Modeling Of Transfer Function Characteristic of Rlc-Circuit

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Abstract: *The performances of a transfer function characteristic of RLC-circuit is investigated and modeled in this paper. The ever increasing demand for electronics has led to the continuous search for the most readily available means of providing better performance of the system. The method of analyzing RLC circuit is never constant and since the resistor (R), inductor (L), and the capacitor (C) are use in every electronic system, a proper understanding of system is necessary to know what happen to system when any of parameter is alter. The design methodology employed in this work involves: the modeling of the equation of the RLC circuit, developing appropriate algorithms to imitate the real life behavior of an RLC circuit, Appling matlab codes in the m-file and carrying out analysis of the designed RLC circuit in the Matlab GUI. Result obtained showed good proper behavior of the system when the input parameters were varied.*

Keywords: *RLC-circuit, matlab, gui, transfer function*

I.   Introduction

RLC circuits in the past and present have been source of error and inaccuracy in many major applications causing poor performance of electrical product and prototype. Recent researches in regard to this aspect seen to provide opportunities in improving the RLC circuit simulation for advancement in technology, it have been analyze in several way, that is separately as resistance (R), inductance (L) and capacitance(C) and also have been analyze together as RLC circuit, making it more confusing to study independently by student, without an instructor and the display of it's transfer function characteristics in the time domain or frequency domain so that one can control the system in order to achieve the desire output.

The transfer function is an important factor to consider in the design of any system because with the help of the transfer function one can describes the behavior of the output as a function of the input frequency. Therefore it's easy for us to control the system to achieve the output we wanted. It helps us decide the output of the system for every inputs, and to decide if the system is stable or not, to finds out the resonance frequencies of the system, to optimize the system and to determine system behavior with all possible inputs. In other hand, modeling is the process of producing a model, that is a representation of the construction and working of propose system or existing it help in evaluation of the system.

The analysis of this component always pose a problem where wrong analysis result to poor performance of the system after long time and capital have been lavish on the design, or doubling of a component not necessary in the design due to uncertainty when analyzing leading to increase in cost. The big question is how can these be avoided or minimized? In this work, a much dimensional solution is proposed by developing a GUI in matlab/guide.

The objective of this paper is to develop and model a transfer function characteristics of an RLC circuits in which one could be able to select capacitor, inductor and resistor used in design process in accordance to the required specification, running analysis with easy having no difficulty in calculation of the parameters and to ensue adequate safety and good performance of electrical circuit design and prototype. The first step is the modeling of the equation of the RLC circuit. The second step is the designing of the RLC circuit model using the equation using matlab/Guide. The final step is carrying out analysis of the designed RLC circuit.

Jaradat (2006) carried out analysis on RLC circuit using Matlab, maxima and Pspice, the analysis was carried out on four circuits namely; DC, AC, Transient, and Frequency Response. The purpose of the analysis was to enable users understand the behavior of RLC circuit under varying component values. The result obtain were good though the GUI of the application lacked some input features.

Mathwork (2007) presented 'Analyzing the Response of an RLC Circuit'. It shows how to use the Control System Toolbox functions to analyze the time and frequency responses of common RLC circuits as a function of their physical parameters, analyzing circuit configurations such as low-pass and high-pass RLC networks. The work was limited to the fact that it cannot analyze angular resonance, phase margin and also damping factor.

in their project work at the university of Port Harcourt, presented 'modeling and simulation of transfer function characteristics of RLC circuit' as an advancement of modeling and simulation of transfer function characteristic of an RLC circuit, model with simulink parameter and simulation enhancer, presented in graphical user interface (GUI), For better understanding and accuracy of electrical product or prototype (Karris, 2003).

II.   Theoretical Background

THE SERIES RLC CIRCUIT AND STEP RESPONSE: Series RLC circuits are classed as second-order circuits because they contain two energy storage elements, an inductance L and a capacitance C. Consider the RLC circuit Fig.1.
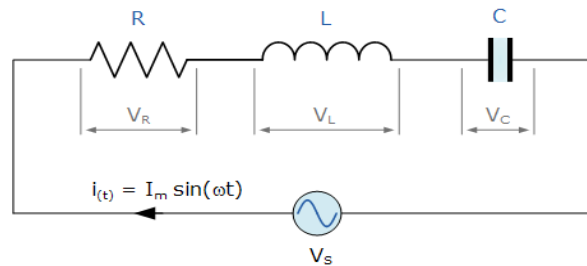

Fig. 1: series RLC circuit (http://www.electronics-tutorials)

Transfer Function Of Rlc Circuit:

A series circuit containing R, L, and C is in **resonance** when the current in the circuit is in phase with the total voltage across the circuit. Depending on the particular values of R, L, and C, resonance occurs at one distinct frequency. Because of its distinct frequency characteristics, the series resonant circuit is one of the most important frequency selective circuits.

An important consideration when designing an RLC circuit is the non ideal nature of the reactive components. Real capacitors closely approximate perfect capacitors so we may neglect the parallel resistance associated with D. Real inductors, however, have a small series resistance which is shown in the circuit diagram as **r**. This cannot normally be neglected since the Q of real inductors is not infinitely large.

The transfer function for this network, V_{out} / V_{in}, which we will call F is, by inspection:

$$F = \frac{R}{(r+R)+j\left(\omega L - \frac{1}{\omega C}\right)}$$

This equation is approximate since r is itself a function of frequency. However, the approximation is fairly good and measured transfer functions do not differ much from it for reasonable ranges of frequency. F has both a phase and a magnitude as a function of frequency. Since the circuit is a series resonant circuit, at resonance, the inductive and capacitive reactances have equal values but opposite signs so the imaginary term in the denominator goes to zero and the value of F just becomes R / r + R.

The graph below shows a series of plots of the transfer function for a series RLC circuit where the inductance had a value of 1.0 mH, the capacitor was 25.3 uF and the series resistor across the output, R, was 1.0 ohm. The three curves are for inductors with three different Q values: 100, 10 and 1. As you can see, the highest Q circuit had the least loss and the narrowest pass band. The pass band is defined as the difference, in Hz, between the two frequencies at which the F is down 3 dB from its peak value. The lowest value of Q resulted in the greatest loss and the broadest pass band.

There is an approximate linear relation between circuit Q and bandwidth which is:

Q = $\square_o$ / $\square$ $\square$

Where $\square$ $\square$is the bandwidth between -3 dB points as previously described.
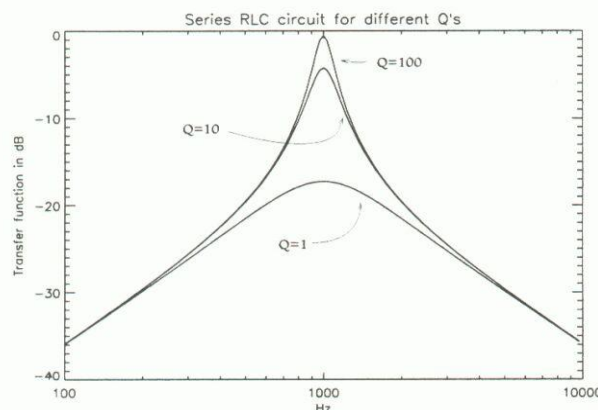

Fig.2:series of plots of the transfer function for a series RLC circuit

Note that the overall Q Fig.2 of the circuit must also take into account the value of R, the output series resistance, since it is part of the circuit. That is, for the circuit as a whole, the total resistance is r+R, not just r. So:

Circuit Q = □ L / (r+R)

At resonance, $X_L. = X_C$, and the resonant frequency is determined using the values of L and C (Omijeh, 2009):

$f_r = 1 / 2$□ L C)$^{1/2}$

Also the step response is obtained by the sudden application of a dc source. Consider the series RLC circuit shown in Fig. 3. Applying KVL around the loop for t>0,
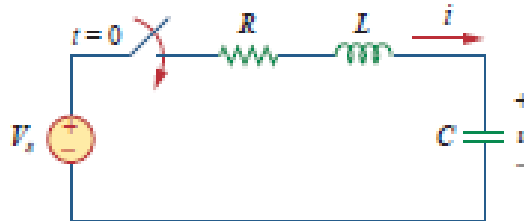

Fig.3 step voltage apply to series rlc circuit (Alexander and Sadiku 4th edition)

But
$$\frac{d^2 v_c(t)}{dt^2} = -\frac{R}{L}\frac{dv_c(t)}{dt} - \frac{1}{LC}v_c(t) + \frac{1}{LC}u(t) \qquad 1.1$$

The solution to Eq. (1.1) has two components: the transient $v_s(t)$ response and the steady-state response $v_{ss}(t)$; that is,

$$V(t) = v_s(t) + v_{ss}(t) \qquad\qquad 1.2$$

The transient response $v_s(t)$ is the component of the total response that dies out with time. Therefore, the transient response $v_s(t)$ for the overdamped, underdamped, and critically damped cases are(Alexander, 2004):

$$v_r(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(Overdamped)}$$
$$v_r(t) = (A_1 + A_2 t)e^{-\alpha t} \qquad \text{(Critically damped)}$$
$$v_r(t) = (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \qquad \text{(Underdamped)}$$

Fig.4 plots the responses for the three cases. From this figure, we observe that the critically damped response approaches the step input of 24 V the fastest.
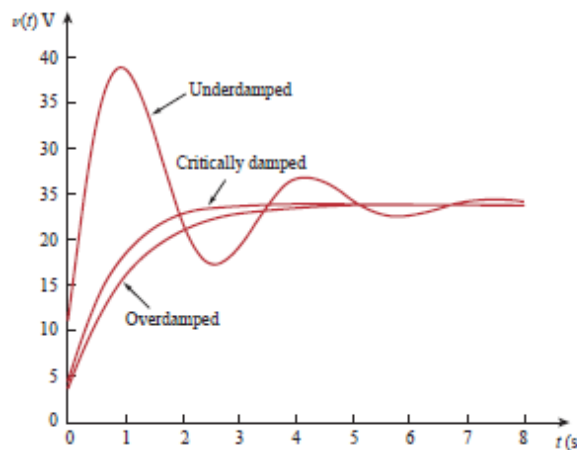

Fig.4 for the three degree of damping ((Alexander and Sadiku 4th edition))

III. Guide Model For The Rlc Circuit

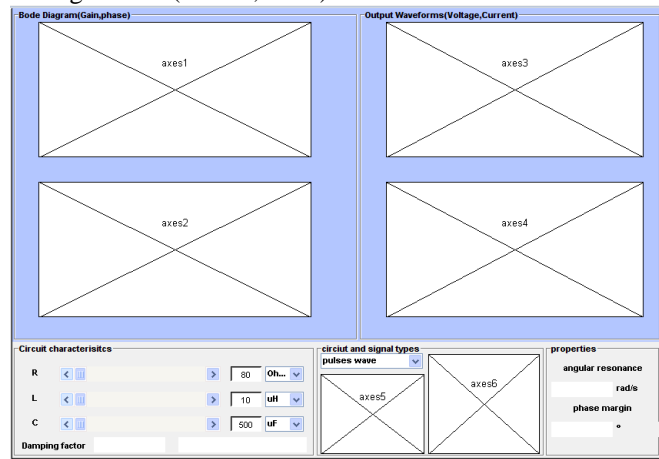The model was developed using Matlab (Steven, 2003)

Fig.5: RLC simulator graphical user interface in the process of development

Fig.5 is the RLC graphical user interface of which the parameters of the resistor (R), inductor (L) and capacitor(C) of their various units can be imputed at a click or drag of each position buttons and the performance of that parameter at the instance is display in the axes in the process of development.
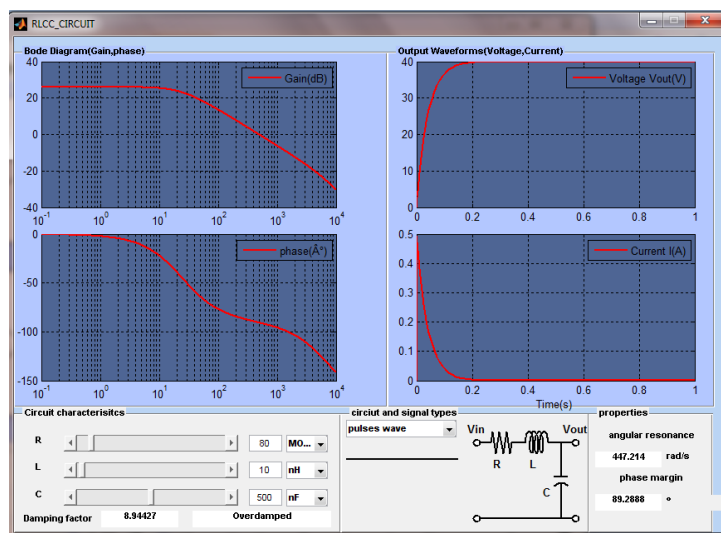Fig.6 is the model for the rlc circuit which shows the various waves forms:

Fig.6 The RLC circuit simulator

IV. Result And Discussion

The figure above shows the system is overdamped, the wave form of the output voltage, current, gain and the phase angle, it can be seen that at the value display above R=80M, L=10nH and C=500nF. The voltage increases from 0-20v without oscillating before it remain constant that is at high value of resistor the system is overdamped, while the current increase from 0-.2A and decrease to a constant value. In the case of the gain at those value display the gain increase to a maximum value at a low frequency, remain constant as the frequency increase then decreases at a high frequency. The phase starts at 0° and decrease to -150^0 at high frequency. Let us take a look at the gain which is the wave of the transfer function verse the frequency phase angle voltage and current, what happen when each parameter is varied. Tables 1, 2, 3,4 show various input parameters for validation the performance efficiency of the developed simulator.

Case1: Table 1: Input Information

Series RLC	
Electric resistance	5hms
Magnetic inductance	100H (henries)
Electric capacitance	220µf (microfarads)

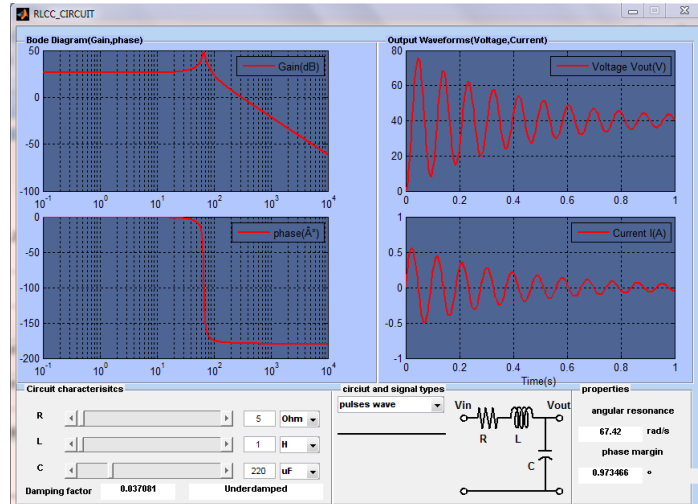**Fig.7** shows the waveform of the input parameter.

Fig.7: waveform of the input parameter

Table 2: Result Fom The Input Above

Angular resonance frequency	67.42rad/s
Phase margin	.973466°
Damping factor	0.037081
Oscillation quality	underdamped

From the input parameter to the output up to figure 7, it can be observe that the system is underdamped which confirm to that in section 2, the Figure also shows the approximate and actual amplitude of the bode plot. Note that the actual plot depends on the damping factor as well as the corner frequency. It is also observe that gain start at a high level for low frequency and remain constant for some range of frequency before decreasing which shows that the system is stable, while the phase angle at low frequency start at 0° before decreasing to -120°. With the help of the transfer function we are able to analysis the circuit in order to achieve a better output or maximum gain and to know when the system is stable.

Case 2;   Table 3: Input Information

Series RLC	
Electric resistance	5mΩ (ohms)
Magnetic inductance	10H (microhenries)
Electric capacitance	10f (microfarads)
Frequency	60 Hz (hertz)

Table 4: Result From The Input Of Table1.3

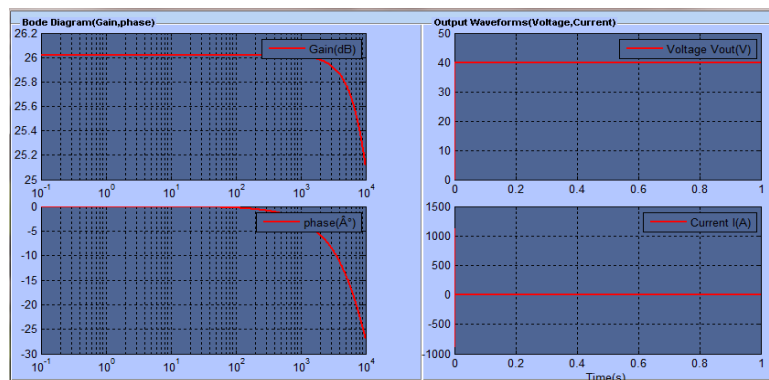|	
Angular resonance frequency	100000rad/s (radians per second)
Phase margin	58.1676(degree)
Damping factor	2.5
Oscillation quality	Overdamped

Fig.8: the gain and the phase characteristic on the left side and pulse representation of current on the right side
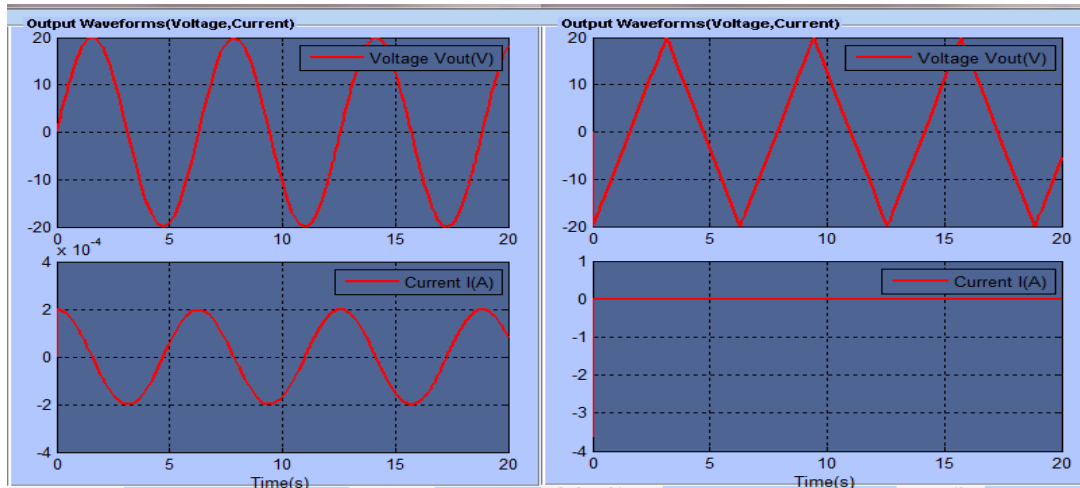
Fig.9: the current and the voltage sine wave characteristic on the left side and current and voltage triangle wave represented on the right side

From the input parameter to the output up to the figure above it can be observe that the system is a pure DC i.e. the system dose not oscillate which means only a DC source can be use at resonance since L=C and at this point frequency (f=0) is equal to zero, overshoot of the peak transient wave form is not detected. It is also observe that gain start at a high level for low frequency and remain constant for some range of frequency before decreasing while the phase angle at low frequency start at 0° before decreasing to -30° and at this point the voltage is practically in phase with the current. It can now be seen that without the transfer function the analysis of the gain and the phase of a system can not be done accurately, i.e. its stability, gain and phase in an industry. So the transfer function helps us to analysis the circuit in order to achieve a better understanding of how the gain and the phase behave at different value of parameter.

Validation of the simulator using
The GUI model also help in accurate evaluation as will be show with different value.
Given a series RLC circuit with the value use above wee the input is step input that is:
R= 5ohms, L =1h, C =220uF, u_t =1

$$w_o = \frac{1}{\sqrt{LC}}$$

$$w_o = \frac{1}{\sqrt{1 \times 220 \times 10^{-6}}} = 67.41998 \text{rad/sec}$$

Quality factor (Q):

$$Q = \frac{wL}{R}$$

$$Q = \frac{67.41998 \times 1}{5} = 13.483997$$

Damping factor (ζ):

$$\zeta = \frac{1}{2Q}$$

$$\zeta = \frac{1}{2 \times 13.483997} = 0.03708099$$

Phase Margin

$$\tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{(4\zeta^4+1)} - 2\zeta^2}} \right)$$

$$\tan^{-1}\left(\frac{2 \times 0.03708099}{\sqrt{\sqrt{(4 \times 0.03708099^4+1)} - 2 \times 0.03708099^2}} \right) = .973466^o$$

The formular for manual calculations are shown in Table 5.

Table 5: The Formula For Computing Manually   The Analysis Of The Above Input.

$w_o = \dfrac{1}{\sqrt{LC}}$	$Q = \dfrac{wL}{R}$
$\zeta = \dfrac{1}{2Q}$	$\emptyset = \tan^{-1}\left(\dfrac{2\zeta}{\sqrt{\sqrt{(4\zeta^4 + 1)} - 2\zeta^2}} \right)$

w_o = Resonance Angular frequency
R = Electric resistance
L = Magnetic inductance
C = Electric Capacitance
F = frequency
Q = Quality Factor
ζ = Damping factor
$\emptyset$ = Phase Margin

The result from the above calculation of input parameter is the same as that which is obtains from the graphical user interface (GUI) simulator. Show in table 6 below.

Table 6: Result Comparism Of The Simulator And Formula Application From Input Parameter

	Graphical user interface (GUI) simulator	Formula Application
Angular resonance frequency	67.42.rad/s	**67.41998**Rad/s
Phase margin	.97346(degree)	.973466°
Damping factor	0.037081	**.03708099**
Oscillation quality	Underdamped	Underdamped

V.   Conclusion

The component resistance (R), inductance(L) and capacitance(C) are available in every electronic either integrated or not and are use in bandpass filter, bandstop filter, highpass filter and lowpass filter also in application such as variable tuned circuit, voltage multiplier, pulse discharge and oscillators. Due to this wild range of application the performance of this component as an RLC circuit have to be understood before use. This is done at ease by simply inputting the parameter in to the graphical user interface (GUI) simulator as carried out in this work. This work shows the graphical user interface (GUI) simulator which display the transfer characteristic of rlc circuit where the behavior of the parameters can be study and understand as it vary. This simulator also helps us to know when the system is stable in order to achieve a better performance and accuracy in electrical design and to avoid waist of resources and also to make learning more visible to student. The methodology used in this work, if fully implemented, will not only reduce the cost of design but will also reduce the cost of maintenance, making it economical and affordable.

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