

Comparative Study between DCT and Wavelet Transform Based Image Compression Algorithm

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Abstract: Discrete cosine transform emerged as a popular mathematical tool in past decade and widely used in image compression algorithm due to its high energy compaction capacity. But this energy compaction has some limitation in blocking artifacts that overcome by another mathematical tool wavelet transform based compression which have both frequency and spatial component study at same time. In this paper we study a comparison between DCT based image compression and wavelet based image compression using CDF9/7 wavelet as used in JPEG-2000 Standards based on common encoding scheme known as Huffman encoding for both. We also analyzed a trend based on different level of wavelet transform on image size and image quality based on mean square error.

Keywords: Wavelet transform, Image Compression, Discrete Cosine Transform, Fast wavelet algorithm, Huffman encoding.

I. Introduction

Image compression has a number of steps including conversion of analog signals into digital form. The first step involve in this process is sampling. The points we get after digitization of a continuous image function are called sampling points. These sampling points are ordered in a plane in a grid manner. Therefore, we can call our digital image a geometrical structure, commonly a matrix. After sampling even that the pixel values contain real values. When we change these real values to digital values, this transition and its digital equivalent is called quantization. The number of levels of quantization should be high so that the boundaries in the image can be easily distinguished and then only the digital image can be closely approximated to original continuous image function. The third step is transform step using any popular transform technique. So basic steps in image compression includes Sampling, Quantization and Transform step [1][2].

II. Preliminaries

1.1 Discrete Cosine Transform

One Dimensional cosine transform is a function which is a linear combination of cosines with growing frequencies which are called basis functions. DCT operates on function samples of finite length and to perform DCT expansion, periodic extension of this function is needed. DCT converts finite set of points into a sum of cosine functions oscillating at different frequencies. DCT uses only real numbers with even symmetry, where in some variations the input and/or output data are shifted by half a sample. There are eight standard variations of DCT, of which four are common. The most common variation of DCT is the type-II DCT, its inverse DCT is called type-III DCT. In signal and image processing, DCT-II is used. Because of its strong energy compaction property, it is used in lossy data compression. Energy compaction means most of the signal information is concentrated in a few low-frequency components of the DCT. DCTs are also widely used in solving partial differential equations by spectral methods [1]. DCT II can be given by

$$y_k = \sum_{n=0}^{N-1} x_n \cos\left(\frac{\pi(n+1/2)k}{N}\right)$$

1.2 Wavelets

Mathematically is wavelet if a is scaling and b is translation parameter [3]

$$\psi_{a,b}(t) = \psi\left(\frac{t-b}{a}\right) \frac{1}{\sqrt{|a|}} \quad \text{And satisfy following admissibility condition.}$$

$$C_\psi = 2\pi \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$$

1.3 Discrete Wavelet

We can change continuous wavelet $\psi_{a,b}$ as discussed above into discrete wavelet $\psi_{m,n}$ by changing scaling coefficient $a = a_0^m$ such that $a \neq 0, 1$ and $b = ab_0 a_0^m$ such that $b \neq 0$. Then we get discrete wavelet as [4]

$$\psi_{(m,n)}(t) = a_0^{(-m/2)} \psi(a_0^{-m} t - nb_0)$$

1.4 Discrete Wavelet Transform of Digital Images

Let scaling function be $\phi(x)$ and wavelet function be $\psi(x)$ having translation and dilation property stated by equation (1),

$$\phi(x) = \sum_n h_\phi(n) 2^{-1/2} \phi(2x - n),$$

$$\psi(x) = \sum_n h_\psi(n) 2^{-1/2} \psi(2x - n).$$

Then four component of image data can be generated by convolution of image data $h_\psi(-n)$ followed by $h_\psi(-m)$, $h_\psi(-n)$ followed by $h_\phi(-m)$, $h_\phi(-n)$ followed by $h_\psi(-m)$, $h_\phi(-n)$ followed by $h_\phi(-m)$ and each step followed by downsampling by 2 which generate four lower resolution component of same digital image [5]. It can be viewed in following diagram [3].

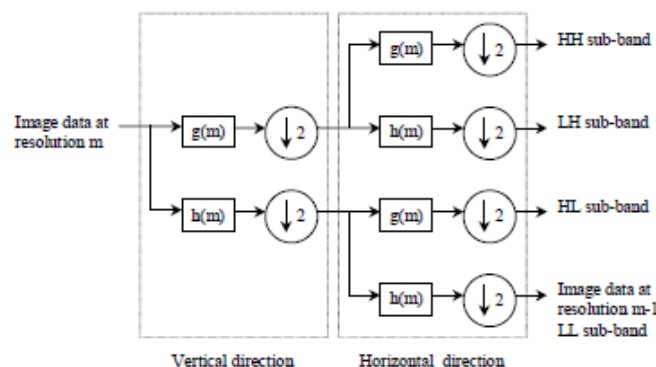


Figure 1 DWT Process in Images

III. Methodology

We have taken DCT and Wavelet transform based image compression based on common encoding technique that is Huffman encoding. Wavelet chosen for this case is CDF9/7 as used by JPEG2000 committee [6]. The result is analyzed on the basis of compression ratio and mean square error and also trend is analysed on basis of different levels of wavelet transform. Huffman encoding is used in both algorithm to get symmetry in the compression process so that efficient comparison will occur only on basis of DCT and wavelet. Huffman codes are specific codes which are called prefix codes. In prefix codes, the bit code representing some code symbol is never a prefix of the bit code representing some other symbol. The bit codes with shorter lengths are used to represent most frequent symbols and bit codes with longer lengths are used to represent less frequent symbols [7].

IV. Experiments and Results

In this experiment we have taken four images namely brain image, livingroom, woman and cameraman and analyzed respectively decompressed images via DCT, Wavelet 1-level, Wavelet 2-level, Wavelet 3-level transform. We find error image that is pixel difference of original and decompressed one in each case along with error histogram which is shown respectively below.

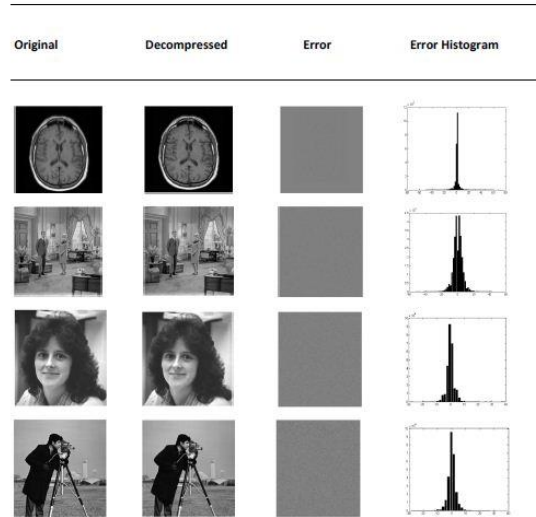


Figure 2 Analysis of Original Image with Decompressed Image using DCT

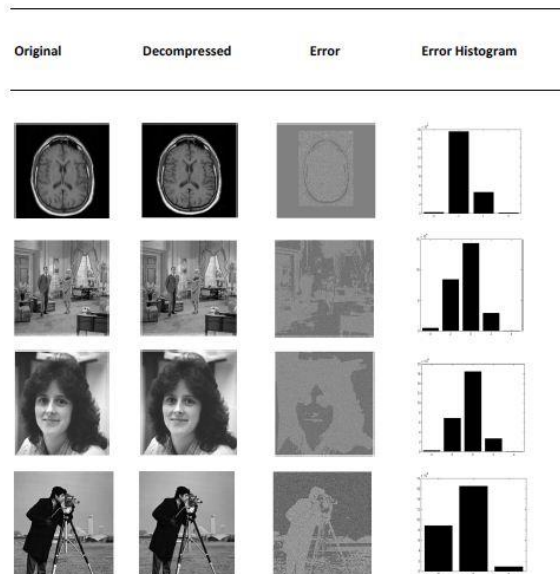


Figure 3: Analysis of Original Image with Decompressed Image using Wavelet 1-Level Transform

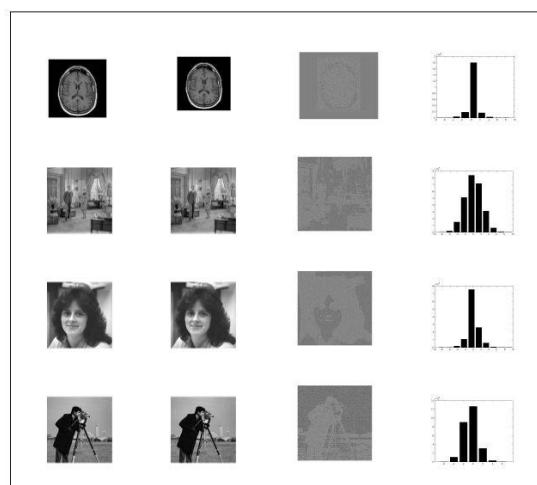


Figure 4: Analysis of Original Image with Decompressed using Wavelet 2-Level transform.

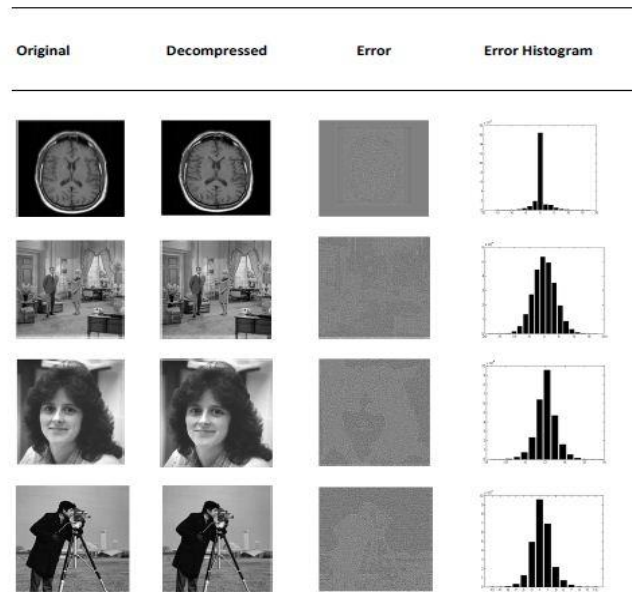


Figure 5: Analysis of Original Image with Decompressed Image Using Wavelet3-Level Transform

We get following mathematical result of Mean Square Error (MSE) and compressed size compression ratio (CR) for all above cases which is shown in tabular form below.

Table 1: Compression Ratio and MSE using DCT

Image (Actual size)	Compressed size	Decompressed size	CR	MSE
Brain (224540)	16398	224540	14.6931	4.3488
Livingroom(262144)	32680	262144	8.3052	5.4872
Woman (262144)	17642	262144	15.8625	2.6251
Cameraman(262144)	22158	262144	12.4581	2.9017

Table 2: Compression Ratio and MSE using Level 1 DWT

Image (Actual size)	Compressed size	Decompressed size	CR	MSE
Brain (224540)	64066	224540	3.5740	0.6627
Livingroom(262144)	162974	262144	1.6208	1.2476
Woman (262144)	117768	262144	2.2496	1.1274
Cameraman(262144)	87082	262144	3.0538	0.9060

Table 3: Compression Ratio and MSE using Level 2 DWT

Image (Actual size)	Compressed size	Decompressed size	CR	MSE
Brain (224540)	38250	224540	6.0670	1.1537
Livingroom(262144)	101230	262144	2.6217	2.3430
Woman (262144)	50816	262144	5.2877	1.8128
Cameraman(262144)	47848	262144	5.6244	1.4772

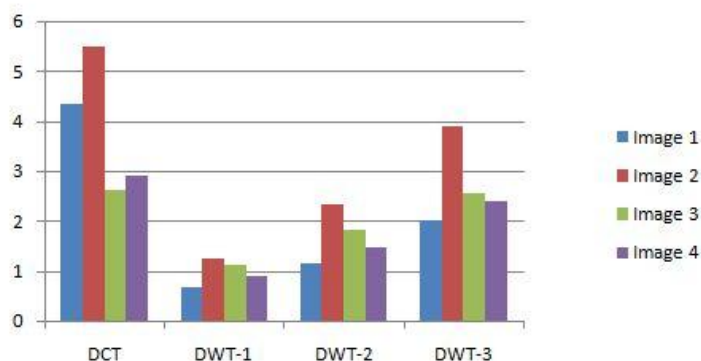
Table 4: Compression Ratio and MSE using 3 Level DWT

Image (Actual size)	Compressed size	Decompressed size	CR	MSE
Brain (224540)	24598	224540	9.6130	1.9971
Livingroom(262144)	57768	262144	4.6374	3.9080
Woman (262144)	21406	262144	12.9993	2.5668
Cameraman(262144)	28666	262144	9.5582	2.3972

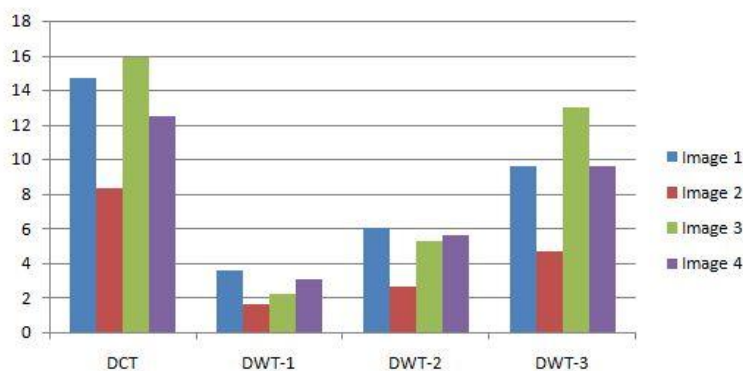
V. Conclusion

By doing experiments on several images, it can be observed that in DWT that we get low Mean Square Error on average compression ratio. If we increase the compression ratio, MSE is greater in comparison to DCT which is because in DWT the range of error is wide but in DCT error range is not that much wide. But in the higher levels of DWT, we get a quite good compression ratio and low mean square error too. One disadvantage of DCT is that correlation across the block is not considered in this case and therefore it cannot be decorrelated. It results in blocking artifacts which is not the case in DWT. It means DWT enables high compression ratios while maintaining good visual quality.

By given two bar charts it is clear that after going to next level in wavelet transform compression ratio increases however MSE up to wavelet 3-level transform found to be less than DCT based compression. Also it is clear that Huffman algorithm is more efficient in DCT than DWT.



Comparison of MSE in DCT, DWT-1, DWT-2, DWT-3



Comparison of Compression ratio in DCT, DWT-1, DWT-2, DWT-3

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