the choice, or to choose the optimal seed randomly out of a set of several candidates.

A solution. We keep the assumption that $M \cup R$ is finite. Construct the following directed labeled graph Γ. The set of vertices of Γ is 2^M. Two vertices M_1 and M_2 are connected by a directed edge if there exists a set of resources $R_1 \subseteq R$ such that $M_2 = G(M_1, R_1)$ and $R_1 \cap M_2 = \emptyset$. An edge of Γ connecting vertex M_1 to M_2 is labeled by the function $T(G(M_1, R_1))$.

By construction, the connected component Γ_M of the vertex M is the set of all seeds of the set M.

In order to find a seed, optimal with respect to a component of the function T, we consider the set of all simple (without self-intersections) paths, (using any standard algorithm for the purpose [1]), connecting a vertex in Γ_M to M, sum the component of T along the path, and create a list of vertices of Γ_M sorted by the value of this component.

Self-replication and Fixed-point Theory.

Assume that the set of resources R is a singleton $R = \{r\}$. In this case there is no need to mention r explicitly in the definition of the generation function, so we can write $G(M_0, r) = G(M_0)$ for $M_0 \subseteq M$. If $M_0 = \{m\}$ is a singleton, by definition, the machine m is self-replicating$\Leftrightarrow G(m) = m$, which means that m is a fixed-point of the function G. So the study of self-replicating machines in systems with a single resource is the study of fixed points of functions, which is a classical rich and well-developed subject [2]. Consider several examples.

Example 1. *If G is a linear operator and M is a vector space, then 0 is always a fixed point of G. Also $G(m) = m \Leftrightarrow m$ is an eigenvector of G corresponding to the eigenvalue 1, so the set of self-replicating machines is a linear subspace of M.*

Example 2. *Let M be the set of all polynomials of the single variable t with real coefficients, and let $G(m(t)) = \frac{d}{dt}m(t)$. In this case G is a linear operator which does not have eigenvalue 1, and M is a vector space, so the only fixed point of G is the constant zero polynomial $m(t) = 0$. This generation system has a unique self-replicating machine, namely the constant zero polynomial.*

Example 3. *Let M be the set of all analytic functions of the single variable t with real coefficients, and let $G(m(t)) = \frac{d}{dt}m(t)$. In this case G is a linear operator which has an eigenvalue 1, and M is a vector space, so the fixed points of G are the functions $\{Ae^t, A \in R\}$. This system has infinitely many (even uncountably many) self-replicating machines, namely all the exponential functions $\{Ae^t, A \in R\}$.*

Example 4. *Let M be a closed n-dimensional ball in Euclidean space. Brouwer's fixed-point theorem states that any continuous map $G : M \to M$ has at least one fixed point.*

Example 5. *Let (M, ρ) be a metric space. A function G is called a contraction if there exists a constant $0 < \lambda < 1$ such that for any pair of points m_1 and m_2 in M, $\rho(G(m_1), G(m_2)) \leq \lambda\rho(m_1, m_2)$. The contraction mapping theorem states that if M is complete (for example, any closed and bounded subset of Euclidean space is complete) and G is a contraction, then G always has one and only one fixed point.*

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