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Quantum Cellular Automata Circuit Mixed Strategy Game Theoretic Optimization

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Abstract : *Gates and circuits made of Quantum Cellular Automata are subject to manufacturing errors due to imprecision of measurement. Our work examines the use of game theory optimization to correct Gaussian measurement error in Quantum Cellular Automata studies. We examine the use of game theory as a possible tool of optimization, specifically that of mixed strategy games. Simulation results show improvement in standard deviation of noise for mixed strategy games, as well as improvement in average polarity of gate.*
Keywords: *Game theory, Mixed strategy games, Noise removal, Optimization, Quantum Cellular Automata*

I. Introduction

In a Quantum Cellular Automata (QCA) gate, Coulomb interaction works to send data along the path of QCA aligned in a pattern to form a circuit. Data is transmitted along this wire or is transformed by the QCA gate. However, due to errors in manufacturing these gates and wires do not always work as planned. Sometimes the polarity of data is reversed, such as sending a binary 0 instead of a 1. The physics of data transfer involves Coulomb interaction between positively and negatively charged QCA. These Quantum Cellular Automata are made of quantum dots, which are able to hold charge. They are larger than atoms and able to hold positive or negative magnetic charge and are what make the QCA function. Work at the University of Notre Dame first showed the possibility of using Quantum Cellular Automata as replacements for VLSI circuits [1], [2], [3], [4], [5]. One of the first things identified and still studied is the problem with manufacturing at such a small scale. Current manufacturing limits what can be done, as there are imprecision errors that show up in the actual QCA circuit. Our work examines some of what has been done in the field to reduce errors and suggests the use of mixed strategy game theoretic optimization for measurement.

II. Background

QCA transmit binary data based on their magnetic charge. Due to the simplicity of their physics, they are extremely fast and powerful when functional. They work by polarization of tiny pieces of metal within them, called quantum dots. These are small, only a few atoms wide, and dominated by quantum instead of Newtonian physics. However, being larger than atoms they are easier to control yet may be manufactured. QCA research has grown due to the shrinking size circuits. At only a few atoms wide, physical limitations cause errors at this scale. The work in [6] details examples of these errors. Vertical imprecision of one-half a cell width causes error in cell interaction, leading to the opposite logical effect for any gate [7], [6], [8]. The authors in [9] used a neural network model of a QCA gate to identify device uncertainty, while [10], [11] used Bayesian systems to calculate correct output probability of defective cells. They did not examine Gaussian error or game-theoretic decision processes for calculation. Recent work at the University of Notre Dame has been done using probabilistic transfer matrices [12].

Each QCA typically has four quantum dots, with two of the dots possessing magnetic charge. Electron charge is controlled by potential barriers. A charge of binary zero is achieved by electrons being negatively charged; if positively charged, then the QCA contains a binary 1. Data moves by Coulomb interaction, and no electrical currents are needed; thus there is great possibility for powerful computation. Gates such as the block majority gate or AND-gate have been created; wires have also, as in (Fig. 1):

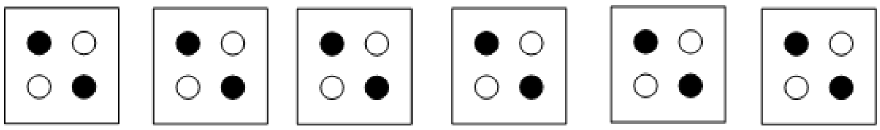

Figure 1: QCA wire

Here in (Fig. 1) we see data of binary 0 transmitted along the wire. The device cell takes the polarity of the input cell on the left and transmits this to the output cell on the right side. It functions because all wires and gates must seek their lowest energy state, or ground state [3]. Gates such as the AND-gate can be represented by fixing the binary value of one of the input cells to 0, representing negative charge.

The use of games whereby each player is aware of the other player seeks to play the game to optimize his own reward, or utility over the possible actions was first proposed by Von Neumann and Morganstern [13]. In an environment where each player, or intelligent agent, seeks to maximize reward in a probabilistic environment, there exists the possibility that by doing a greedy search and maximization of the environment using these more traditional processes that seek individual agent maximization will lead to a suboptimal decision. As such, there exists the possibility that an outcome chosen by agent is suboptimal. An outcome that is optimal in game theory, however, is maximized for all agents where all know that other agents will act to maximize their own reward versus the other agents. This adds an additional probabilistic vector of decisions that must be made, which becomes especially interesting in situations where an agent makes decisions based on a probability distribution over the actions; this is known as a mixed strategy. In [14] the authors use a modified game theoretic rule to allow for agents to collaborate to increase the possibility of maximizing their individual reward in a probabilistic environment for pure or mixed strategies. In fact, it was found that groups of three or more intelligent agents are much more likely to maximize group reward when agents are free to leave or join the group.

III. Methodology

We examine the use of evaluation and correction of QCA gate measurement with Gaussian error over a distribution of values representing a QCA 5x5 block gate. Each QCA that makes up the gate acts as an intelligent agent or player that seeks to maximize utility versus other players, playing a probability distribution over the actions. Utility is determined iteratively until convergence, with agents seeking to maximize utility by playing a probability distribution over the actions. We examined 20%, 30%, and 50% mixed strategy probability distribution for an agent playing a suboptimal strategy. We believe that the use of a game-theoretic adaptive learning algorithm that seeks to maximize reward in an environment that works with unknown existence of Gaussian error and unknown action by the other intelligent agents could be effective. Our proposal will show the model of a mixed strategy game theoretic solution for homogenous players that can be used to improve evaluation of a QCA circuit. Our algorithm removes noise from a signal given prior probabilities; each cell and relative to its neighbors is examined. Once error is removed, any gate with average greater than 0.5 is positive charge. Using this method should most likely find ground state; it exists as an analysis tool for determining actual output of a QCA circuit that possesses fabrication errors that affect gate polarity. We represent error as a Gaussian distribution over the QCA gate. Further complicating evaluation of the gate is due to the physics of electromagnetic charge; charge is never discretely negative or positive, but instead it is a continuous value over (-1, +1). Polarity is affected by both individual and neighbor polarity. For node:

$$x_{i,j} \in X \tag{1}$$

Set X is a distribution of probability values. Set of neighbors of x_{ij}:

$$\{x_{i,j}\} \in X \tag{2}$$

Probability of error is a probability distribution over X used to calculate likelihood of certainty:

$$\{x'_{i,j}\} \in X' \tag{3}$$

Distribution of error is given by a Gaussian noise distribution:

$$\frac{e^{-\left(x\prime-\overline{\{x\prime\}}\right)^2 / 2(\sigma(X))^2}}{\sigma(X)\sqrt{2\pi}} \tag{4}$$

Utility is probability of likelihood minus cost of error:

$$u(x')\left(\frac{P(x\,|x\prime)P(x\prime)}{P(x)} - \frac{e^{-\left(x\prime-\overline{\{x\prime\}}\right)^2 / 2(\sigma(X))^2}}{\sigma(X)\sqrt{2\pi}}\right) \tag{5}$$

IV. Results

We modeled the gate with Gaussian noise added to each representative cell. Groups of agents are of size four. We examined average and standard deviation for mixed strategies in simulated gates with Gaussian noise; tests were repeated for 1000 circuits iterations, each with 100 iterations of game theoretic optimization as well as unique Gaussian noise representing QCA error. Gaussian noise had mean of 0.25 and standard deviation of 0.15 for variance of measured charge between binary 0 (-1 charge) and 1 (+1 charge). Mixed strategy agent exploration of 20%, 30%, and 50% suboptimal strategy was used for respective probability of an agent playing a suboptimal strategy for the probability distribution over the actions. The gate is transmitting a positive value.

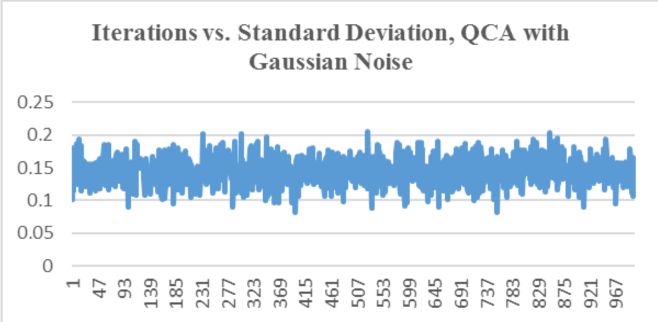

Figure 2: Standard deviation of Gaussian noise of a 5x5 majority gate QCA circuit. 1000 circuits were created in the above iterations.

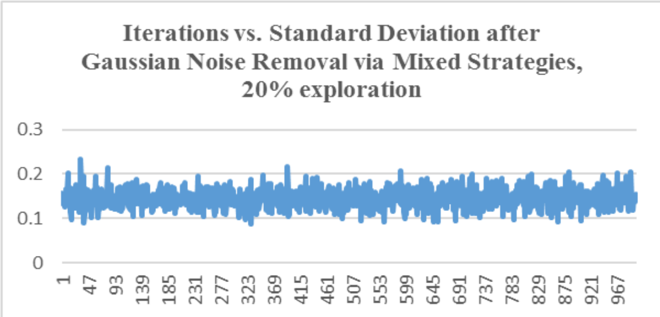

Figure 3: The same 1000 QCA circuits and Gaussian noise from (Fig. 2), after noise removal using mixed strategies for 20% probability of an agent playing a suboptimal strategy.

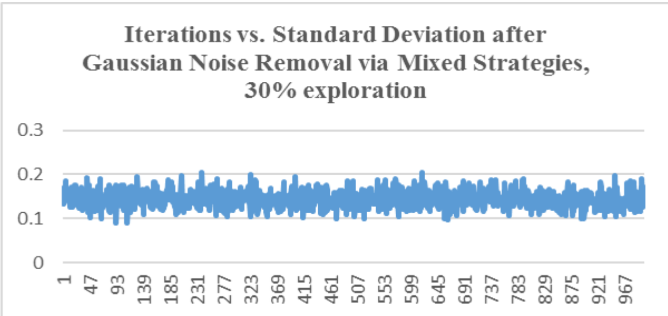

Figure 4: The same 1000 QCA circuits and Gaussian noise from (Fig. 2), after noise removal using mixed strategies for 30% probability of an agent playing a suboptimal strategy.

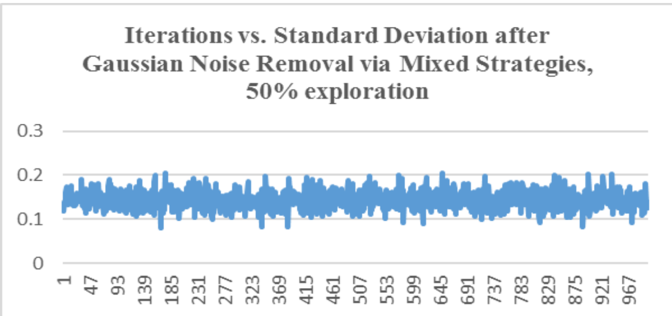

Figure 5: The same 1000 QCA circuits and Gaussian noise from (Fig. 2), after noise removal using mixed strategies for 50% probability of an agent playing a suboptimal strategy.

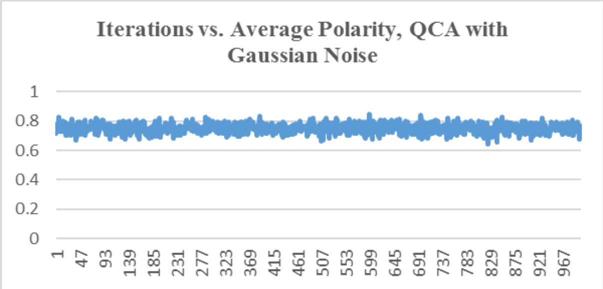

Figure 6: Again the same QCA circuits as in (Fig. 2), but showing average QCA gate polarity for the 1000 circuit models.

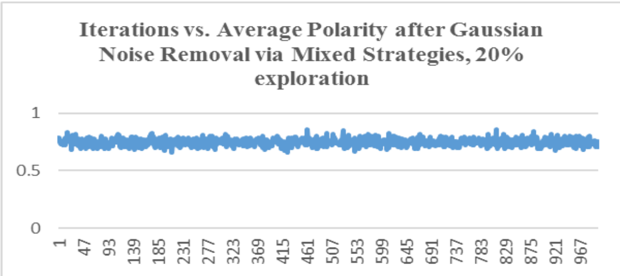

Figure 7: The same average gate polarities for the 1000 QCA circuits after noise removal for (Fig. 3), using mixed strategies for 20% probability of an agent playing a suboptimal strategy.

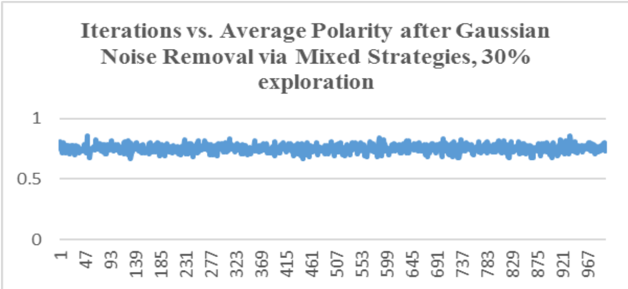

Figure 8: The same average gate polarities for the 1000 QCA circuits after noise removal for (Fig. 4), using mixed strategies for 30% probability of an agent playing a suboptimal strategy.

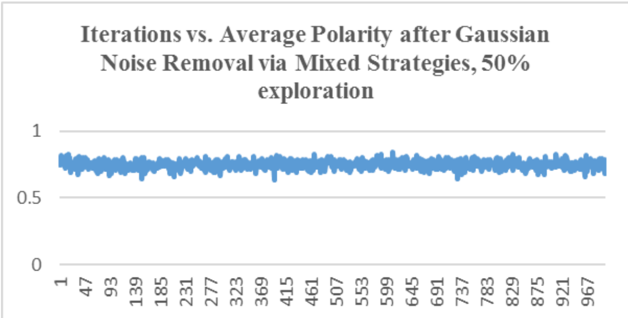

Figure 9: The same average gate polarities for the 1000 QCA circuits after noise removal for (Fig. 5), using mixed strategies for 40% probability of an agent playing a suboptimal strategy.

V. Conclusion

Our solution builds upon previous work using game theoretic optimization and Bayes classification. We were surprised by the average polarity narrowing for 20% and 30% mixed strategies being close to one another. The 50% mixed strategy results being close to the original with Gaussian noise was unsurprising. We did not expect that after noise removal for 20% probability of playing a suboptimal strategy, the range for standard deviation of Gaussian noise fluctuated to higher and lower values for the QCA gates than before noise removal; this strongly indicates that mixed strategies can be more than suboptimal in isolated cases. However overall the average bandwidth of Gaussian noise decreased. We were also surprised that the 30% mixed strategies had fewer high fluctuations than the 20% mixed strategies, giving a better overall average improvement. The 50% probability distribution over the actions, being as close at it is to before Gaussian noise removal, was also unsurprising and correlated to the results for average Gaussian noise. Results are indicative of an optimal percentage for probability distribution over the actions, showing improvement for exploration using suboptimal strategies.

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