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Relation between '3 Utility Problem' And 'Eulerian Trail'

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Abstract: There are two most renowned puzzles whose algorithms have been developed in the computer science (field- Algorithms and design Analysis) that are impossible to solve till date. They are 3 utility problem and Eulerian trail. Both of them have some methods and have to be done according to some rule. A lot of work has been done to find why this happens and what application can be drawn out of it. The answer is though simple, these problems cannot be solved in 2- dimension and we need a 3-dimensional environment to solve them.

But hardly anyone have ever thought that ***Eulerian theorem is not a new thing but a result of 3 utility problem***, means if 3 utility problem exists then Eulerian puzzle too. Actually both puzzles are invented in different time but are related to one other, though their methods may be different. This paper will prove that why some figure are Eulerian Trial and other not. It will give result that if we don't follow rules of 3 utility problem then we cannot make any Eulerian Trail. *The paper gives six conditions/ rules and two theorems that will help in making Algorithms for finding relation between these two puzzles.* Means by following these rules one can make Algorithm easily for showing relation between two puzzles as shown in 'Explanation by an example' section.
Both the problems are giving same result that is we need a 3-dimensional environment. So there must be something that creates a path in between these two problems. Moreover this research work will give answers of some other questions like – Why Eulerian trial occur when there are 3 or more than 3 internal figures exists in any figure with odd number of edges?

Keywords: closed figures, *edges, Eulerian Trail, vertices, 3 utility problem*

I. INTRODUCTION

Eulerian Trail is – to draw a line in such a way that it crosses every edge in a figure shown below without crossing it twice and without leaving any edge untouched. The figure is given below with edges and vertices; it shows that whatever way you will take but at least one edge will not ever be covered by our line.

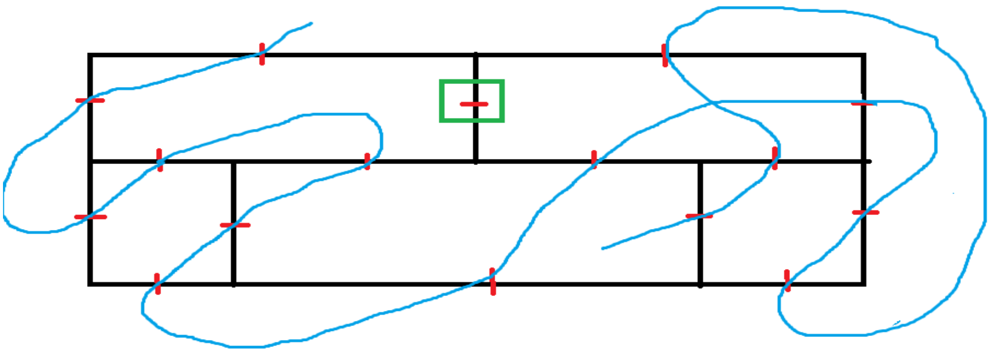


fig. 1

In this example one can clearly see that he has to cover all the red marks by a blue line, without crossing the same edge (or red mark) which he has crossed earlier. But a green rectangle here is showing that our blue line is unable to cross the red mark inside it. One can try any number of ways but he will be unable to cover all the marks.

3 Utility Problem '3 utility problem' says that if we have 3(or more) service providers and 3(or more) houses that need services, then we need to connect each house by every service provider such that no two lines drawn between service provider and house cross or overlap each other.

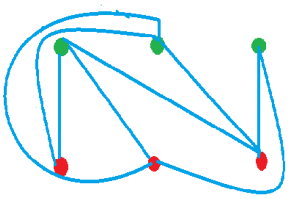


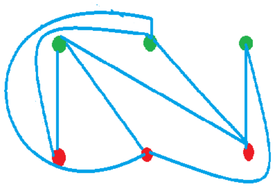
fig.2

Here green points are service provider and red are houses that need services. So, one can see that the 3rd green point is unable to touch the 1st red point.

II. THE PROPOSED METHOD

Relation between '3 Utility problem' and 'Eulerian trial'.

Now we know that what actually are these problems, so let's start with '3 utility problem'. Let's draw diagram of 3 utility problem for 3 houses and 3 service providers, here you have to try to connect each service provider to each home, until all routes are blocked or no more connection between service provider and home can be established according to rules.

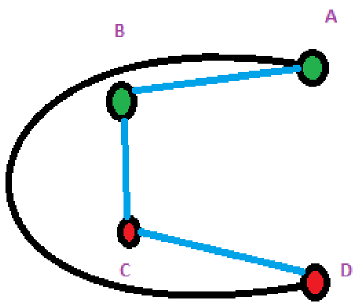


Fig(2)

Here green points are three service providers and red points are houses that need services, so one can see that the 3rd green point is unable to touch the 1st red point. Now to draw a relation between 2 puzzles, we need to convert the 3 utility problem in form of Eulerian puzzle and *after whole discussion you will be able to know that why some diagrams cannot form Eulerian puzzle* -

Condition 1- any line from service provider to houses that covers some points under it and if 2 of those points are consecutive to each other (not diagonally), make one straight line between those consecutive points or can draw odd number of discrete lines or a combination of both as-

Case-1- any line from service provider to houses that covers 2 points under it such that those points are consecutive to each other (not diagonally), make one straight line between those consecutive points



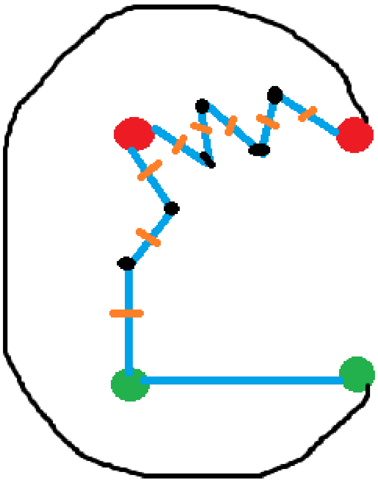
Rule (1)

Here according to rule if service provider A is touching home D by crossing service provider B and home C as shown in figure (shown by black line), then it is covering points A, B ,C, D and point A and B, B and C and C and D are consecutive to each other so draw a line A-B, B-C, C-D(shown by blue line). If any of line is already connected no need to connect is twice as if let's say BC was already connected and when a black curved line between A and D is drawn we need to connect only A-B and C-D.

Now Condition 1 also says one can draw odd number of discrete lines between consecutive points. To understand this, we need to understand what is discrete line? Look at diagram below-

Rule (2)

Here you can see that from 1st green node a blue line is connecting 1st red node, but the blue line is neither straight nor continuous, it is discrete or have 3 discrete parts, so you can make this kind of discrete lines too, between service provider and home, but their number must be odd, here between 2 (red and green) nodes thee blue discrete lines are three, shown by orange marks.
We have also said that it can be a combination of above mention ways-

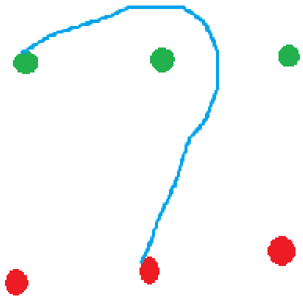


Rule(3)

- * In every 3 utility problem we means, to draw every lines between service provider and home till it become impossible to draw any other line

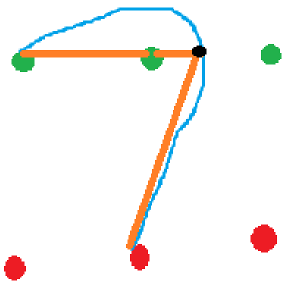
Here it must be clear that instead of straight lines between two consecutive red points or between red and green point we have drawn odd number of discrete lines. You cannot intersect any discrete lines (follow 3 utility rule, any line that has been formed earlier can never be crossed)

(ii) If any line in 3 utility goes in the way given below-



Rule (4)

Join the consecutive points (not diagonally) between the blue curve by condition 1 and also the make one more point between the closed green node and the open green node as-

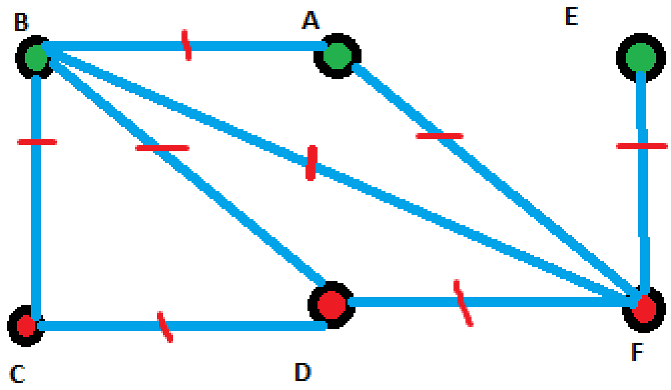


Here orange line is showing the way you have to connect points for drawn blue line. And we have made one more node shown by black point. One can also go for odd number of discrete points along with straight line as condition 1 remains true here too, just we have to make one more node.

Condition 2- All internal (diagonal points) lines between 2 nodes will be drawn straight or can be an odd number of internal discrete lines, such that no two internal discrete lines can intersect each other at any point, and if any two of the discrete lines are parallel, we can coincide that, though it is not necessary.

Condition2-part-1→ all internal lines will be drawn straight. Internal lines are lines like-B-D, B-F, A-F in diagram below, and such that no two lines can intersect each other at any point (basic condition of 3 utility diagram

So by combining Condition 1 and condition 2 part-1 we can make the 3 utility problem in figure 2 as



Rule (5)

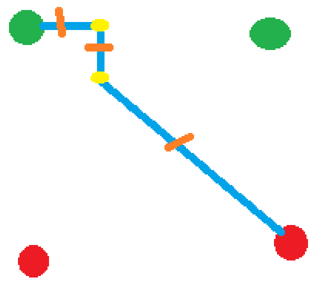
Try to solve this problem by Eulerian method, you will be unable.

This is the proof that both of the puzzle are same, just condition is different. Means **the closed figure formed out of 3 utility problem (here ABCDF) is the smallest Eulerian puzzle because any problem less than 3 utility can be solved. So Eulerian is actually derived from 3 utility problem. Means E is unable to meet C and so it is a '3 utility problem' and no line can cross exactly once to every edge of the figure drawn above so it is an 'Eulerian trail'.**

Condition-2-part-2→ all internal lines (diagonal points) will be odd number of internal discrete lines such that no two internal discrete lines can intersect each other at any point and if any two of the discrete lines are parallel, we can coincide that, though it is not necessary.

Before starting this condition we must understand the meaning if internal discrete lines between 2 points.

See the diagram below-

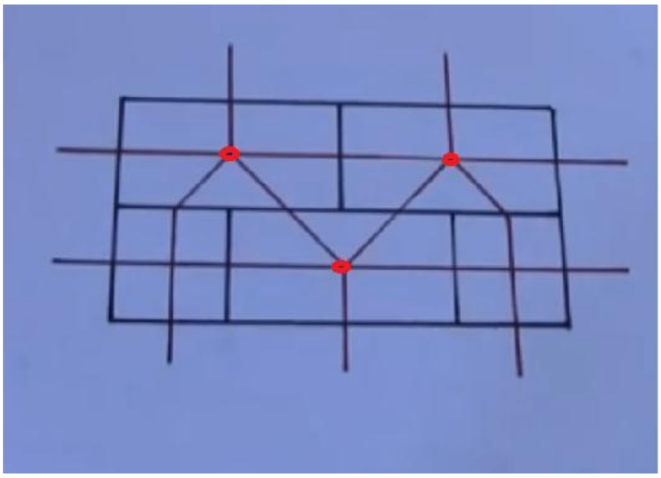


Rule (6)

Here you can see that from 1st green node a blue line is connecting 2nd red node, but the blue line is neither straight nor continuous, it is discrete or have 3 discrete parts, so you can make this kind of discrete lines too, between service provider and home, but their number must be odd, here between 2 nodes this blue internal discrete lines are three, shown by orange marks, it is same as discrete lines mentioned in condition 1 just we have added word internal to show that here lines are diagonal always.

Why we have to draw odd number of discrete lines are used between two vertices in 3 utility problem to make it an Eulerian puzzle?

Eulerian trail are all those figures which have 3 or more internal figures such that from their center, odd lines go outside as shown-



Here the rectangular blocks inside this whole Eulerian trail, having red points are the 3 internal figure because of which we can make this figure as Eulerian Trail, but question arises, why?
Is there any other simpler method for explanation point of view? The answer is YES.
<u>This paper also gives a new idea of understanding that why Eulerian trail occur?</u>
The actual answer that is easier to understand is hidden in edges of those internal figures.
Look the figure below it is like triangle BCD of the figure 3 drawn above-

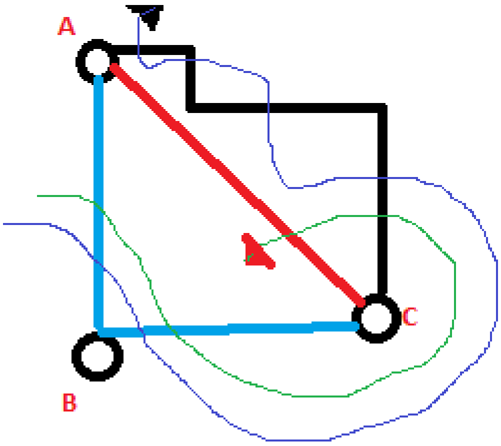
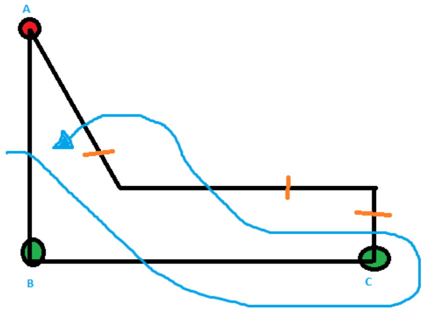


fig.4

Here you can see that if we draw all straight line like AB, BC, CA (red one) then **one end** of our green line traversing all edges AB, BC, CA with red arrow is trapped inside triangle ABC, while if we go for internal **discrete line**s between AC (black one) then both the ends of our dark blue line with black head are free.
Now 3 utility problem says, you cannot cross any line drawn earlier, means once you create lines CA, AB and BC no line can come inside this triangle, that's what you need to do in any closed figure to make it an Eulerian trail. So according to rule if we have drawn that black line between A and C in **3 utility problem**, then no line from outside can come inside the figure ABC (with black line) because here black though is looking as 4 discrete lines but for 3 utility problem it is just a line dividing a plane into two parts and the plane inside the black line cannot be used. But according to **Eulerian Trail** discrete lines are not a single line so we have to traverse every discrete line (as method of proving same thing is different) shown by dark blue line whose both ends are free, means **if you will not follow condition of 3 utility problem you cannot make Eulerian Trail .**
<u>So you can clearly see that meaning of both the puzzles are same, once you crossed or make any line you cannot traverse it back and that is why if you make even number of internal discrete lines in any 3 utility problem, it takes it as one single (odd) line not as 3 or 4 different discrete lines but Eulerian trail takes, and that is if you make any figure with internal figures having even number of edges, it actually never makes any Euler trail, because you are not following 3 utility condition, so any Eulerian trail is an outcome of 3 utility problem only.</u>
Moreover 3 utility problem needs at least 3 service providers and 3 homes that is why Euler trail needs 3 closed internal figure as this world 3 as if telling 3-D environment is needed to solve both of the puzzles.

So here if we draw that non linear black line with and odd edges we can make one end of the black line trapped inside the figure formed by nodes ABC as shown below-



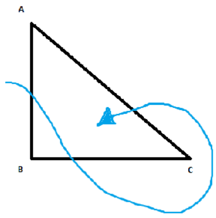
Rule (7)

Means discrete lines between A and C must be odd, **shown by orange marks**. Here discrete lines between A and C are 3. This is simple to explain that any line traversing any closed figure can do 2 things to go outside of the closed region or to come inside the closed region, means 2 jobs and if number of lines in closed figure is odd, then if one end point initially moves from outside to inside the second end of same line when traverse the last remaining edge of the closed figure will also move from outside to inside, and get trapped and vice versa (here also you will be trapped), means atleast one end of line is trapped.

"So, *now the answer for-how just by looking any figure you can declare that any figure is an Eulerian Trail and thus we have prooved what Eulerian trail theorem does means –*

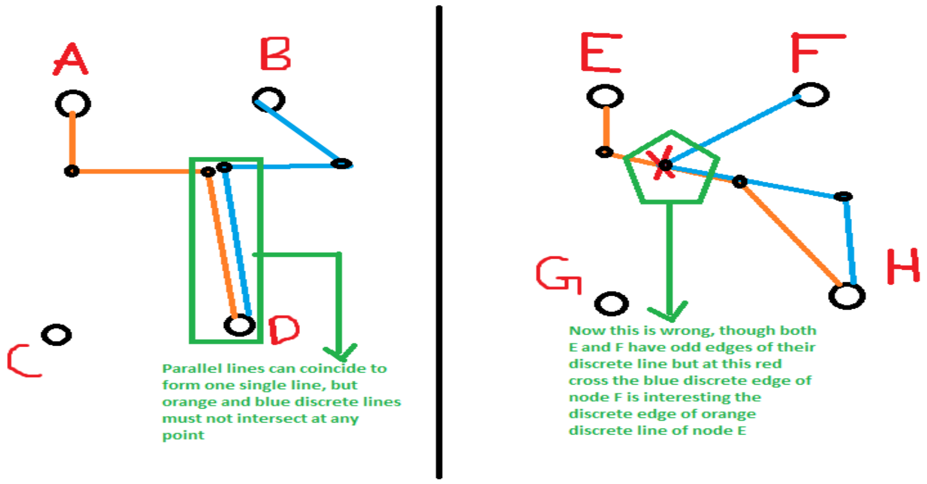
Simply check for atleast 3 closed figures inside the diagram given to you, and each of them having odd number of edges, the diagram will always be an Eulerian Trail and thus 3 utility problem.

Here one end of the blue line is trapped inside the drawn figure. Means here once you crossed all the discrete line between A and C you cannot go outside of region ABC as actually was happening when we were drawing straight line between A-C in 3 utility problem-



Both the puzzles have one main aim that is "TRAP". In 3 utility problem we have to "Trap the vertices so that at least 2 of them remained untouched or non connected" (one of service provider and other of house)While in Eulerian puzzle we have to "trap the two ends of the line that has to cross all the edges without leaving any of them and without crossing them twice"

Both the puzzles are depicting same meaning by using different method and any Eulerian trail can only be formed if it follow the condition of 3 utility problem. *So this means the final condition of both the puzzles are same though methods are different, and that is why some condition must be followed to convert a 3 utility problem into Eulerian trail, so that different methods of both puzzles gives same result.* Now look at the diagram below it will explain the meaning that how two discrete lines cannot intersect each other-



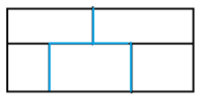
Rule (8)

Here the left figure is showing that 2 of the six internal discrete lines between AD and BD are parallel (inside green rectangular box), means you can combine to make them a single line to make 3 utility problem an Eulerian trail, even if you do not make them coincide, still that 3 utility figure will form some other kind of an Eulerian trail. But you cannot intersect internal discrete lines (rule of 3 utility problem not to override any drawn line earlier) from two different points as shown in right side diagram, with a red cross.

THEOREM-"Y1"

Every 3 utility problem can generate an Eulerian trail if and only all **lines are drawn straight or odd number of discrete lines are drawn between service providers and homes or a combination of both, such that no two internal discrete lines or discrete lines can intersect each other at any point, and if any two of the discrete lines are parallel, we can coincide them, though it is not necessary means if** ***diagram follows the conditions of 3 utility problem then only can make any Eulerian Trail.***

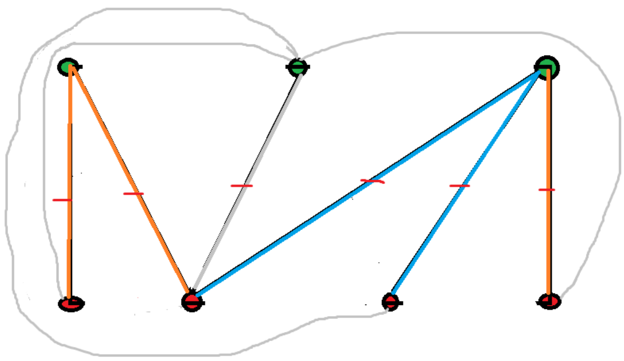
Let's test for inverse, that if figure is Eulerian puzzle it must be somehow related to 3 utility problem.



This figure actually is 3 service providers and 4 houses problem, for sure if number of service provider>=3 and number of homes >=3 then any combination between them holds true for 3 utility problem. Means 3 service provider and 4 houses is also a 3 utility problem.

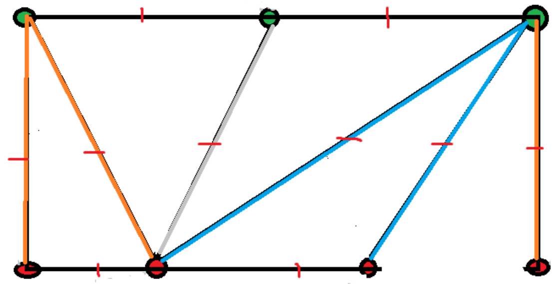
Now we have already discussed that any 3 utility problem will also generate an Eulerian puzzle if all edges are continuous and straight as-

Look this 3 service provider and 4 houses problem. Green points are service providers and red are houses.



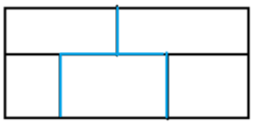
By applying condition discussed above make this figure as-

By following condition explained above we can make the above 3 utility problem as an Eulerian Trail.

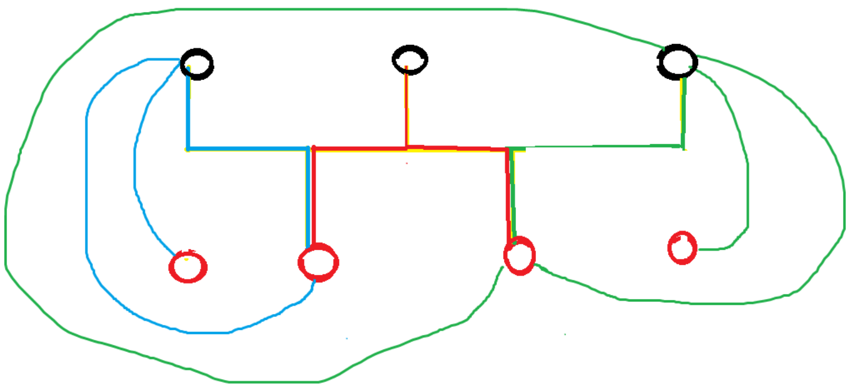


Again you cannot solve this problem by Eulerian Trial which comes out by 3 utility problem. It is for 3 service provider and 4 houses problem.

But we can also make something as this for 3 service provider and 4 houses problem by using condition 2 part-2→ All internal lines will be a combination of odd number of discrete lines such that no two internal discrete lines can intersect each other at any point and if any two of the discrete lines are parallel, we can coincide that, though it is not necessary.



Try to make this diagram as a 3 utility problem by reversing the rules discussed, the 3 utility problem for 3 service providers and 4 houses will look like-



So we can conclude all Eulerian puzzle are related to or originated from 3 utility problem.

THEOREM 'Y2'

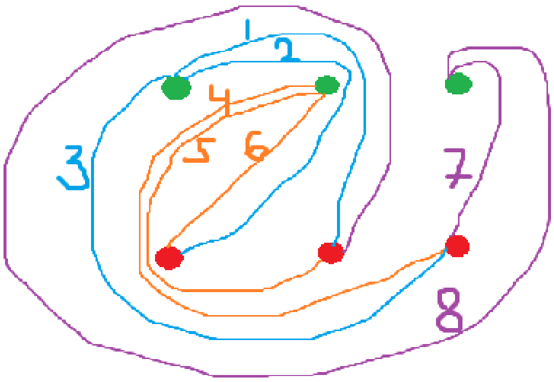
<u>If any figure for Eulerian puzzle exists, it is an output of 3 utility problem.</u>

BY theorem Y1 AND Y2 we conclude that- ***All Eulerian Trail is the result of an unsolved 3 utility problem, and all unsolved 3 utility problem can form Eulerian trail if '3 utility problem's' method matched with Eulerian trail. Both the puzzles are depicting same meaning by using different method and any Eulerian trail can only be formed if it follow the condition of 3 utility problem.***

III. Explaination By An Example

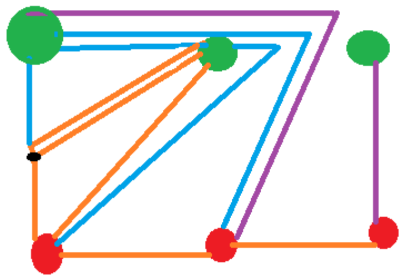
Now we will try to solve it by an example

Let us make a 3utility diagram as shown below

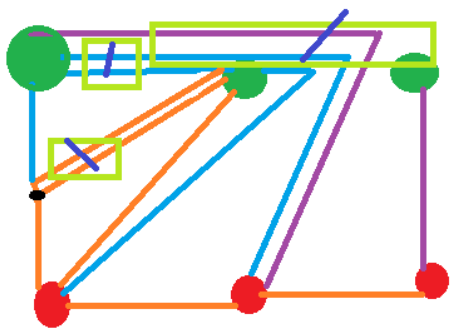


Here you can see that the third green point is unable to meet 1st red point. And we have drawn every line that can be drawn without breaking rules of 3 utility problem.
Now follow conditions, the conditions are in numbers serially as shown in diagram, here lines 1,2,3 are blue so it is for leftmost service provider, and lines numbered 4,5,6 are orange for middle service provider and finally 7,8 are for rightmost service provider, their answer are also having number with the same color and number, to make it more clear.

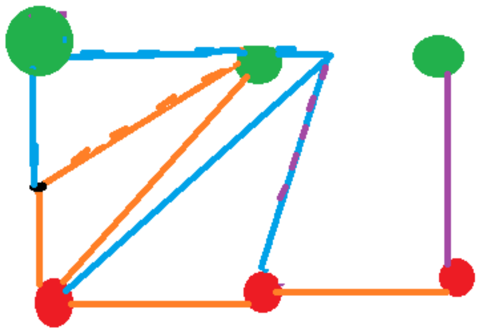
Algorithm by conditions discussed earlier:-
Condition for 1-follow diagram with rule (4)
Condition for 2-follow diagram with rule (4)
Condition for 3-follow diagram with rule (1)
Condition for 4-follow diagram with rule (1)
Condition for 5-follow diagram with rule (1)
Condition for 6-follow diagram with rule (5)
Condition for 7-follow diagram with rule (1)
Condition for 8-follow diagram with rule (4)
You can go for other rules, like instead of rule (5) you can go for rule (6), but we will Eulerian Trial by rules discussed above
By all this we can make



Try to solve it; it is again by an Eulerian Trial. It won't get solved.
Now apply rule (8) and make parallel lines to coincide, parallel lines are shown by light green rectangle and dark blue line.



The new diagram after coinciding parallel lines will be-



Try to solve it, again you will be unable.
So in this way we can make Eulerian Trail by any 3 utility problem.

IV. CONCLUSION

The research paper gives a new way of understanding both of the puzzles; it tells that every Eulerian trail is derived from 3 utility puzzle. It also tells us that why only 3 or more inner figures having odd number of edges in any diagram can form an Eulerian Trail. The thing is though methods of solving both the problems are different but the condition is same that is if one follows rules of 3 utility problem, by condition discussed above one can make a diagram accordingly, it will always result into an Eulerian Trail.

Both the puzzles have one solution that is 3-Dimension and they are using a tricky approach that is trapping, 3 utility uses trapping of nodes while Eulerian uses trapping of both ends of a line trying to traverse all edges of any diagram.

So this research paper gives two theorems that proves relation between 3 utility problem and Eulerian trial and also gives a method to explain Eulerian Trail concept.

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