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Fuzzy mathematical models for the analysis of fuzzy systems with application to liver disorders

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***Abstract**: The main objective of this model is to focus on how to use the model of fuzzy system to solve fuzzy mathematics problems. Some mathematical models based on fuzzy set theory, fuzzy systems and neural network techniques seem very well suited for typical technical problems. We have proposed an extension model of a fuzzy system to N-dimension, using Mamdani's minimum implication, the minimum inference system, and the singleton fuzzifier with the center average defuzzifier. Here construct two different models namely a fuzzy inference system and an adaptive fuzzy system using neural network. We have extended the theorem for accuracy of the fuzzy system to N- dimensions, and provided a medical application of the fuzzy mathematics models. Since, liver is the largest internal member in the human body, so diagnosing liver disorder disease is a high interest to researchers of the fuzzy modeling and the fuzzy system. Therefore, the fuzzy mathematical models are applied on a real data to the Liver Disorder disease. Consequently, a comparison between three models: the FS with Mamdani model, ST model, and the ANFIS is made. Therefore, we have obtained the best result with the ANFIS. Finally, the programs of these models by using MATLAB created and performed.*

***Keywords:** Fuzzy system; FIS; ANFIS; Neural networks; Accuracy of the fuzzy system; Liver disorders.*

I. Introduction

Fuzzy mathematics provides the starting point and basic language for fuzzy systems (FSs) and fuzzy modeling (FM) [Ruan and Wang (1997)], while the fuzzy mathematical principles are developed by replacing the sets in classical mathematical theory with fuzzy sets (FSs) [Samandar (2011)]. The concepts and principles in fuzzy mathematics are useful in FSs and adaptive neuro-fuzzy systems (ANFSs) [Singh et al. (2009)], [Shing and Jang (1993)]. Fuzzy variables are processed using a system called a fuzzy inference system (FIS) which involves fuzzification, fuzzy inference, and defuzzification [Wang (1997)], [Aik and Jayakumar (2008)]. The FIS collects the rules in the fuzzy rule-base into a mapping from fuzzy set $\hat{A} \in X$ to fuzzy set $\hat{B} \in Y$ [Sivanandam and Deepa (2008)]. We must construct interfaces that are the fuzzifier and defuzzifier, between the FIS and the environment because in most applications the input and output of the FS are real valued numbers such our application in this model to Liver Disorders [Sug (2012) and Gulia et al. (2014)]. The reason to represent a fuzzy system in terms of a neural network is to utilize the learning ability of neural networks to improve performance, such as adaptation of FS [Rameshkumar and Arumugam (2011)]. When the expert is demonstrating, we measure the inputs and the outputs; that is, we can collect a set of input-output data pairs [Nayak (2004)], [Hndoosh et al. (2012) and (2013)]. Therefore, the knowledge is transformed into a set of input-output pairs. The task in this work is to model a FS that describes the input-output behavior represented by the input-output pairs and apply the model to Liver Disorders. We will model the FS by first assigning its structure and then tuning its parameters [Jose et al. (1999)]. To simulate the modeling system, need a mathematical model of the Liver Disorders that is described by linguistic variables and its membership functions (MFs) [Chai et al. (2009)]. We note that the fuzzy modeler can successfully control and handle the real data of any problem. As well as, the prediction accuracy is improved by defining more FSs for each input variable [Marza and Seyyedi (2008)]. The advantage of using the FS is that the parameters of MFs have clear physical meanings and we have models to choose good initial values for them [Doğan et al. (2007). We can recover the fuzzy if-then rules that model the FS [Belohlavek and Klir (2011)]. These recovered fuzzy if-then rules may help to demonstrate the modeled FS in a user-friendly manner. The work is divided into four Sections. Section 1 introduces the fundamental concepts and principles in the general field of fuzzy theory that are particularly useful in FSs and ANFIS [Sivanandam and Deepa (2008)]. In Section 2, we have provided the detailed mathematical formulas of the FIS, and we construct interfaces between the FIS and the environment using fuzzifier and defuzzifier models [Jandaghi et al. (2010)].

In the first part of this Section, we propose and extend the model of the FS, the work of Hndoosh et al. (2013) and Wang (1998), from 2-dimention to *N*-dimension using Mamdani's minimum implication with the minimum inference system, the singleton fuzzifier, and center average defuzzifier [Rojas (1996)]. In the second part, we have provided the theorem for accuracy of a proposed model [Hndoosh et al. (2012), (2013), and (2014)]. This approach requires N-pieces of information in order to model FS to satisfy any pre-specified degree of accuracy [Kamel and Hassan (2009)]. As well as, we have adapted the structure of the FS and modeled of an adaptive FS using a neural network through the third part of this Section [Singh et al. (2009)], [Shing, and Jang (1993)]. In Section 3, we have applied all the previous concepts and models on a real application to Liver Disorders [Sug (2012) and Gulia et al. (2014)], and we have structured of the applied model at the first part. Second and third subsection, provided discussion and results for the model of the FS with Mamdani and ST models and the adaptive FS using neural network, respectively. Consequently, obtained good results of the models, and created programs for the different models using 'MATLAB'. Concluding remarks are present in Section 4. Finally, Appendix is provided the representation of results of the FIS with the Mamdani and ST models, and the results of the ANFIS with their errors through Table 3.

II. Proposal Of A New Model Of A Fuzzy System On N-Dimensions

In this section, proposed a model of a fuzzy system, that is extension of the work of Hndoosh et al. (2013) and Wang (1997), from 2-dimention to N-dimension [Ruan and Wang (1997)]. Consider the general membership function of fuzzy set, A, is a continuous function in R given by:

$$\mu(x;a,b,c,d)=\begin{cases}0 & if x<a\\ a(x) & if a\le x<b\\ 1 & if b\le x\le c\\ d(x) & if c<x\le d\\ 0 & if d<x\end{cases} \qquad (1)$$

where $[a,d]\subset R$ and $a\le b\le c\le d, 0\le a(x)\le 1$ a non-decreasing function is $\in[a,b)$ and $0\le d(x)\le 1$ is a non-increasing function $\in(c,d]$.

If fuzzy sets $A^1,A^2,\dots,A^N\in W\subset R$ then, they are called:

1. Complete on W, if there exists A^k such that $\mu_{A^k}(x)>0$, for any $x\in W$.
2. Consistent on W if $\mu_{A^k}(x)=1$ for some $x\in W$ implies that $\mu_{A^j}(x)=0$, for all $k\neq j$.
3. Normal, consistent and complete with general MFs, $\mu_{A^j}\left(x;a_j,b_j,c_j,d_j\right)$. If $A^1<A^2<\dots<A^N$, then $c_j\le a_{j+1}<d_j\le b_{j+1}, for\ j=1,2,\dots,N-1$.

In the next section, we will mode a particular type for medical application of Liver Disorders that have some properties [Belohlavek and Klir (2011)], and consider N-inputs fuzzy systems. Now the proposed model is as the following:

2.1 The Proposed Model

Let $G(x)$ be defined $G(x): X\subset R^n\to R$, that is, a function on the compact set $X=[\alpha_1,\beta_1]\times\dots\times[\alpha_n,\beta_n]$ and the analytic formula of $G(x)$ be unknown. Suppose that for any $x\in U$, we can obtain $G(x)$. Now, to model a fuzzy system that approximates $G(x)$ is main task and model of a fuzzy system as follows:

Step 1:

Define N_j $(j=1,2,\dots,n)$ fuzzy sets $A_j^1,A_j^2,\dots,A_j^{N_j}\in[\alpha_j,\beta_j]$, which are normal, consistent, and complete with triangular MFs $\mu_{A_j^1}\left(x_j;\ a_j^1,b_j^1,c_j^1\right),\dots,\mu_{A_j^{N_j}}\left(x_j;\ a_j^{N_j},b_j^{N_j},c_j^{N_j}\right)$, and $A_j^1<A_j^2<\dots<A_j^{N_j}$ with $a_j^1=b_j^1=\alpha_j$ and $b_j^{N_j}=c_j^{N_j}=\beta_j$, which,

- $e_1^1=\alpha_1, e_1^{N_1}=\beta_1,$ and $e_1^j=b_1^j$ for $j=2,3,\dots,N_1-1,$
- $e_2^1=\alpha_2, e_2^{N_2}=\beta_2,$ and $e_1^j=b_2^j$ for $j=2,3,\dots,N_2-1,$ (2)
- $\vdots$
- $e_n^1=\alpha_n, e_n^{N_n}=\beta_n,$ and $e_1^j=b_1^j$ for $j=2,3,\dots,N_n-1.$

Step 2:

Construct $I=N_1\times N_2\times\dots\times N_n$ fuzzy if-then rules in the following form:

$R_X^{j_1\dots j_n}$: IF x_1 is $A_1^{j_1}$ and x_2 is $A_2^{j_2}$ and ... and x_n is $A_n^{j_n}$ Then y is $B^{j_1\dots j_n}$, (3)

where $j_1=1,2,\dots,N_1$, $j_2=1,2,\dots,N_2$, ... , $j_n=1,2,\dots,N_n$, and the center of the fuzzy set $B^{j_1\dots j_n}$, denoted by $\bar{y}^{j_1\dots j_n}$, is chosen as:

$$\bar{y}^{j_1\dots j_n}=G\left(e_1^{j_1},\dots,e_n^{j_n}\right) \qquad (4)$$

This is the case when (3) depends on the Mamdani fuzzy rule [Chai et al. (2009)], and the antecedent of our model is connected by "and" [Sivanandam and Deepa (2008)], [Marza and Seyyedi (2008)], then the truth-value evaluation is given by:

$$\vartheta_i = \tau\left(\mu_{A_1^{j_1\ldots j_n,i}}(x_1),\mu_{A_2^{j_1\ldots j_n,i}}(x_2),\ldots,\mu_{A_n^{j_1\ldots j_n,i}}(x_n)\right) \quad (5)$$

Therefore, from $\mu_{\overline{B^i}}(y) = t\left(\vartheta_i, \mu_{B^i}(y)\right), \forall y \in R$, the fuzzy inference produces the fuzzy set of output by:

$$\mu_{\overline{B^{j_1\ldots j_n,i}}}(y) = t\left(\vartheta_i, \mu_{B^{j_1\ldots j_n,i}}(y)\right) \forall y \in R \quad (6)$$

The consequents of all the rules are aggregated in the consequents by the 'max' function as:

$$\mu_{\overline{B^{j_1\ldots j_n}}}(y) = s(\mu_{B^{j_1\ldots j_n,1}}(y), \mu_{B^{j_1\ldots j_n,2}}(y), \ldots, \mu_{B^{j_1\ldots j_n,i}}(y)) \quad (7)$$

However, when the consequent of rule is a linear function, then the output of the Sugeno rule depends on function as follows:

$$\bar{y}^{j_1\ldots j_n} = f_i(x_1, x_2, \ldots, x_n), \quad (8)$$

where f_i is linear function base on x_j, that is defined as:

$$f_i(x_1, x_2, \ldots, x_n) = a_1^i x_1 + a_2^i x_2 + \cdots + a_n^i x_n + a_{n+1}^i, \quad (9)$$

where a_j^i are the parameters, and can be computed by the least square model.

Step3:

Constructing the fuzzy system $f(x)$ from the $N_1 \times N_2 \times \ldots \times N_n$ rules of (4) using Mamdani's minimum implication (MMI) (10a) with the minimum inference system (MIS) (10b), the Singleton fuzzifier (SF) (11), and the Center Average Defuzzifier (CAD) (12), are as follows:

$$\left(\mu_{Q_{IM}}(x,y) = min[\mu_{A_1}(x), \quad \mu_{A_2}(y)], Q_{IM} \in X \times Y\right) \quad (10a)$$

$$\mu_{B'}(y) = \max_{\forall i}\left[\sup_{x\in X} min\left(\mu_{A'}(x), \mu_{A_1^i}(x_1), \ldots, \mu_{A_n^i}(x_n), \mu_{B^i}(y)\right)\right] \quad (10b)$$

$$\mu_{A'}(x) = \begin{cases} 1 & if \quad x = x^* \\ 0 & otherwice \end{cases} \quad (11)$$

$$y^* = \frac{\sum_{i=1}^{I} \bar{y}^i w_i}{\sum_{i=1}^{I} w_i} \quad (12)$$

Therefore, we obtain:

$$f(x) = \frac{\sum_{j_1=1}^{N_1}\cdots\sum_{j_n=1}^{N_n} \bar{y}^{j_1\ldots j_n}\left(min\left(\mu_{A_1^{j_1}}(x_1),\ldots,\mu_{A_n^{j_n}}(x_n)\right)\right)}{\sum_{j_1=1}^{N_1}\cdots\sum_{j_n=1}^{N_n}\left(min\left(\mu_{A_1^{j_1}}(x_1),\ldots,\mu_{A_n^{j_n}}(x_n)\right)\right)} \quad (13)$$

Since the fuzzy sets $A_j^1, \ldots, A_j^{N_j}$ are complete at every $x \in X$, then there exist $j_1, j_2, \ldots, j_n$ such that:

$$min\left(\mu_{A_1^{j_1}}(x_1), \mu_{A_2^{j_2}}(x_2), \ldots, \mu_{A_n^{j_n}}(x_n)\right) \neq 0. \quad (14)$$

Consequently, the fuzzy system (13) is well defined. From step 2, we note that the antecedent of the rules (4) constitute all the possible sets of the fuzzy sets defined for each input variable [Hndoosh (2013)]. The total number of rules is N_n, that increases exponentially with the dimension of the input space [Jandaghi (2010)]. In the second part, we explain the accuracy of the f(x) modeled above on the unknown function G(x), is explained [Nayak (2004)], [Kamel and Hassan (2009)]. Here extended the accuracy of the fuzzy system from 2-dimention to N-dimention, as well as changed the type of fuzzy inference system to a minimum inference system to suit any application [Wang (1997)].

1.2. Theorem (The Fuzzy System Accuracy for the Proposed Model)

Let $f(x)$ be the fuzzy system in (13) and $G(x)$ be the unknown function in (4). If $G(x)$ is continuously differentiable on $X = [\alpha_1, \beta_1] \times [\alpha_2, \beta_2] \times \ldots \times [\alpha_n, \beta_n]$, then:

$$\|G - f\|_\infty \leq \left\|\frac{\partial G}{\partial x_1}\right\|_\infty h_1 + \left\|\frac{\partial G}{\partial x_2}\right\|_\infty h_2 + \cdots + \left\|\frac{\partial G}{\partial x_n}\right\|_\infty h_n, \quad (15)$$

where the infinite norm $\|.\|_\infty$ is defined as: $\|d(x)\|_\infty = sup_{x\in X}|d(x)|$ *and* $h_j = max_{1\leq k\leq N_j}\left|e_j^{k+1} - e_j^k\right|$, *(j = 1, 2, …,n.*

Proof:

Let $X^{j_1\ldots j_n} = [e_1^{j_1}, e_1^{j_1+1}] \times [e_2^{j_2}, e_2^{j_2+1}] \times \ldots \times [e_n^{j_n}, e_n^{j_n+1}]$, where $j_1 = 1,2,\ldots,N_1 - 1$, $j_2 = 1,2,\ldots,N_2 - 1$, …, $j_n = 1,2,\ldots,N_n - 1$. Since $[\alpha_j, \beta_j] = [e_j^1, e_j^2] \cup [e_j^2, e_j^3] \cup \ldots \cup [e_j^{N_j-1}, e_j^{N_j}]$, $j = 1,2,\ldots,n.$, then:

$$X = [\alpha_1, \beta_1] \times [\alpha_2, \beta_2] \times \ldots \times [\alpha_n, \beta_n] = \bigcup_{j_1=1}^{N_1-1}\bigcup_{j_2=1}^{N_2-1}\cdots\bigcup_{j_n=1}^{N_n-1} X^{j_1 j_2\ldots j_n}, \quad (16)$$

which implies that for any $x \in X$, there exists $X^{j_1 \cdots j_n}$ such that $x \in X^{j_1 \cdots j_n}$.
Now suppose $x \in X^{j_1 \cdots j_n}$, that is $x_1 \in [e_1^{j_1}, e_1^{j_1+1}]$, $x_2 \in [e_2^{j_2}, e_2^{j_2+1}]$,…, $x_n \in [e_n^{j_n}, e_n^{j_n+1}]$ (since x fixed, j_n are also fixed in the sequel). Since the fuzzy sets $A_1^1, A_1^2, \dots, A_1^{N_1}$ are normal, consistent, and complete, at least one and at most two, $\mu_{A_1^{k_1}}(x_1)$ are non-zero for $k_1 = 1, \dots, N_1$. From the definition of $e_1^{j_1}(j_1 = 1, 2, \dots, N_1 - 1)$, these two possible non-zero $\mu_{A_1^{k_1}}(x_1)'$s are $\mu_{A_1^{j_1}}(x_1)$ and $\mu_{A_1^{j_1+1}}(x_1)$. Similarly upto, the two possible non-zero $\mu_{A_n^{k_n}}(x_n)'s\ (for\ k_n = 1, \dots, N_n)$ are $\mu_{A_n^{j_n}}(x_n)$ and $\mu_{A_n^{j_n+1}}(x_n)$. Hence, the fuzzy system $f(x)$ of (16) is simplified as the following:

$$f(x) = \frac{\sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1} \bar{y}^{k_1 \dots k_n}\left(min\left(\mu_{A_1^{k_1}}(x_1),\ \mu_{A_2^{k_2}}(x_2), \dots,\ \mu_{A_n^{k_n}}(x_n)\right)\right)}{\sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1} min\left(\mu_{A_1^{k_1}}(x_1),\ \mu_{A_2^{k_2}}(x_2), \dots,\ \mu_{A_n^{k_n}}(x_n)\right)} \tag{17}$$

From (4), we obtain:

$$f(x) = \sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1}\left[\frac{min\left(\mu_{A_1^{k_1}}(x_1), \dots,\ \mu_{A_n^{k_n}}(x_n)\right)}{\sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1} min\left(\mu_{A_1^{k_1}}(x_1), \dots,\ \mu_{A_n^{k_n}}(x_n)\right)}\right] * G\left(e_1^{k_1}, \dots, e_n^{k_n}\right) \tag{18}$$

$$\because \sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1}\left[\frac{min\left(\mu_{A_1^{k_1}}(x_1),\ \dots,\ \mu_{A_n^{k_n}}(x_n)\right)}{\sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1} min\left(\mu_{A_1^{k_1}}(x_1), \dots,\ \mu_{A_n^{k_n}}(x_n)\right)}\right] = 1 \tag{19}$$

we have:

$$\begin{aligned}|G(x) - f(x)| &\le \sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1}\left[\frac{min\left(\mu_{A_1^{k_1}}(x_1), \dots,\ \mu_{A_n^{k_n}}(x_n)\right)}{\sum_{k_1=j_1}^{j_1+1} \dots \sum_{k_n=j_n}^{j_n+1} min\left(\mu_{A_1^{k_1}}(x_1), \dots,\ \mu_{A_n^{k_n}}(x_n)\right)}\right] * \left|G(x) - G\left(e_1^{k_1}, \dots, e_n^{k_n}\right)\right| \\ &\le \max_{\substack{k_1=j_1:j_1+1 \\ \vdots \\ k_n=j_n:j_n+1}} \left|G(x) - G\left(e_1^{k_1}, \dots, e_n^{k_n}\right)\right| \end{aligned} \tag{20}$$

From the Mean Value model, may be written (20) as:

$$|G(x) - f(x)| \le \max_{\substack{k_1=j_1,j_1+1 \\ \vdots \\ k_n=j_n,j_n+1}} \left(\left\|\frac{\partial G}{\partial x_1}\right\|_\infty \left|x_1 - e_1^{k_1}\right| + \left\|\frac{\partial G}{\partial x_2}\right\|_\infty \left|x_2 - e_2^{k_2}\right| + \dots + \left\|\frac{\partial G}{\partial x_n}\right\|_\infty \left|x_n - e_n^{k_n}\right|\right) \tag{21}$$

Since $x \in X^{j_1 \cdots j_n}$, means that $x_1 \in [e_1^{j_1}, e_1^{j_1+1}]$, $x_2 \in [e_2^{j_2}, e_2^{j_2+1}]$ … $x_n \in [e_n^{j_n}, e_n^{j_n+1}]$, we have,
$\left|x_1 - e_1^{k_1}\right| \le \left|e_1^{j_1+1} - e_1^{j_1}\right|, \left|x_2 - e_2^{k_2}\right| \le \left|e_2^{j_2+1} - e_2^{j_2}\right| \dots, and \left|x_n - e_n^{k_n}\right| \le \left|e_n^{j_n+1} - e_n^{j_n}\right|$ for $k_1 = j_1, j_1 + 1, k_2 = j_2, j_2 + 1$, …, and $k_n = j_n, j_n + 1$
Then (21) becomes:

$$|G(x) - f(x)| \le \left\|\frac{\partial G}{\partial x_1}\right\|_\infty \left|e_1^{j_1+1} - e_1^{j_1}\right| + \left\|\frac{\partial G}{\partial x_2}\right\|_\infty \left|e_2^{j_2+1} - e_2^{j_2}\right| + \dots + \left\|\frac{\partial G}{\partial x_n}\right\|_\infty \left|e_n^{j_n+1} - e_n^{j_n}\right| \tag{22}$$

Since $\|d(x)\|_\infty = \sup_{x \in X} |d(x)|$ then $\|G - f\|_\infty = \sup_{x \in X} |G - f|$, we get:

$$\|G - f\|_\infty \le \left\|\frac{\partial G}{\partial x_1}\right\|_\infty \max_{1 \le j_1 \le N_1 - 1} \left|e_1^{j_1+1} - e_1^{j_1}\right| + \dots + \left\|\frac{\partial G}{\partial x_n}\right\|_\infty \max_{1 \le j_n \le N_n - 1} \left|e_n^{j_n+1} - e_n^{j_n}\right|$$

$$\therefore \|G - f\|_\infty \le \left\|\frac{\partial G}{\partial x_1}\right\|_\infty h_1 + \left\|\frac{\partial G}{\partial x_2}\right\|_\infty h_2 + \dots + \left\|\frac{\partial G}{\partial x_n}\right\|_\infty h_n \tag{23}$$

From (22), we can conclude that fuzzy systems in the form of (17).
Specifically, since $\left\|\frac{\partial G}{\partial x_1}\right\|, \left\|\frac{\partial G}{\partial x_2}\right\|, \dots, \left\|\frac{\partial G}{\partial x_n}\right\|$ are finite numbers for any given $\varepsilon > 0$, we can choose $h_1, h_2, \dots, h_n$ small enough such that $\left\|\frac{\partial G}{\partial x_1}\right\|_\infty h_1 + \left\|\frac{\partial G}{\partial x_2}\right\|_\infty h_2 + \dots + \left\|\frac{\partial G}{\partial x_n}\right\|_\infty h_n < \varepsilon$. Hence from (15) we have:

$$sup_{\ x \in X} \ |G - f| = \|G - f\|_\infty < \varepsilon \tag{24}$$

From (23), we can show that, in order to model a fuzzy system with a pre-specified accuracy, we must know the bounds of the derivatives of $G(x)$ with respect to $x_1, x_2, \dots, x_n$, that is $\left\|\frac{\partial G}{\partial x_1}\right\|_\infty, \left\|\frac{\partial G}{\partial x_2}\right\|_\infty, \dots, \left\|\frac{\partial G}{\partial x_n}\right\|_\infty$. In the model process, we need to know the value of $G(x)$ at x, where

$$x = \left(e_1^{j_1}, e_2^{j_2}, \dots, e_n^{j_n}\right) \text{ for } j_1 = 1, 2, \dots, N_1, \quad j_2 = 1, 2, \dots, N_2, \dots, j_n = 1, 2, \dots, N_n. \tag{25}$$

Therefore, this approach requires these N pieces of information in order for the model fuzzy system, to satisfy any pre-specified degree of accuracy.

1.3. Model of an Adaptive Fuzzy System Using a Neural Network

In this section, we adapt the structure of the fuzzy system that is specified with the structure of some parameters [Singh et al. (2009)], [Shing and Jang (1993)]. We specify the structure of the fuzzy system to be modeled. Here, we choose the fuzzy system with a MIS, a SF, a CAD, and a Triangular MF [Samandar (2011)], then, we obtain:

$$f(x)=\frac{\sum_{i=1}^{I}\bar{y}^i\left(\min_{\forall j}\mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^{I}\left(\min_{\forall j}\mu_{A_j^i}(x_j)\right)}=\frac{\sum_{i=1}^{I}\bar{y}^i\left[\min_{\forall j}\left(max\left(\min_{\forall j}\left(\frac{x_j-a_j^i}{b_j^i-a_j^i},\ \frac{c_j^i-x_j}{c_j^i-b_j^i}\right),\ 0\right)\right)\right]}{\sum_{i=1}^{I}\left[\min_{\forall j}\left(max\left(\min_{\forall j}\left(\frac{x_j-a_j^i}{b_j^i-a_j^i},\ \frac{c_j^i-x_j}{c_j^i-b_j^i}\right),\ 0\right)\right)\right]} \tag{26}$$

where I is fixed, and $\bar{y}^i, a_j^i, b_j^i, c_j^i$ are free parameters. The fuzzy system (26) has not been modeled because the parameters $\bar{y}^i, a_j^i, b_j^i, c_j^i$ are not specified [Wang (1997)]. In order to determine these parameters in some optimal manner, it is helpful to represent the fuzzy system $f(x)$ of (26) as a feed-forward network [Jose et al. (1999)]. Specifically, the mapping from the input $x \in U \subset R^n$ to the output $f(x) \in V \subset R$ can be performed according to operations [Doğan et al. (2007)]. Note that, the input x is passed through a minimum triangular operator to become:

$z^i = max\left(\min_{\forall j}\left(\frac{x_j-a_j^i}{b_j^i-a_j^i},\ \frac{c_j^i-x_j}{c_j^i-b_j^i}\right),\ 0\right)$, where z^i are passed through a summation operator.

Let $K=\sum_{i=1}^{I} z^i$ and $L=\sum_{i=1}^{I}\bar{y}^i z^i$; therefore, the output of the fuzzy system is computed as $f(x)=L/K$. Consequently, we summarize the procedures to model a fuzzy system that depends on layers of network as the following:

<u>Step1:</u> Structure specification and initial parameters

Select the fuzzy system (26) and determine the number of rule [Rameshkumar and Arumugam (2011)].

The larger number of rule, results more parameters and more computation, but gives better accuracy. Specify the initial parameters $\bar{y}^i(0), a_j^i(0), b_j^i(0), c_j^i(0)$, then the initial fuzzy system becomes as in (27). These initial para-meters may be determined according to the linguistic rules from human experts as in our application.

$$f(x)=\frac{\sum_{j_1=1}^{N_1}\cdots\sum_{j_n}^{N_n}\bar{y}^{j_1\cdots j_n}(0)\left[min\left(max\left(\min_{\forall k}\left(\frac{x_{k0}^p-a_k^{j_1\cdots j_n}(0)}{b_k^{j_1j_2j_3}(0)-a_k^{j_1j_2j_3}(0)},\ \frac{c_k^{j_1\cdots j_n}(0)-x_{k0}^p}{c_k^{j_1\cdots j_n}(0)-b_k^{j_1\cdots j_n}(0)}\right),\ 0\right)\right)\right]}{\sum_{j_1=1}^{N_1}\cdots\sum_{j_n}^{N_n}\left[min\left(max\left(\min_{\forall k}\left(\frac{x_{k0}^p-a_k^{j_1\cdots j_n}(0)}{b_k^{j_1\cdots j_n}(0)-a_k^{j_1\cdots j_n}(0)},\ \frac{c_k^{j_1\cdots j_n}(0)-x_{k0}^p}{c_k^{j_1\cdots j_n}(0)-b_k^{j_1\cdots j_n}(0)}\right),\ 0\right)\right)\right]} \tag{27}$$

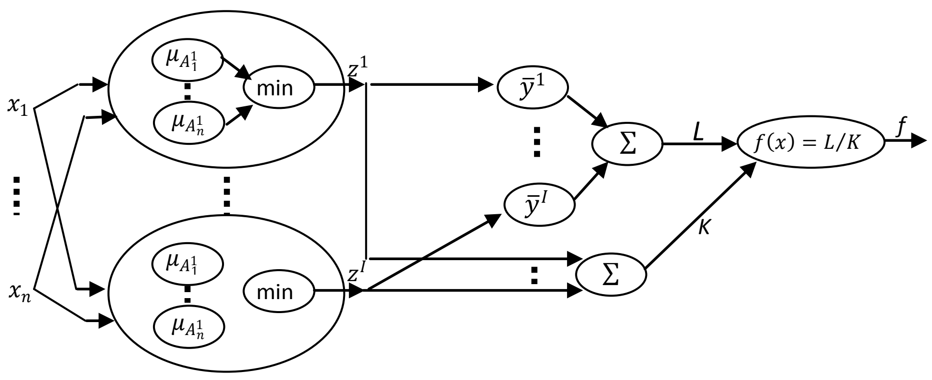


Figure 1: Network representation of the fuzzy system

Step2: Calculating the outputs of the fuzzy system

For a given inputs-output pair $\left(x_{k0}^{p};\, y_{0}^{p}\right),\ p = 1,2,..\ ,\ k = 1,2,..,n$ and at the q^{th} training stage, $q = 0, 1, ...$, present x_{k0}^{p} to the input layer of the fuzzy system in Figure 1 and compute the outputs of layers, and therefore, we compute:

First output: Every node produces MF of an input parameter. The node output $o_1^{j_i}$ is explained by:

$$o_1^{j_1} = \mu_{A_1^{j_1}}(x_1);\ o_1^{j_2} = \mu_{A_2^{j_2}}(x_2), \dots ,\ and\ \ o_1^{j_n} = \mu_{A_n^{j_n}}(x_n), \tag{28}$$

where $x_1, x_2, \dots, x_n$ are the inputs, $\mu_{A_1^{j_1}}, \mu_{A_2^{j_2}}, \dots, \mu_{A_n^{j_n}}$ are linguistic fuzzy sets related with nodes, and $o_1^{j_i}$ is the degree of MFs of a fuzzy set.

Second output: Every node is a fixed node, whose output is the minimum of all MFs:

$$o_2^{j_1 \dots j_n} = z^{j_1 \dots j_n}, z^{j_1 \dots j_n} = \min_{\forall j}\left(\mu_{A_j^{j_1 \dots j_n}}(x_j)\right), \tag{29}$$

where $\mu_{A_1^{j_1 \dots j_n}}$ is declared by triangular MF, and then we obtain:

$$z^{j_1 \dots j_n} = \min_{\forall j}\left(max\left(\min_{\forall k}\left(\frac{x_{k0}^{p} - a_k^{j_1 \dots j_n}(q)}{b_k^{j_1 \dots j_n}(q) - a_k^{j_1 \dots j_n}(q)},\ \frac{c_k^{j_1 \dots j_n}(q) - x_{k0}^{p}}{c_k^{j_1 \dots j_n}(q) - b_k^{j_1 \dots j_n}(q)} \right),\qquad 0 \right)\right) \tag{30}$$

Third output: Depending on (30), the $j_1 \dots j_n{}^{th}$ node calculates all rules as:

$$o_3^{j_1 \dots j_n} = \bar{z}^{j_1 \dots j_n}(x) = \frac{z^{j_1 \dots j_n}}{\sum_{j_1=1}^{N_1} \dots \sum_{j_n=1}^{N_n} z^{j_1 \dots j_n}} \tag{31}$$

Fourth output: Every node $j_1 \dots j_n$ is an adaptive node with a node MF of output.

$$o_4^{j_1 \dots j_n} = \bar{z}^{j_1 \dots j_n}\,\bar{y}^{j_1 \dots j_n}, \tag{32}$$

where $\bar{y}^{j_1 \dots j_n} = f_{j_1 \dots j_n}(x_1, x_2, \dots, x_n)$, and from (9), we get:

$$\bar{y}^{j_1 \dots j_n} = \alpha_1^{j_1 \dots j_n} x_1 + \alpha_2^{j_1 \dots j_n} x_2 + \dots + \alpha_n^{j_1 \dots j_n} x_n + \alpha_{n+1}^{j_1 \dots j_n}, \tag{33}$$

where $\alpha_j^{j_1 \dots j_n}, (j = 1, \dots, n+1.)$, is the parameter set of the node.

Fifth output: The single node is a fixed node labeled $\sum$, which computes the final output as the summation of all result $o_4^{j_1 \dots j_n}$

$$o_5^{j_1 \dots j_n} = \sum_{j_1=1}^{N_1} \dots \sum_{j_n=1}^{N_n} \frac{z^{j_1 \dots j_n}}{\sum_{j_1=1}^{N_1} \dots \sum_{j_n}^{N_n} z^{j_1 \dots j_n}} \left(\alpha_1^{j_1 \dots j_n} x_1 + \alpha_2^{j_1 \dots j_n} x_2 + \alpha_n^{j_1 \dots j_n} x_n + \alpha_{n+1}^{j_1 \dots j_n}\right) \tag{34}$$

Suppose,

$$K = \sum_{j_1=1}^{N_1} \dots \sum_{j_n=1}^{N_n} z^{j_1 \dots j_n}$$

$$L = \sum_{j_1=1}^{N_1} \dots \sum_{j_n=1}^{N_n} z^{j_1 \dots j_n}\left(\alpha_1^{j_1 \dots j_n} x_1 + \alpha_2^{j_1 \dots j_n} x_2 + \alpha_n^{j_1 \dots j_n} x_n + \alpha_{n+1}^{j_1 \dots j_n}\right) \tag{35}$$

Consequently, the final output is obtained as:

$$f(x) = \frac{L}{K}. \tag{36}$$

Here, noted that $\bar{y}^{j_1 \dots j_n}$ are free parameters to be modeled. When select the initial parameters $\theta(1)$, and there are linguistic rules from experts, then choose $\bar{y}^{j_1 \dots j_n}(1)$ to be the centers of the then part fuzzy sets in these linguistic rules; otherwise, choose $\theta(1)$ arbitrarily in the output space $Y \subset R$. In this way, we can say that the initial fuzzy system is constructed from experts.

Step3: Update the parameters

Use the training algorithm to compute the updated parameters $\bar{y}^{j_1 \dots j_n}(q+1), a_k^{j_1 \dots j_n}(q+1),\ b_k^{j_1 \dots j_n}(q+1)$, $c_k^{j_1 \dots j_n}(q+1)$, where $y = y_0^{p}$, and $z^{j_1 \dots j_n}, K, L$ and f equal to those that computed in step2, i.e., compute the new parameters θ using the least squares model as:

$$\theta(p+1) = \theta(p) + t(p+1)\left[y_0^p - \left(z(x_{k0}^p)\right)^T \theta(p)\right] \quad (37)$$

in which,

$$t(p+1) = \frac{P(p+1)\, z(x_{k0}^p)}{\left[P(p+1)\, z(x_{k0}^p)\left(z(x_{k0}^p)\right)^T + 1\right]}, \quad (38)$$

$$P(p+1) = P(p) - \frac{P(p) z(x_{k0}^p)}{\left[P(p) z(x_{k0}^p)\left(z(x_{k0}^p)\right)^T + 1\right]} P(p)\left(z(x_{k0}^p)\right)^T \quad (39)$$

when $p = 1$, note that $\theta(1)$ is chosen using step 2**,** and $P(1)$ is a large constant. The modeled fuzzy system in (26) with the parameters $\bar{y}^{j_1 \dots j_n}$ is equal to the corresponding elements in $\theta(p)$.

Step4: Repeat by going to step 2 with $q = q + 1$, until the error $\left|f - y_0^p\right| < \varepsilon$, or until the q equals a pre-specified number.

Step5: Repeat by going to step 2 with $p = p + 1, p = 1,2, \dots$; that is, update the parameters using the next input-output pair $\left(x_{k0}^{p+1};\ y_0^{p+1}\right)$.

Keep in mind that the parameters $\bar{y}^{j_1 \dots j_n}$ are the centers of the fuzzy sets in the consequent parts of the rules, and the parameters $a_j^{j_1 \dots j_n}$ and $c_j^{j_1 \dots j_n}$ are the left and right base points, $b_j^{j_1 \dots j_n}$, the centers of the triangular fuzzy sets in the antecedent parts of the rules [Rojas (1996)]. We can improve the fuzzy if-then rules that modeled the fuzzy system, and improved fuzzy if-then rules may help to explain the model fuzzy system in a user-friendly manner [Aik and Jayakumar (2008)].

III. Application

Liver is the largest internal member in the human body, and it is known that the member is responsible for more than one hundred functions of human body. The complexity of this member makes it easily affected by disease of disorder. Therefore, diagnosing liver disorder disease (LDD) is a high interest to researchers and doctors [1], and fuzzy system has been a good intelligent model to diagnose such disease [Sug (2012) and Gulia et al. (2014)]. The fuzzy system has very good property that the model is easy to understand. This property of fuzzy system is important in case that human should understand the knowledge structures fully. This is one of the main reasons why fuzzy system is accepted in medical domain. There are six continuous attributes as dependent attributes, (Table 1 for detail of the attributes). The first five variables are all blood tests that are thought to be sensitive to liver disorders that might result from excessive alcohol consumption. Each line in the LDD_data constitutes the record of a single male individual.

Table 1: The meaning of variables

Variable	**Variable name**	**Meaning**	**Range**
x_1	mcv	mean corpuscular volume	[79,103]
x_2	alkphos	alkaline phosphotase	[35,109]
x_3	sgpt	alamine aminotransferase	[5,155]
x_4	sgot	aspartate aminotransferase	[11,68]
x_5	gammagt	gamma-glutamyl transpeptidase	[5,297]
x_6	drinks	number of half-pint equivalents of alcoholic beverages drunk per day	[0,20]

3.1. Structure of the Model

Consider the multi-inputs x_i, $(i = 1,2, \dots, 6)$, with output y (disorder types of liver that contains simple liver disorder or acute liver disorder). A fuzzy inference system (FIS) can be defined as:

$$FIS: X \to Y, where\ X \subset R^n\ and\ Y \subset R. \quad (40)$$

The fuzzy system is composed of a fuzzifier, fuzzy rule-base, fuzzy inference, and defuzzifier. In order to apply a steps of FIS systematically, the inputs $x_1, x_2, \dots, x_6$ with output y must be described as the following:

1. Inputs

Inputs-data have treated and measured, and it becomes restricted between zero and one. The first five variables have five different degrees of linguistic variables: Low (L), Low Medium (LM), Medium (M), High Medium (HM) and High (H), while sixth input is represented by five linguistic variables, Less (Le), Less Average (LA), Average (A), More Average (MA) and More (Mo).

2. Output

Output-data are represented the disorder types of liver that contains two types: simple liver disorder (SLD) or acute liver disorder (ALD), see Figure 2.

3.2. Discussion and Results of the Fuzzy System with Mamdani and ST Models

In this part, we build the proposed fuzzy system step by step on our application:

Step 1:

Define $N_j, (j = 1,2, \dots, 6)$, where $N_j = 1,2,3$. Let $A_i^1, \dots, A_i^6$ is a fuzzy sets for x_i, with triangular MFs, $\mu_{A_1^1}(x_1;\, a_1^1, b_1^1, c_1^1), \dots, \mu_{A_6^5}(x_6;\, a_6^5, b_6^5, c_6^5)$, and $A_j^1 < A_j^2 < \cdots < A_j^{N_j}$ with $a_j^1 = b_j^1 = 0$ and $b_j^{N_j} = c_j^{N_j} = 1$.

Therefore, we can define:

- $e_1^1 = 0, e_1^5 = 1$, and $e_1^2 = b_1^2$, $e_1^3 = b_1^3$, $e_1^4 = b_1^4$
- $e_2^1 = 0, e_2^5 = 1$, and $e_2^2 = b_2^2$, $e_2^3 = b_2^3$, $e_2^4 = b_2^4$,
- $e_3^1 = 0, e_3^5 = 1$, and $e_3^2 = b_3^2$, $e_3^3 = b_3^3$, $e_3^4 = b_3^4$, (41)
- $e_4^1 = 0, e_4^5 = 1$, and $e_4^2 = b_4^2$, $e_4^3 = b_4^3$, $e_4^4 = b_4^4$,
- $e_5^1 = 0, e_5^5 = 1$, and $e_5^2 = b_5^2$, $e_5^3 = b_5^3$, $e_5^4 = b_5^4$,
- $e_6^1 = 0, e_6^3 = 1$, and $e_6^2 = b_6^2$.

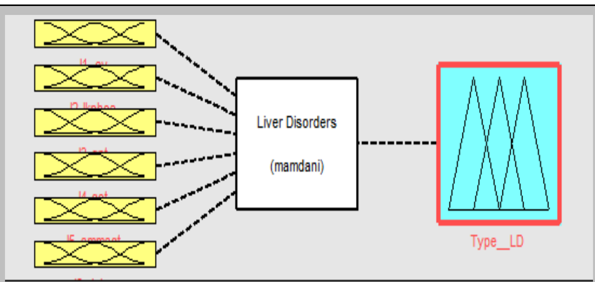


Figure 2a: The FIS of LD using Mamdani model

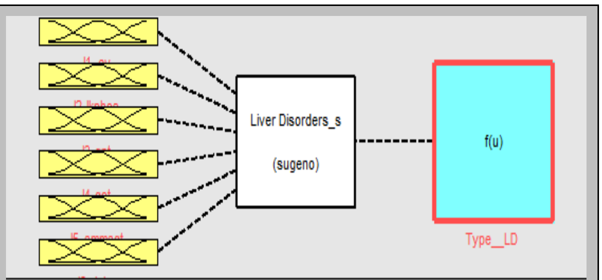


Figure 2b: The FIS of LD using ST model

Here the Singleton fuzzifier is defined as: $SF: x^* \to A'$, where $x^* = \{x_1, \dots, x_6\}$, and the softwaer 'MATLAB' is used to create the programs of our application. Six vectors of inputs with one vector of output are loaded for 100 observations. For example, the first observation is represented as [mcv=0.8544, alkphos= 0.5088, sgpt=0.12903, sgot= 0.25, gammagt=0.0303, and drinks =0.025]. Consequently, we have built the first rule such as [If (mcv is M) and (alkphos is M) and (sgpt is L) and (sgot is LM) and (gammagt is L) and (drinks is L) Then y is SLD].

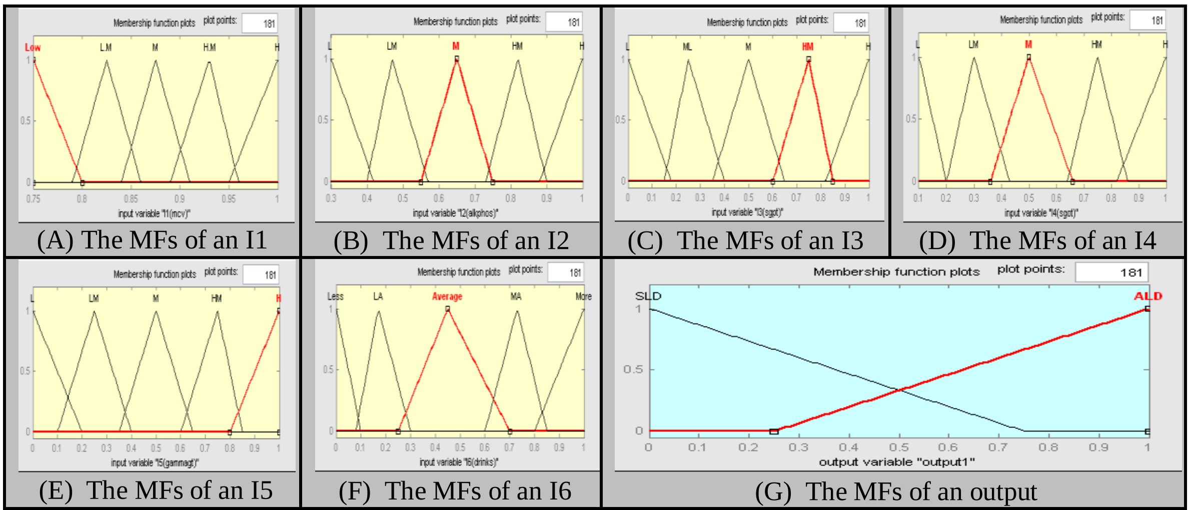
Step2:

Note that, the fuzzy rule-base consists of $(\mathrm{I} = 5^6)$ fuzzy if-then rules and the centers of $(f^{j_1 \dots j_6, i}(x_1, \dots, x_6))$ are evaluated at the 729 points, and therefore, we obtain:

$$R^i: \begin{cases} \text{If } x_1 \text{ is } A_1^{j_1,1} \text{ and } x_2 \text{ is } A_2^{j_2,1} \text{ and} \dots \text{and } x_6 \text{ is } A_6^{j_6,1} \text{ Then y is } B^{j_1 \dots j_6,1} \\ \text{If } x_1 \text{ is } A_1^{j_1,2} \text{ and } x_2 \text{ is } A_2^{j_2,2} \text{ and} \dots \text{and } x_6 \text{is } A_6^{j_6,2} \text{ Then y is } B^{j_1 \dots j_6,2} \\ \vdots \\ \underbrace{\text{If } x_1 \text{ is } A_1^{j_1,I} \text{ and } x_2 \text{ is } A_2^{j_2,I} \text{ and} \dots \text{and } x_6 \text{ is } A_6^{j_6,I}}_{\text{antecedent}} \text{ Then } \underbrace{\text{y is } B^{j_1 \dots j_6,I}}_{\text{consequent}} \end{cases} \quad (42a)$$

$$R^i: \begin{cases} \text{If } x_1 \text{ is } A_1^{j_1,1} \text{ and } x_2 \text{ is } A_2^{j_2,1} \text{ and} \dots \text{and } x_6 \text{ is } A_6^{j_6,1} \text{ Then y is } f^{j_1 \dots j_6,1}(x_1, \dots, x_6) \\ \text{If } x_1 \text{ is } A_1^{j_1,2} \text{ and } x_2 \text{ is } A_2^{j_2,2} \text{ and} \dots \text{and } x_6 \text{ is } A_6^{j_6,2} \text{ Then y is } f^{j_1 \dots j_6,2}(x_1, \dots, x_6) \\ \vdots \\ \underbrace{\text{If } x_1 \text{ is } A_1^{j_1,I} \text{ and } x_2 \text{ is } A_2^{j_2,I} \text{ and} \dots \text{and } x_6 \text{ is } A_6^{j_6,I}}_{\text{antecedent}} \text{ Then } \underbrace{\text{y is } f^{j_1 \dots j_6,I}(x_1, \dots, x_6)}_{\text{consequent}} \end{cases} \quad (42b)$$

The model (42a) is a system of Mamdani fuzzy rules, while the model (42b) is a system of Sugeno fuzzy rules. The sets A and B are a fuzzy sets, x_h $(h = 1, \dots, 6)$ are an input variables, y is the output variable, and i, $(i = 1, \dots, I)$, is the number of rules. The fuzzy set A consists of, $\left(A_1^{j_1}, \dots, A_6^{j_6}\right)$, fuzzy subsets. It is called linguistic terms that represented by triangular MFs such as (1) with b = c. The fuzzy operator "and" (t-norm) is used to connecting between linguistic terms in each rule of the model. The function $f^{j_1 \dots j_6, i}(x_1, \dots, x_6)$ is a linear function depends on inputs x_k that defined using (9). The first five linguistic terms are represented by $A_h^{j_i}$ $(j_i = 1, 2, \dots, 5)$ that depends on linguistic variable x_h $(h = 1, \dots, 5)$, see Figure 3, for e.g. the linguistic term for first input variable defined as the following:



(A) The MFs of an I1 (B) The MFs of an I2 (C) The MFs of an I3 (D) The MFs of an I4

(E) The MFs of an I5 (F) The MFs of an I6 (G) The MFs of an output

Figure 3: The representation of membership functions of inputs and output

$A_1^1 \equiv$ "Low"$\equiv \left(\mu_{A_1^1}(x_1; 0,0,0.18)\right)$, $A_1^2 \equiv$ "Low Mediam"$\equiv \left(\mu_{A_1^2}(x_1; 0.15,0.25,0.4)\right)$, $A_1^3 \equiv$ "Mediam", $\equiv$ $\left(\mu_{A_1^3}(x_1; 0.35,0.25,0.65)\right)$, $A_1^4 \equiv$ "High Medium"$\equiv \left(\mu_{A_1^4}(x_1; 0.6,0.73,0.85)\right)$ and $A_1^5 \equiv$ "High"$\equiv \left(\mu_{A_1^5}(x_1; 0.82,1,1)\right)$.
While, we have represented the sixth linguistic term $A_6^{j_i}$ $(j_i = 1, \dots, 5)$ that depends on linguistic variable x_6 as the following:
$A_6^1 \equiv$ "Less"$\equiv \left(\mu_{A_6^1}(x_6; 0,0,0.1)\right)$, $A_6^2 \equiv$ "Less Average"$\equiv \left(\mu_{A_6^2}(x_6; 0.08,0.2,0.3)\right)$, $A_6^3 \equiv$ "Average"
$\equiv \left(\mu_{A_6^3}(x_6; 0.25,0.5,0.7)\right)$, $A_6^4 \equiv$ "More Average"$\equiv \left(\mu_{A_6^4}(x_6; 0.6,0.7,0.83)\right)$ and $A_6^5 \equiv$ "More"$\equiv \left(\mu_{A_6^5}(x_6; 0.8,1,1)\right)$.
Moreover, the output is described by triangular MFs, $(\mu_{B^{j_1 \dots j_6}}(y))$, and its linguistic term that represented as the following:
$B^1 \equiv$ "SLD" $\equiv (\mu_{B^1}(y; 0,0,0.75))$ and $B^2 \equiv$ "ALD"$\equiv (\mu_{B^2}(y; 0.25,1,1))$.
The fuzzy inference process defines as the following:

$$FI: A' \to \overline{B^{j_1 \dots j_6, i}}, \tag{43}$$

where A' is an input fuzzy set in the input space *X*, and $\overline{B^{j_1 \dots j_6, i}}$ the fuzzy sets in the output space *Y*. Each one of the rules specifies a fuzzy set $\overline{B^{j_1 j_2 j_3, i}} \subseteq Y$ that is given by the compositional rule of inference:

$$\overline{B^{j_1 \dots j_6, i}} = A' \circ \left(A_j^{j_1 \dots j_6, i} \to B^{j_1 \dots j_6, i}\right), \text{where } A_j^{j_1 \dots j_6, i} = A_1^{j_1 \dots j_6, i} \times A_2^{j_1 \dots j_6, i} \times \dots \times A_6^{j_1 \dots j_6, i} \tag{44}$$

Therefore, from $\mu_{A_1^i \times \dots \times A_n^i}(x_1, \dots, x_n) = \mu_{A_1^i}(x_1) * \dots * \mu_{A_n^i}(x_n)$, we obtain

$$\mu_{A_j^{j_1 \dots j_6, i}}(x_j) = \mu_{A_1^{j_1 \dots j_6, i} \times A_2^{j_1 \dots j_6, i} \times \dots \times A_6^{j_1 \dots j_6, i}}(x_1, \dots, x_6), (j = 1,2, \dots, 6) \tag{45}$$

From $\left(\mu_{B'}(y) = \sup_{x \in X} \{\mu_{A'}(x) \overset{t}{*} \mu_{Q_I}(x, y)\}\right)$, the fuzzy sets $\overline{B^{j_1 \dots j_6, i}}$ are described by MF:

$$\mu_{\overline{B^{j_1 \dots j_6, i}}}(y) = \sup_{x \in X} \left\{\mu_{A'}(x) \overset{t}{*} \mu_{\left(A_j^{j_1 \dots j_6, i} \to B^{j_1 \dots j_6, i}\right)}(x, y)\right\} \tag{46}$$

Consequently, can be re-express (46) as the following

$$\mu_{\overline{B^{j_1 \dots j_6, i}}}(y) = \mu_{A_j^{j_1 \dots j_6, i} \to B^{j_1 \dots j_6, i}}(x_j, y) = Im\left(\mu_{A_j^{j_1 \dots j_6, i}}(x_j), \mu_{B^{j_1 \dots j_6, i}}(y)\right), \tag{47a}$$

where $Im(.)$ is an "implementation". Since, we used Mamdani's minimum implication (10a), therefore, we obtained:

$$Im\left(\mu_{A_j^{j_1\dots j_6,i}}(x_j),\mu_{B^{j_1\dots j_6,i}}(y)\right)=min\left\{\mu_{A_j^{j_1\dots j_6,i}}(x_j),\mu_{B^{j_1\dots j_6,i}}(y)\right\} \tag{47b}$$

or (47) may be written as:

$$\mu_{\overline{B^{j_1\dots j_6,i}}}(y)=min\left\{\mu_{A_j^{j_1\dots j_6,i}}(x_j),\mu_{B^{j_1\dots j_6,i}}(y)\right\} \tag{48}$$

The aggregation operator has been applied in order to get the fuzzy set B' that uses the functions *'max'* (s-norm) or *'min'* (t-norm) depending on the type of fuzzy implication. The aggregation operator is denoted by:

$$B'=\cup_{i=1}^{I}\overline{B^{j_1\dots j_6,\ i}}\ , \tag{49}$$

Therefore, the membership function of B' is computed using the 'max' function as the following:

$$\mu_{B'}(y)=\max_{\forall i}\ \{\mu_{\overline{B^{j_1 j_2 j_3,\ i}}}(y)\} \tag{50}$$

Stepe3:

The defuzzifier performs a mapping as the following:

$$def=B'\rightarrow f(x), \tag{51}$$

where B' is a fuzzy set, $f(x)$ is a crisp point $f(x)\in(Y\subset R)$. The center of the area is a final system output that is defined by the following formula:

$$f(x)=\frac{\sum_{j_1=1}^{5}\dots\sum_{j_6=1}^{5}\bar{y}^{j_1\dots j_6}\left(min\left(max\left(\min_{j=1:6}\left(\frac{x-a_j^{j_1\dots j_6}}{b_j^{j_1\dots j_6}-a_j^{j_1\dots j_6}},\ \frac{c_j^{j_1\dots j_6}-x}{c_j^{j_1\dots j_6}-b_j^{j_1\dots j_6}}\right),\ 0\right)\right)\right)}{\sum_{j_1=1}^{5}\dots\sum_{j_6=1}^{5}\left(min\left(max\left(\min_{j=1:6}\left(\frac{x-a_j^{j_1\dots j_6}}{b_j^{j_1\dots j_6}-a_j^{j_1\dots j_6}},\ \frac{c_j^{j_1\dots j_6}-x}{c_j^{j_1\dots j_6}-b_j^{j_1\dots j_6}}\right),\ 0\right)\right)\right)} \tag{52}$$

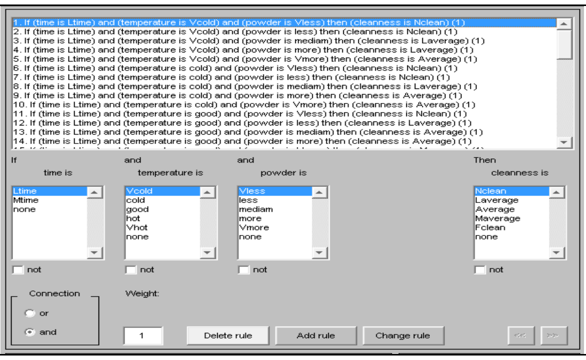


Figure 4a: Representation of the rules editor

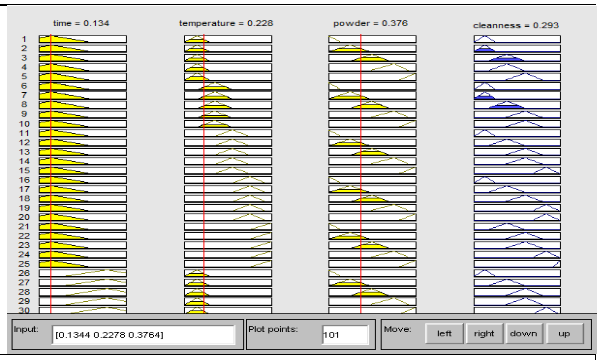
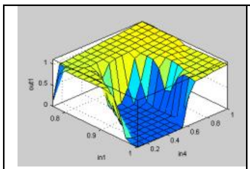


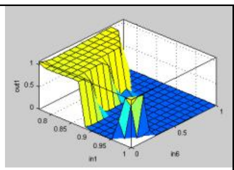
Figure 4b: The rule view of the FIS

The model of fuzzy system f(x) has been built from the I rules of (42a) using Mamdani's minimum implication (10a) with the MIS (10b), the SF (11), and the CAD (12) (see Figure 4). We have created two programs; the first program depends on the Mamdani model, while the second program depends on the ST model. The programs of FIS have applied, that is given by (52), and obtained a good result of target. Therefore, we have applied the measure for accuracy of all data. The deference between the actual and target outputs is given by the formula as the following:

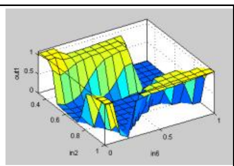
$$Error=|Rv-FISv|\ , \tag{53}$$



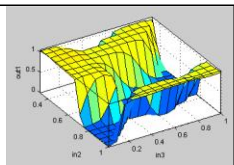
(a) between I1 & I4



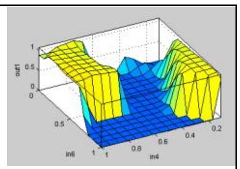
(b) between I1 & I6



(c) between I6 & I2



(d) between I2 & I3



(e) between I6 & I4

Figure 5: The output surface for the different inputs

where *Rv* is actual output values and *FISv* is the output target values. The value of accuracy is a very small, where the average error of Simple LD=0.2047 and the average error of Acute LD=0.1158, when FIS is used the Mamdani fuzzy rule model. While, when the FIS is used the ST fuzzy rule model, then the average error of Simple LD=0.1463 and the average error of Acute LD=0.0061. Table 2 represents all the results. The 'surf view' tool is the surface viewer that helps view the input-output surface of the FIS. This conception is very helpful to understand how the system is going to behave for the entire range of values in the inputs space. Figure 5 shows the output surface for the different inputs.

3.3. Discussion and Results of the Adaptive Fuzzy System Using Neural Network

In this Section, specify the structure of the fuzzy system to be modeled. Here, we choose the fuzzy system with a MIS, a SF, the CAD, and a Triangular MF that given by the model (52). Model (52) has not been modeled, because the free parameters $\bar{y}^{j_1 \dots j_6}, a_j^{j_1 \dots j_6}, b_j^{j_1 \dots j_6}$, and $c_j^{j_1 \dots j_6}$ are not specified. These parameters should be determined in order to represent the $f(x)$. The Model of the adaptive fuzzy system using neural network may be express as the following:

Step1: Determine the initial parameters $\bar{y}^{j_1 \dots j_6}(0), a_j^{j_1 \dots j_6}(0), b_j^{j_1 \dots j_6}(0), and\ c_j^{j_1 \dots j_6}(0)$ according to the linguistic rules from experts, such as when $j = j_h = 1, (h = 1, \dots, 6)$, then $a_1^1(0) = 0, b_1^1(0) = 0, c_1^1(0) = 0.18$, similarly for $\forall\ j_h$, and j and for each input.

Step2: From a given inputs-output pair $\left(x_{1,0}^p, \dots, x_{6,0}^p;\ y_0^p\right)$, $\forall\, p$ (observe), compute the outputs of layers as the following:

(i) The node of the first output $o_1^{j_i}$ is represented by:
$o_1^{j_1} = \mu_{A_1^1}(x_1), (j_1 = 1, \dots, 5); o_1^{j_2} = \mu_{A_1^1}(x_1), (j_2 = 5, \dots, 10);\ o_1^{j_3} = \mu_{A_1^1}(x_1), (j_3 = 10, \dots, 15); o_1^{j_4} = \mu_{A_1^1}(x_1), (j_4 = 15, \dots, 20);\ o_1^{j_5} = \mu_{A_1^1}(x_1), (j_5 = 20, \dots, 25)$ and $o_1^{j_6} = \mu_{A_1^1}(x_1), (j_6 = 25, \dots, 30)$,
where $o_1^{j_i}$ is the degree of MFs of a fuzzy set.

(ii) To compute the second output, we should use the minimum function of all MFs as:

$$o_2^{j_1 \dots j_6} = \min_{\forall j_1 \dots j_6} \left(\mu_{A_1^{j_1 \dots j_6}}(x_1), \mu_{A_2^{j_1 \dots j_6}}(x_2), \dots, \mu_{A_6^{j_1 \dots j_6}}(x_6) \right). \quad (54)$$

Since $\mu_{A_1^{j_1 \dots j_6}}$ is a Triangular MF, therefore, we obtain:

$$o_2^{j_1 \dots j_6} = \min_{\forall j_1 \dots j_6} \left(max \left(\min_{\forall k} \left(\frac{x_{k0}^p - a_k^{j_1 \dots j_6}(q)}{b_k^{j_1 \dots j_6}(q) - a_k^{j_1 \dots j_6}(q)}, \frac{c_k^{j_1 \dots j_6}(q) - x_{k0}^p}{c_k^{j_1 \dots j_6}(q) - b_k^{j_1 \dots j_6}(q)} \right), 0 \right) \right). \quad (55)$$

(iii) For all rules, we have calculated the $j_1 \dots j_6{}^{th}$ node as:

$$o_3^{j_1 \dots j_6} = \left(\frac{\min_{\forall j} \left(max \left(\min_{\forall k} \left(\frac{x_{k0}^p - a_k^{j_1 \dots j_6}(q)}{b_k^{j_1 \dots j_6}(q) - a_k^{j_1 \dots j_6}(q)}, \frac{c_k^{j_1 \dots j_6}(q) - x_{k0}^p}{c_k^{j_1 \dots j_6}(q) - b_k^{j_1 \dots j_6}(q)} \right), 0 \right) \right)}{\sum_{j_1=1}^5 \dots \sum_{j_6}^5 \min_{\forall j} \left(max \left(\min_{\forall k} \left(\frac{x_{k0}^p - a_k^{j_1 \dots j_6}(q)}{b_k^{j_1 \dots j_6}(q) - a_k^{j_1 \dots j_6}(q)}, \frac{c_k^{j_1 \dots j_6}(q) - x_{k0}^p}{c_k^{j_1 \dots j_6}(q) - b_k^{j_1 \dots j_6}(q)} \right), 0 \right) \right)} \right). \quad (56)$$

(iv) The fourth output depends on a node MF of output $\bar{z}^{j_1 \dots j_6}$ with an adaptive node $\bar{y}^{j_1 \dots j_6}$, therefore, we obtain

$$o_4^{j_1 \dots j_6} = o_3^{j_1 \dots j_6} f_{j_1 \dots j_6}(x_1, x_2, \dots, x_6) \quad (57)$$

Since $f_{j_1 \dots j_6}(x_1, x_2, \dots, x_6) = \alpha_1^{j_1 \dots j_6} x_1 + \alpha_2^{j_1 \dots j_6} x_2 + \dots + \alpha_6^{j_1 \dots j_6} x_6 + \alpha_7^{j_1 \dots j_6}$, then we should determine the initial parameters of $\left(\alpha_j^{j_1 \dots j_6}, \forall\, j = 1, \dots, 7.\right)$.

(v) In order to compute the final output, we must take the summation of all results $o_4^{j_1 \dots j_6}$ as the following:

$$o_5^{j_1 \dots j_6} = \sum_{j_1=1}^{5} \dots \sum_{j_6=1}^{5} \frac{o_2^{j_1 \dots j_6}}{\sum_{j_1=1}^5 \dots \sum_{j_6}^5 o_2^{j_1 \dots j_6}} * \left(\alpha_1^{j_1 \dots j_6} x_1 + \alpha_2^{j_1 \dots j_6} x_2 + \dots + \alpha_6^{j_1 \dots j_6} x_6 + \alpha_7^{j_1 \dots j_6} \right) \quad (58)$$

Step3: In step 2, it is specified the initial parameters of $\bar{y}^{j_1..j_6}(q), a_k^{j_1..j_6}(q),\ b_k^{j_1..j_6}(q), c_k^{j_1..j_6}(q)$ and $q=0$. Consequently, we update these parameters at $q+1$, using the least squares model and repeat all the procedure in step 2 for compute K, L and f. From (37) to (39), we compute the new parameters as follows:

$$\theta(2) = \theta(1) + t(2)\left[0.462 - \left(o_2^{j_1..j_6}\right)^T \theta(1)\right]$$
$$t(2) = \frac{P(2)\ o_2^{j_1..j_6}}{\left[P(2)\ o_2^{j_1..j_6}\left(o_2^{j_1..j_6}\right)^T + 1\right]} \qquad (59)$$
$$P(2) = 1 - \frac{P(1) o_2^{j_1..j_6}}{\left[P(1)\ o_2^{j_1..j_6}\left(o_2^{j_1..j_6}\right)^T + 1\right]}\ 1 * \left(o_2^{j_1..j_6}\right)^T$$

Similarly, we can update all parameters of θ for all training data. Repeat all procedures in step 2 with $q = q + 1$ ttil the last specific number of q is reached.

From the program of the AFS using a Neural Network that is given by model (52), we have obtained better results with the AFS using a Neural Network than the result of the FIS. We have applied the measure of accuracy for all data, and obtained the average error of Simple LD=0.000943, while the average error of Acute LD=0.0055, (see Table 2). Additionally, obtained the best average testing error of training data (0.00000679) with epoch 100, and the average testing error of checking data (0.03802), see Figure 6. As well as, Table 3 presents the representation of results of the FIS with Mamdani model, the FIS with ST models, and the ANFIS with their errors through Appendix.

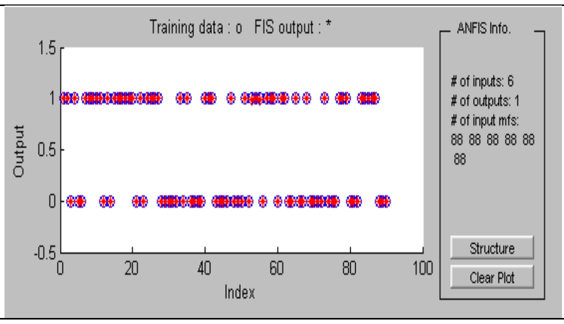


Figure 6a: Representation of the errors of training data with 100 epoch

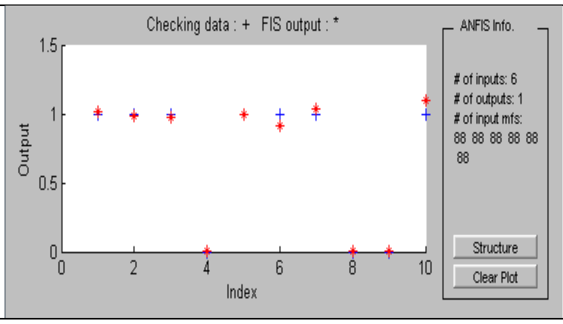


Figure 6b: Representation of the errors of checking data with 100 epoch

Table 2: Representation errors of Simple, and Acute Liver Disorder and average error for different models.

Type of model	Average error of Simple LD	Average error of Acute LD	Average error of training data
FIS (Mamdani model)	0.2047	0.1158	0.1603
FIS (ST model)	0.1463	0.0061	0.0762
ANFIS (ST model)	0.000943	0.0055	0.0032

IV. Conclusion

This work focused on how to use the fuzzy models to solving fuzzy mathematics problems. Here constructed two different models, namely fuzzy inference system, and adaptive neuro-fuzzy inference system. Further, suggested an extension FS and ANFS at N-dimension those depended on the MMI with the MIS, the SF, and the CAD. It is provided the theorem for accuracy of proposed models, as well as, adapted the model of an adaptive FS using a neural network. In addition, we have provided a medical application of the fuzzy mathematics models. We have diagnosed the liver disorder disease that is a high interest to researchers of fuzzy modeling and fuzzy system because the liver is the largest internal member in the human body. Therefore, we have applied the fuzzy mathematical models on the real data to the liver disorder disease. We have presented discussion and results for the model of the FS with Mamdani and ST models, and the ANFIS, respectively. Additionally, we have used the software 'MATLAB' in order to perform the results of different models. Consequently, we have gotten good results for accuracy of these models. The comparison between the three models the FS with Mamdani model, the ST model, and the ANFIS had presented. We have obtained the best result with the ANFIS. Finally, we have presented the representation of results of the different models with their errors through Appendix. In the future work, we can develop these models of fuzzy system to generate many outputs or extending the number of input variables. As well as, we can change the fuzzy inference system with another types, or with different type of fuzzifier or defuzzifier.

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APENDIX

In this Section, present the representation of results of the FIS with Mamdani model, the FIS with ST models, and the ANFIS with their errors.

Table 3: Representation of results of FIS and ANFIS with their errors

N0.	Real	FIS with (Mamdani model)	$E(FIS_M)$	FIS with (ST model)	$E(FIS_{ST})$	ANFIS with (ST model)	$E(ANFIS_{ST})$
1.	1	0.915	0.085	1.27	-0.27	1.00E+00	0.00E+00
2.	1	0.916	0.084	1.24	-0.24	1.00E+00	0.00E+00
3.	0	0.197	-0.197	0.225	-0.225	0.00E+00	0.00E+00
4.	0	0.222	-0.222	0.202	-0.202	0.00E+00	0.00E+00
5.	0	0.186	-0.186	0.2	-0.2	0.00E+00	0.00E+00
6.	1	0.907	0.093	1.2	-0.2	1.00E+00	0.00E+00
7.	0	0.2	-0.2	0.192	-0.192	0.00E+00	0.00E+00
8.	1	0.909	0.091	1.19	-0.19	0.998	0.002
9.	0	0.197	-0.197	0.188	-0.188	3.04E-03	-3.04E-03
10.	0	0.208	-0.208	0.183	-0.183	-5.85E-07	5.85E-07
11.	0	0.238	-0.238	0.177	-0.177	1.81E-05	-1.81E-05
12.	0	0.196	-0.196	0.177	-0.177	0.00E+00	0.00E+00
13.	0	0.208	-0.208	0.172	-0.172	0	0
14.	1	0.912	0.088	1.17	-0.17	9.99E-01	1.00E-03
15.	0	0.184	-0.184	0.165	-0.165	1.21E-06	-1.21E-06
16.	1	0.187	0.813	1.16	-0.16	1.00E+00	0.00E+00
17.	0	0.197	-0.197	0.156	-0.156	0.00E+00	0.00E+00
18.	0	0.202	-0.202	0.155	-0.155	-4.40E-08	4.40E-08
19.	0	0.224	-0.224	0.151	-0.151	3.60E-07	-3.60E-07
20.	0	0.205	-0.205	0.151	-0.151	-3.10E-08	3.10E-08
21.	0	0.202	-0.202	0.149	-0.149	4.98E-05	-4.98E-05
22.	0	0.185	-0.185	0.149	-0.149	-1.00E-08	1.00E-08
23.	0	0.205	-0.205	0.148	-0.148	0.00E+00	0.00E+00
24.	0	0.21	-0.21	0.147	-0.147	1.70E-08	-1.70E-08
25.	0	0.197	-0.197	0.144	-0.144	2.06E-07	-2.06E-07
26.	0	0.203	-0.203	0.144	-0.144	2.28E-03	-2.28E-03
27.	0	0.199	-0.199	0.143	-0.143	5.26E-04	-5.26E-04
28.	0	0.197	-0.197	0.141	-0.141	1.20E-08	-1.20E-08
29.	0	0.216	-0.216	0.141	-0.141	0.0384	-0.0384
30.	0	0.2	-0.2	0.14	-0.14	-1.10E-08	1.10E-08
31.	1	0.905	0.095	1.14	-0.14	0.948	0.052
32.	0	0.202	-0.202	0.139	-0.139	-1.10E-08	1.10E-08
33.	0	0.199	-0.199	0.138	-0.138	-7.54E-07	7.54E-07
34.	0	0.196	-0.196	0.138	-0.138	0	0
35.	0	0.185	-0.185	0.136	-0.136	-4.60E-08	4.60E-08
36.	0	0.254	-0.254	0.135	-0.135	-1.07E-07	1.07E-07
37.	0	0.21	-0.21	0.135	-0.135	6.10E-08	-6.10E-08
38.	0	0.2	-0.2	0.134	-0.134	0.00E+00	0.00E+00
39.	0	0.204	-0.204	0.132	-0.132	-3.33E-07	3.33E-07
40.	0	0.192	-0.192	0.131	-0.131	8.91E-07	-8.91E-07
41.	0	0.174	-0.174	0.13	-0.13	9.00E-08	-9.00E-08
42.	1	0.908	0.092	1.13	-0.13	1.00E+00	0.00E+00
43.	1	0.917	0.083	1.13	-0.13	0.992	0.008
44.	1	0.908	0.092	1.13	-0.13	1.00E+00	0.00E+00
45.	0	0.405	-0.405	0.128	-0.128	-2.37E-07	2.37E-07
46.	0	0.205	-0.205	0.126	-0.126	5.00E-09	-5.00E-09

47.	0	0.204	-0.204	0.126	-0.126	2.48E-07	-2.48E-07
48.	0	0.192	-0.192	0.126	-0.126	0	0
49.	0	0.178	-0.178	0.125	-0.125	-0.000193	0.000193
50.	0	0.172	-0.172	0.125	-0.125	0.000215	-0.000215
51.	0	0.216	-0.216	0.124	-0.124	2.32E-07	-2.32E-07
52.	0	0.178	-0.178	0.123	-0.123	-2.55E-05	2.55E-05
53.	0	0.17	-0.17	0.122	-0.122	-2.00E-08	2.00E-08
54.	0	0.2	-0.2	0.117	-0.117	4.70E-08	-4.70E-08
55.	0	0.214	-0.214	0.117	-0.117	-4.30E-08	4.30E-08
56.	0	0.192	-0.192	0.116	-0.116	-6.00E-07	6.00E-07
57.	0	0.2	-0.2	0.113	-0.113	4.70E-07	-4.70E-07
58.	1	0.913	0.087	1.11	-0.11	1	0
59.	1	0.91	0.09	1.11	-0.11	1	0
60.	1	0.909	0.091	1.11	-0.11	0.993	0.007
61.	1	0.91	0.09	1.1	-0.1	1	0
62.	1	0.904	0.096	1.1	-0.1	1	0
63.	1	0.911	0.089	1.09	-0.09	1	0
64.	1	0.907	0.093	1.09	-0.09	1	0
65.	1	0.912	0.088	1.06	-0.06	1	0
66.	1	0.909	0.091	1.05	-0.05	1	0
67.	1	0.91	0.09	1.04	-0.04	1	0
68.	1	0.907	0.093	1.04	-0.04	1	0
69.	1	0.911	0.089	1.03	-0.03	1	0
70.	1	0.907	0.093	1.01	-0.01	1	0
71.	1	0.928	0.072	0.995	0.005	1	0
72.	1	0.909	0.091	0.99	0.01	1	0
73.	1	0.916	0.084	0.989	0.011	1	0
74.	1	0.904	0.096	0.983	0.017	1	0
75.	1	0.916	0.084	0.979	0.021	1	0
76.	1	0.912	0.088	0.965	0.035	1	0
77.	1	0.909	0.091	0.963	0.037	1	0
78.	1	0.909	0.091	0.959	0.041	1	0
79.	1	0.917	0.083	0.956	0.044	1	0
80.	1	0.905	0.095	0.956	0.044	1	0
81.	1	0.913	0.087	0.955	0.045	1	0
82.	1	0.901	0.099	0.954	0.046	1	0
83.	1	0.907	0.093	0.951	0.049	1	0
84.	1	0.909	0.091	0.918	0.082	1	0
85.	1	0.909	0.091	0.915	0.085	1	0
86.	1	0.909	0.091	0.903	0.097	1	0
87.	1	0.909	0.091	0.903	0.097	1	0
88.	1	0.909	0.091	0.896	0.104	1	0
89.	1	0.909	0.091	0.89	0.11	1	0
90.	1	0.907	0.093	0.884	0.116	1	0
91.	1	0.925	0.075	0.883	0.117	1	0
92.	1	0.914	0.086	0.882	0.118	0.925	0.075
93.	1	0.912	0.088	0.881	0.119	1	0
94.	1	0.905	0.095	0.88	0.12	0.856	0.144
95.	1	0.915	0.085	0.878	0.122	1	0
96.	1	0.2	0.8	0.872	0.128	1	0
97.	1	0.918	0.082	0.87	0.13	1	0
98.	1	0.921	0.079	0.864	0.136	1	0
99.	1	0.918	0.082	0.859	0.141	1	0
100.	1	0.921	0.079	0.85	0.15	1	0

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