IOSR Journal of Business and Management (IOSR-JBM)
e-ISSN:2278-487X, p-ISSN: 2319-7668. Volume 26, Issue 8. Ser. 1 (August. 2024), PP 50-58
www.iosrjournals.org

Visualizing Self-Intersecting Truncated Polyhedra Through A Virtual Reality Web Environment

Paulo Henrique Siqueira[1]
[1](*Graphic Expression Department, Federal University Of Paraná, Brazil)*

Abstract:
This paper shows the use of web resources to create environments for visualizing self-intersecting truncated polyhedra and their respective duals. These environments were created using Virtual Reality (VR) resources, which allow the visitor to manipulate the polyhedra and compare the polyhedra and their duals. Geometric transformations of homothety (scale), translation and rotation were used to build the virtual rooms, using HTML page hierarchies structures, and inserting each polyhedron in its respective virtual room. The resources presented in this work can be used in the classroom to visualize polyhedra with immersive goggles and even in Augmented Reality (AR) using smartphones and tablets. Other studies that can be developed with the polyhedra modeled in this article are areas, volumes, plane sections and the Euler relation. This article shows the possibility of creating teaching materials with a simple, free technology that makes a great contribution to improving the teaching of Geometry, as well as other areas that use graphic representations of 3D objects.
Keywords*: virtual reality; self-intersecting truncated polyhedra; augmented reality; duality; geometry.*

Date of Submission: 22-07-2024 Date of Acceptance: 02-08-2024

I. Introduction

The topics presented in some disciplines that involve three-dimensional concepts can be understood, by students, more efficiently by using auxiliary resources. The concrete materials, made with 3D printers, can be used in Biology classes [1], Mathematics [2], or disciplines with content that involve development and spatial skills [3]. The web environments creation or applications for teaching can collaborate in the teaching of Physics [4, 5], Biology [6] and has been used as an attractive alternative to aid student learning.

Modeling objects using virtual technologies can also help in learning content that involves concepts in three dimensions (3D). Virtual Reality (VR) allows the creation of an immersive and interactive environment with the objects manipulation through controls and immersive goggles [7]. Environments developed in VR can help with the visualization of physical, biological or chemical phenomena [8], simulations of training situations [9], educational games [10], Medicine [11], Nursing [12], construction simulations [13] and other areas related to education [14, 15, 16]. Learning content involving polyhedra in disciplines such as Technical Drawing, Euclidean Geometry or Descriptive Geometry almost always requires auxiliary materials, such as planned polyhedra, printed in 3D, or assembled with alternative materials [17]. Content that involves calculating areas and volumes, visualizing faces and edges, or exploring the properties of solids, can be explored with manipulable materials or modeled in virtual environments.

The virtual environments programmed in VR can complement the use of traditional courseware in teaching polyhedra, as students can interact and visualize solids and their properties in a more effective and meaningful way. VR can help students interact with representations of modeled polyhedra, facilitating visualization and understanding of object properties.

This article presents web resources for creating immersive environments that enable the manipulation of self-intersecting truncated polyhedra and their respective duals with VR and Augmented Reality (AR) technologies. When viewing polyhedra using AR, links to pages developed in VR are available to visitors. A DNA ribbon was modeled in the virtual environments, establishing connections between the elements of the polyhedra and their respective duals. On the page programmed in AR, students view and manipulate the polyhedra from different points of view and access the pages programmed in VR to manipulate the representations of solids with mobile devices, computers or they can even immerse themselves in the scene using VR goggles.

The objective of this article is to show the construction of teaching materials that use VR and AR resources, which help to visualize polyhedra in the teaching of Spatial Geometry. The teaching resources shown in this article can be used in the classroom to manipulate and visualize self-intersecting truncated polyhedra, which represent a set of non-convex and non-trivial polyhedra. The development of these materials can contribute to student learning and enrich classes on these polyhedra using virtual classrooms as a complement to traditional teaching materials.

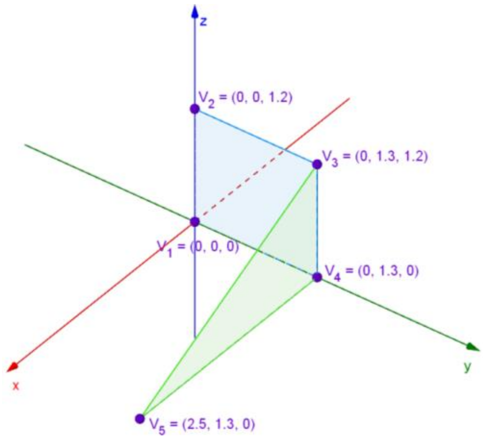
II. Modeling Of Self-Intersecting Truncated Polyhedra

A polyhedron is convex when all edges belong to the same subspace defined by all faces of this polyhedron. Non-convex polyhedra have some faces that "separate" some edges into two subspaces. Self-intersecting truncated polyhedra represent a set of non-convex polyhedra.

A property of regular truncated polyhedra is that they are vertex-transitive with isosceles triangular vertex figures. When we perform a rotation, translation and/or reflection of any two vertices of a polyhedron, and the external appearance remains unchanged, we can say that there is a vertex transitivity. The vertices of the self-interseting truncated polyhedra are combinations of two or more different regular or stellated polygons [18, 19]. These polyhedra can be represented using the coordinates of their vertices or through the constructions of polygons that represent the solids faces.

In [20] the codes used to represent each Archimedes polyhedron are presented using the symmetries and angles between edges and faces that define these solids. Therefore, the modeling of the self-intersecting truncated polyhedra can be done using the vertices coordinates, defining which vertices form each solid's face. This information is inserted into obj format files [21], which work in a very intuitive way.

For example, a quadrilateral that has vertices 1, 2, 3 and 4 and a triangle that has vertices 3, 4, and 5 are defined in an obj file (Figure 1). Faces are defined with f commands, which have sequences of vertices defined in ordered v commands with 3D coordinates. Colors and textures can be defined in the materials file with mtl extension and the $usemtl$ command.




```
v 0 0 0
v 0 0 1.2
v 0 1.3 1.2
v 0 1.3 0
v 2.5 1.3 0
usemtl Cor1
f 1 2 3 4
usemtl Cor2
f 3 4 5
```

Figure 1. Excerpt from a file code in obj format that defines a quadrilateral and a triangle

The obj files with the edges are created in the same way, using the line commands l. The HTML programming codes with the A-Frame [22] resources of each polyhedron are made by inserting the files links that contain the polyhedra modeled in obj format files.

Figure 2 shows the code to model a stellated truncated hexahedron in VR, a solid that has 14 equilateral triangles and 6 regular octagrams. Between lines 10 and 12, the links to the files containing the polyhedron and edges are defined. In lines 16 and 17, the polyhedra modeled in the scene are inserted. Finally, between lines 18 and 41, the *a-sphere* commands are used to represent the 24 vertices of the stellated truncated hexahedron. The other self-intersecting truncated polyhedra are defined using the same HTML file type.

```
1  <!DOCTYPE html>
2  <html>
3   <head>
4    <script src="https://aframe.io/releases/1.3.0/aframe.min.js"></script>
5   </head>
6   <body>
7   <a-scene>
8   <a-entity camera></a-entity>
9   <a-assets>
10   <a-asset-item id="poliedro" src="poli/StellatedTruncatedHexahedron.obj"></a-asset-item>
11   <a-asset-item id="linhas" src="poli/StellatedTruncatedHexahedronL.obj"></a-asset-item>
12   <a-asset-item id="material" src="poli/solidos.mtl"></a-asset-item>
13  </a-assets>
14  <a-sky color="aliceblue"></a-sky>
15  <a-entity scale="8,8,8" position="0,0,0">
16   <a-obj-model src="#poliedro" mtl="#material"></a-obj-model>
17   <a-obj-model src="#linhas" mtl="#material"></a-obj-model>
18   <a-sphere position="-0.207107,0.5,0.207107" radius="0.017"></a-sphere>
19   <a-sphere position="-0.207107,-0.5,-0.207107" radius="0.017"></a-sphere>
20   <a-sphere position="-0.207107,-0.5,0.207107" radius="0.017"></a-sphere>
21   <a-sphere position="0.207107,0.5,-0.207107" radius="0.017"></a-sphere>
22   <a-sphere position="0.207107,0.5,0.207107" radius="0.017"></a-sphere>
     …
40   <a-sphere position="-0.5,0.207107,0.207107" radius="0.017"></a-sphere>
41   <a-sphere position="-0.207107,0.5,-0.207107" radius="0.017"></a-sphere>
42  </a-entity>
43  </a-scene>
44  </body>
45 </html>
```

Figure 2. Code for modeling a stellated truncated hexahedron in VR using A-Frame resources

Figure 3 presents the modeling of the stellated truncated hexahedron and the great truncated icosahedron with obj files. Self-intersecting truncated polyhedra models, virtual rooms and QR codes for viewing each polyhedron using AR technology are available on the page:

https://paulohscwb.github.io/polyhedra/selfintersecttruncated/

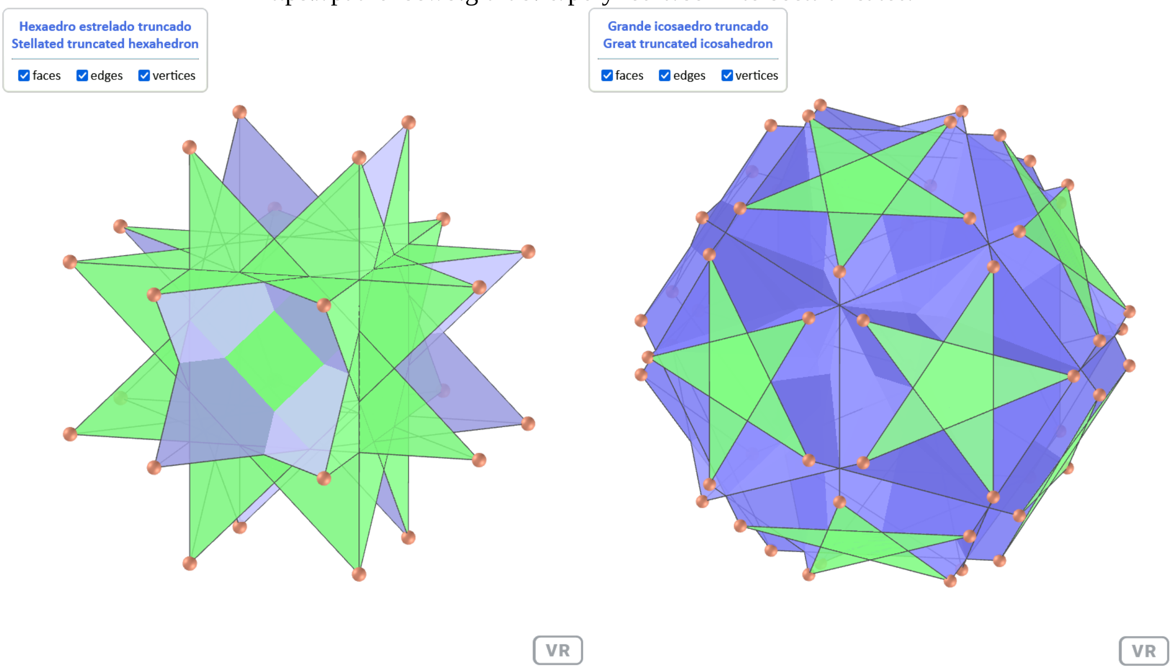


Figure 3. Modeling of the stellated truncated hexahedron and the great truncated icosahedron

III. Duality Of Polyhedra

The duality concept in Geometry is associated with a secondary structure of a polyhedron. According to Wenninger [23], this structure is defined by the one-to-one correspondence between the vertices of one polyhedron with the faces of the other polyhedron. The edges between the pairs of vertices of a polyhedron have a one-to-one correspondence with the edges between the pairs of faces of the dual polyhedron.

Consider the great truncated cuboctahedron, which has 8 regular hexagons faces, 12 squares faces, 6 regular octagrams faces, 72 edges and 48 vertices (Figure 4). The correspondence between each face of this polyhedron with the vertices of the dual determines a solid with 26 vertices and 48 triangular faces called great disdyakis dodecahedron.

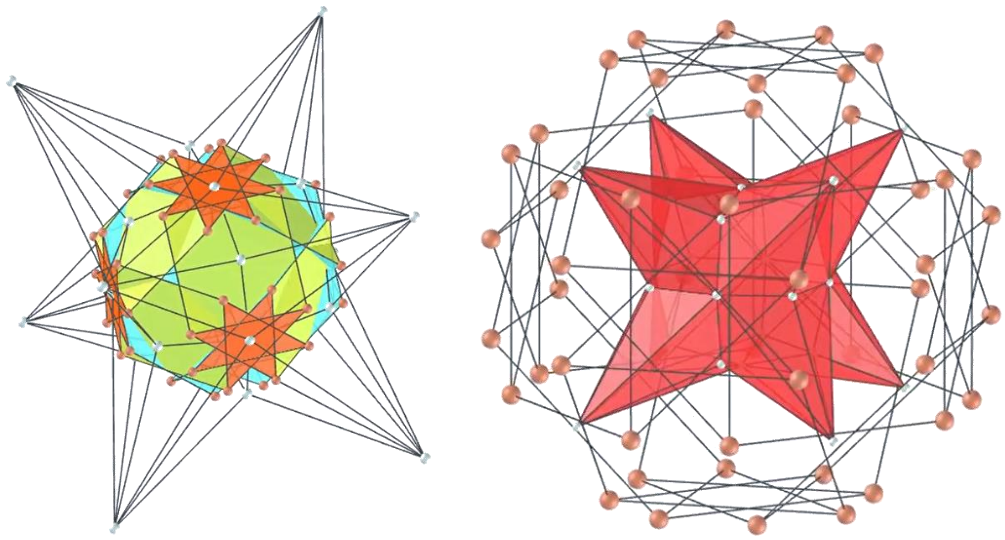


Figure 4. Correspondence between edges, faces and vertices of the great truncated cuboctahedron and the great disdyakis dodecahedron

The edges of the dual polyhedron are orthogonal to the primal polyhedron edges, and the dual of the dual is always the primal polyhedron [18, 23]. Figure 5 presents the modeling of the polyhedra great triakis octahedron and great stellapentakis dodecahedron with obj files, which represent the duals of the stellated truncated hexahedron and the great truncated icosahedron polyhedra shown in Figure 3.

The relations between self-intersecting truncated polyhedra elements and their respective duals, can be understood more efficiently when connections are established between these solids. In this article, a DNA ribbon serve as a basis for visualizing the connections between the elements of dual polyhedra.

The construction of a DNA ribbon can be done using two cylindrical helices symmetric about the z axis. Consider the helix radius $r = 3$, and the first pair with an self-intersecting truncated polyhedron at point A and its respective dual polyhedron in the symmetric position A' (Figure 6).

Consider that each pair of polyhedra is located with a rotation of 45º, about the z axis in relation to the previous pair. Therefore, the second pair of polyhedra has the same x and y coordinates, which can be found using the Pythagorean theorem in the right triangle with hypotenuse r and equal legs:

$$x^2 + x^2 = r^2 \Rightarrow 2x^2 = 3^2 \Rightarrow x^2 = 4{,}5 \Rightarrow x \cong 2.12.$$

Using the vertical distance $z = 1.5$, we have the second pair of duals at points B and B'. The third pair of polyhedra has the coordinate $z = -1.5$ and is located in the symmetric positions of B and B' about the x axis. The other pairs of polyhedra can be positioned following the same reasoning. To improve the visualization of this DNA ribbon in the virtual room, a rotation of the helices with an angle of 90º around the y axis is considered.

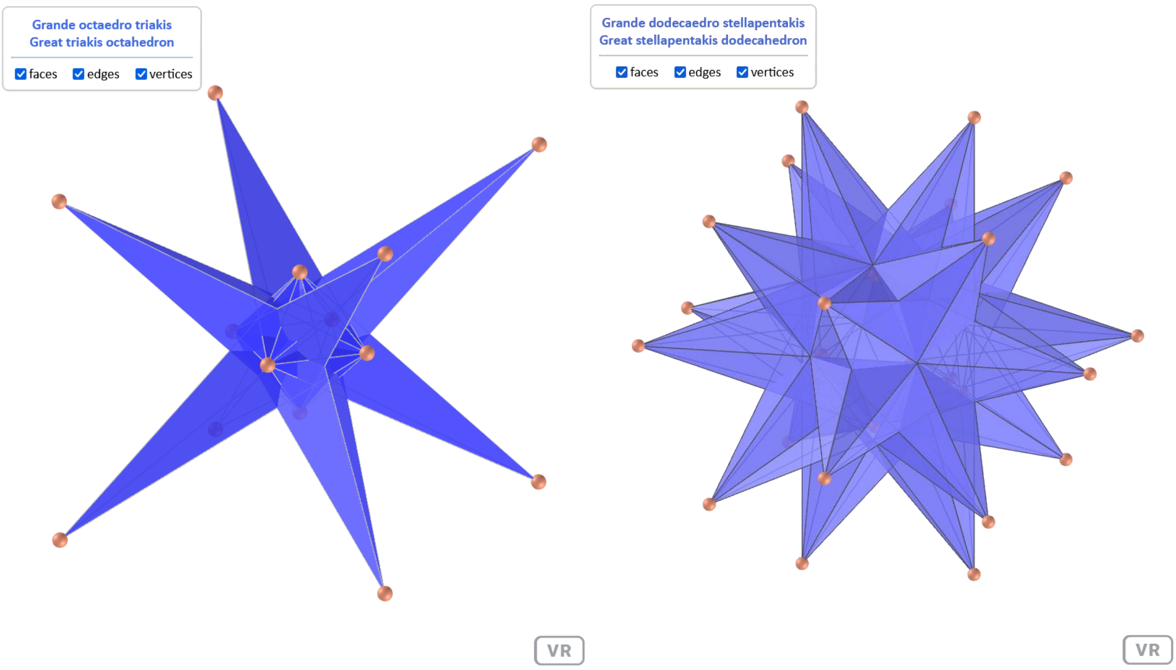


Figure 5. Modeling of the great triakis octahedron and the great stellapentakis dodecahedron

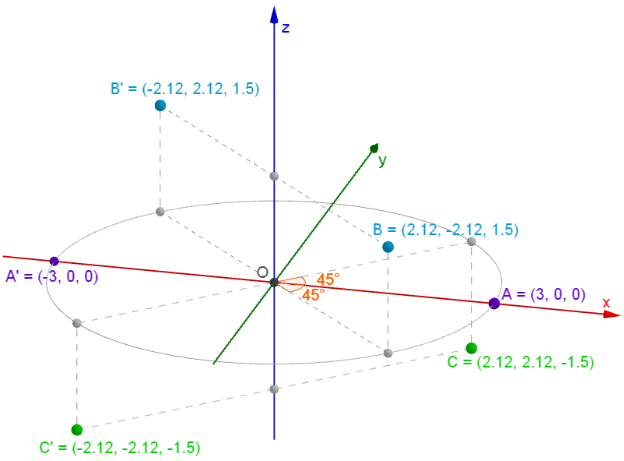


Figure 6. Construction of a DNA ribbon with two cylindrical helices

Modeling the DNA ribbon allows visitors to visualize the overlapping vertices of a polyhedron, which determine the centers of the faces of their respective dual. Figure 7 shows the modeling of this DNA ribbon with two cylindrical helices. In the same way presented in the polyhedra viewed individually, visitors can choose to view only the edges or only the faces of the polyhedra, in addition to manipulating the visualization of the DNA ribbon. The spheres that represent the vertices of the polyhedra were omitted in the cylindrical helices, to make the processing of the graphical representation faster.

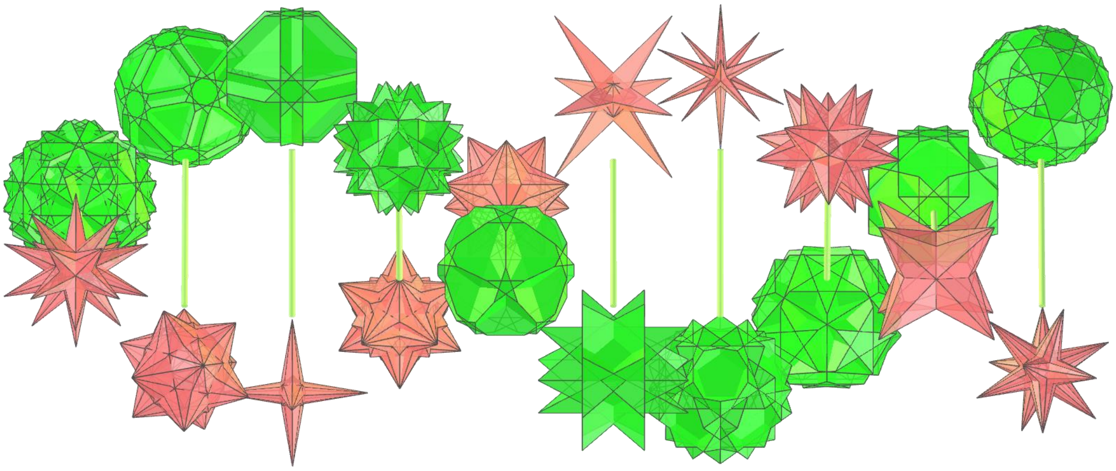


Figure 7. Modeling of the DNA ribbon with the connections between the self-intersecting truncated polyhedra and their respective duals

IV. Immersive Rooms With Modeled Polyhedra

The modeled self-intersecting truncated polyhedra and their duals were inserted into VR immersion rooms, which can be accessed using any device with internet access. Support tables for the polyhedra, equirectangular background photos and a projection screen with the properties of some solids were inserted into the programmed environments.

The A-Frame's gravity and shadow effects properties were programmed into the virtual rooms, with the aim of improving the feeling of immersion [22]. Figure 8 shows an overview of the environment of a virtual room with modeled solids, without the equirectangular image in the background.

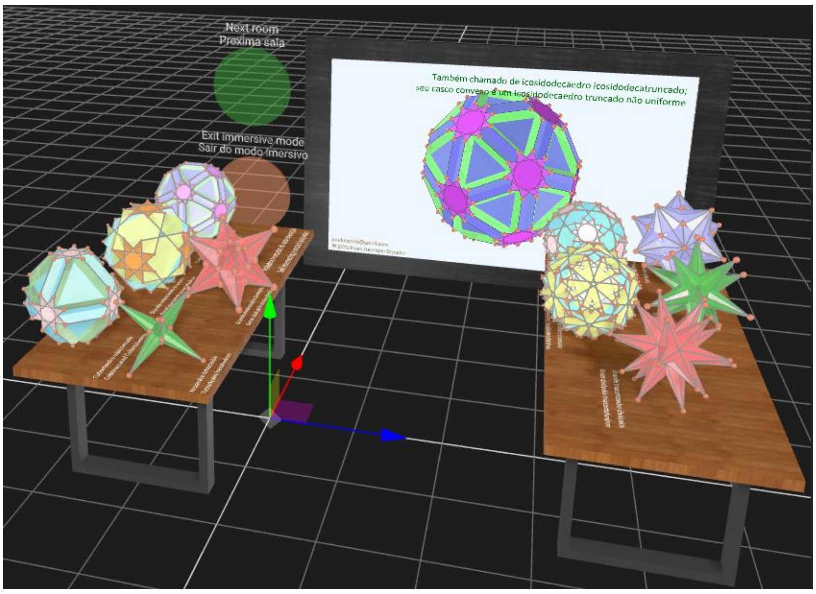


Figure 8. Overview of a virtual room with self-intersecting truncated polyhedra and their duals

The polyhedra models are supported on the tables, with labels containing the respective names in English and Portuguese of each solid. Figure 9 shows one of the tables with modeled polyhedra, presenting the background image in an equirectangular format [24].

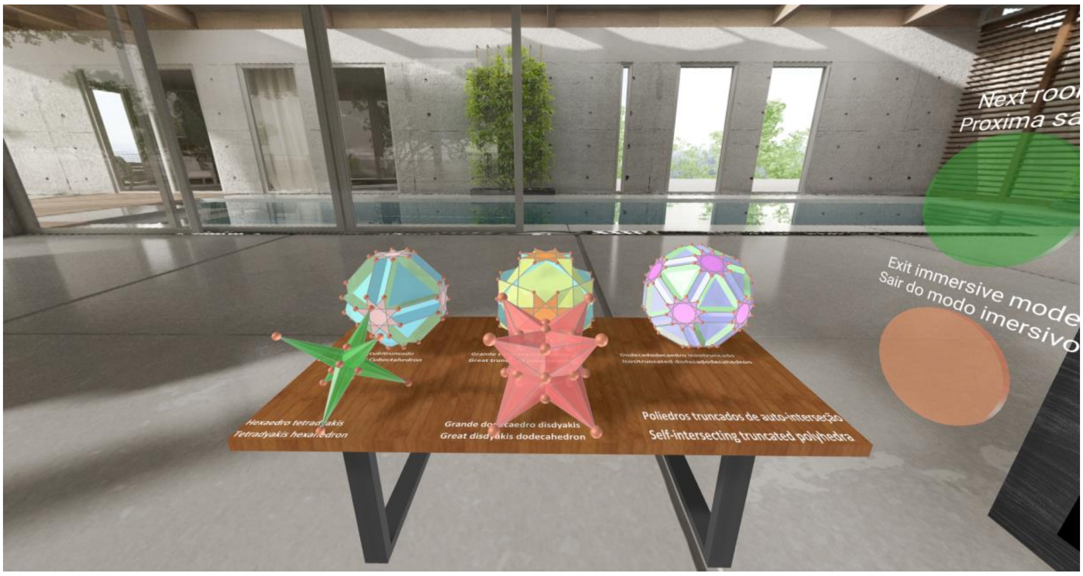


Figure 9. Detail of the virtual room with one of the tables containing self-intersecting truncated polyhedra and their duals

Using controls from immersive goggles, the click of a mouse or the touch of a smartphone or tablet, the visitor can move the polyhedra, change virtual rooms and leave the immersive environment. Figure 10 shows one of the tables with self-intersecting truncated polyhedra, with the background image in equirectangular format. Figure 11 shows the use of Virtual Reality goggles manipulation controls in virtual rooms with self-intersecting truncated polyhedra and their duals.

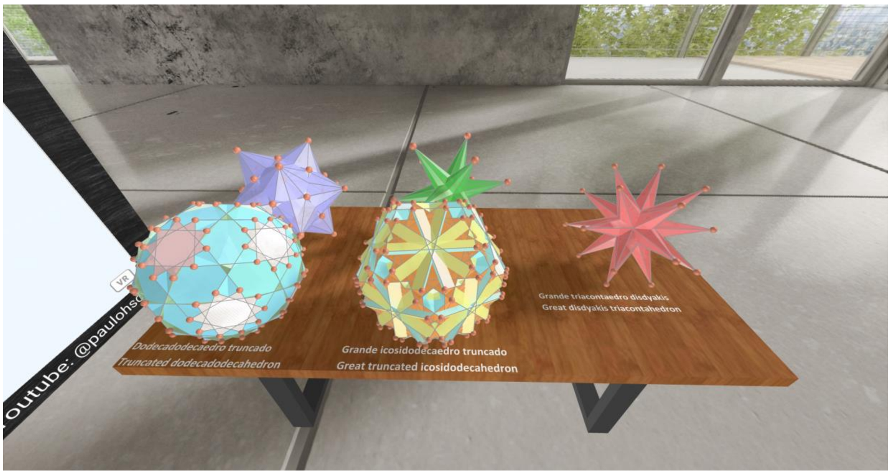


Figure 10. Detail of the virtual room with one of the tables containing self-intersecting truncated polyhedra and their duals

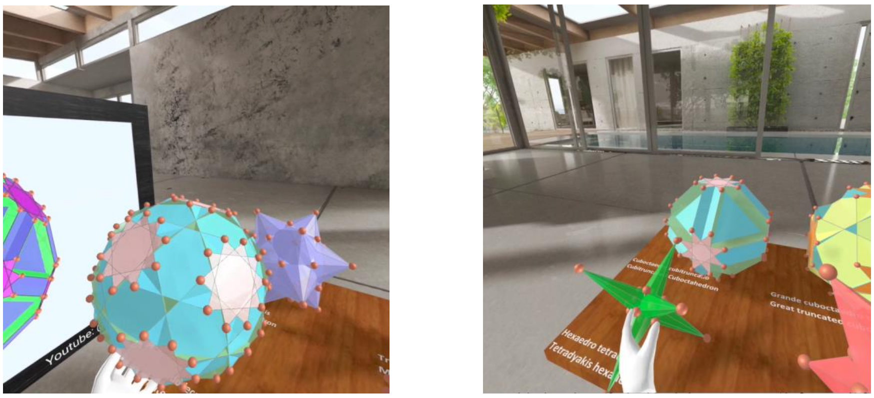


Figure 11. Details of the immersive rooms with polyhedron manipulations using VR goggles controls

An immersive virtual room was created to visualize these connections between the self-intersecting truncated polyhedral and their duals. Figure 12 shows the detail with the table, the DNA ribbon and the equirectangular image [25] in the created virtual environment.

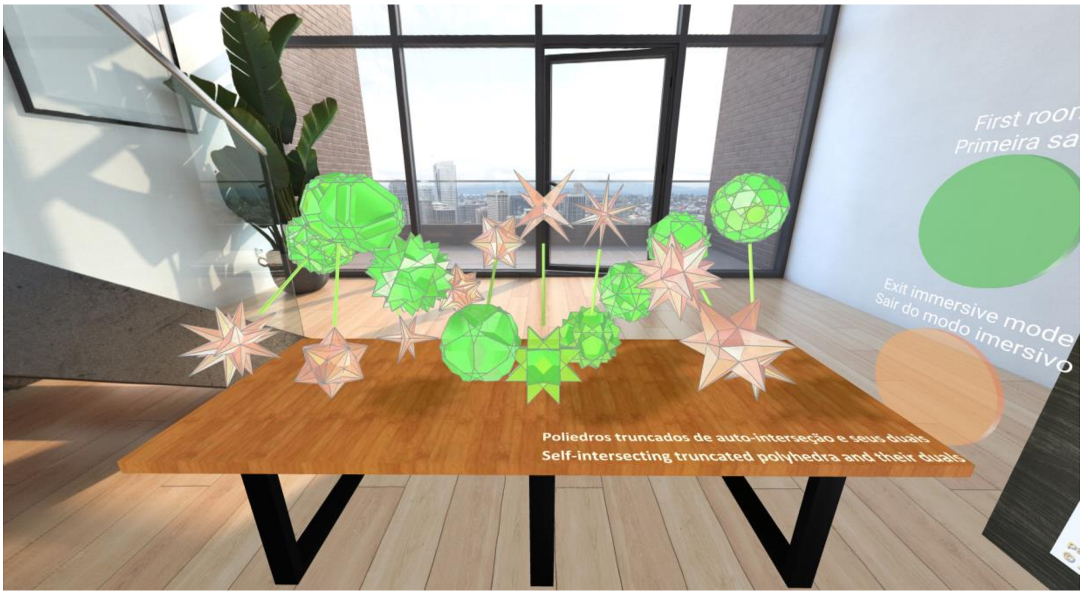


Figure 12. Virtual room with the DNA ribbon showing the connections between the self-intersecting truncated polyhedra and their duals

In addition to virtual rooms and individual polyhedron visualization capabilities, Augmented Reality technology can be used to visualize each polyhedron and the DNA ribbon with the connections between the dual polyhedra.

The programming codes for using AR are practically the same as those shown in Figure 2, with the indication of the QR code marker for each polyhedron model [20]. Figure 13 shows using the AR feature to visualize some self-intersecting truncated polyhedra and their duals.

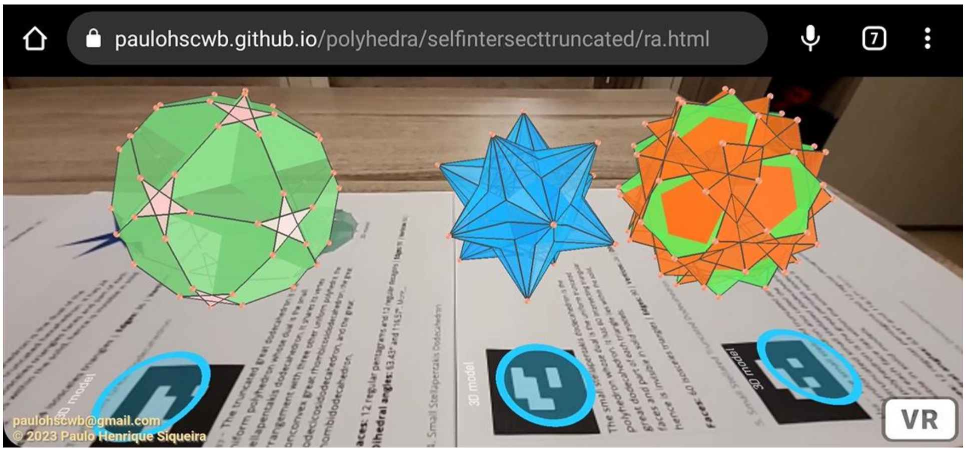


Figure 13. Visualization of some self-intersecting truncated polyhedra and their duals using the Augmented Reality feature

V. Conclusions

This article shows the use of web pages to visualize self-intersecting truncated polyhedra and their duals using Virtual Reality and Augmented Reality technologies. By viewing printed markers, students can view solids in AR on any device with a webcam and internet access, with links to the VR views.

The polyhedra modeling and the virtual rooms creation showed in this article use the hierarchical structures of web page programming with A-Frame scripts, facilitating the insertion of several polyhedra on the same page. The result shows that it is a useful tool for use in the classroom, as it allows students to view and manipulate graphic representations of polyhedra on their devices or to use VR goggles for complete immersion in virtual rooms.

The programmed environments can be explored in Geometry classes, helping to understand the polyhedra elements and properties or topics such as calculating areas and volumes, Euler's relation, plane sections or simply visualizing each modeled solid. The DNA ribbon modeled in this article allows visualization of the connections existing between the elements of self-intersecting truncated polyhedra and their respective duals.

All elements of the polyhedra can be viewed in VR and AR and visitors can move the camera around the scene to find the best views of the solids with A-frame developed tools to orbit the camera around the objects. The web page programming tools shown in this article are simple and intuitive, and can be used in classrooms with printed materials or with immersive goggles. Students access the page programmed in AR, view the solids with their respective printed markers and can interact with the polyhedra programmed in VR. In this way, students can explore the geometric concepts involved in a more efficient and dynamic way.

The teaching resources developed in AR and VR for visualizing self-intersecting truncated polyhedra serve to improve the quality of teaching in the area of Spatial Geometry, as they help in visualizing these polyhedra and understanding their properties. As these are non-trivial polyhedra, auxiliary teaching resources are essential for students to understand these solids. It is worth remembering that teaching resources are complementary instruments to traditional teaching materials, and that they help transform ideas into facts.

Some advantages of creating AR and VR environments as web pages for use in the classroom are practicality, low cost, excellent performance, simple programming and operation on all types of devices with internet access. Another advantage of this tool is the almost immediate loading of pages, as they are programmed in HTML with references from VR libraries developed with the JavaScript language.

The visitors do not need to download applications and several markers can be used on the same HTML page, which allows the creation of teaching materials with different themes programmed in AR and VR. This tool can be used in other disciplines, such as Statistics, Biology, Differential and Integral Calculus, Physics, Geography, Chemistry, Engineering and other areas that use 3D graphical representations.

The modeling presented in this article can be used in other sets of non-trivial polyhedra, such as toroidal polyhedra or polyhedra fractals. Suggestions for future work include modeling objects that use non-convex polyhedra, inserting gamification tools and programming other forms of interactions with visitors in virtual rooms.

References

[1]. Olumorin, Co, Babalola, Eo, Ayoola, Da. Design And Development Of Human Excretory System Model To Teach A Biology Concept In Ilorin, Nigeria. Indonesian Journal Of Teaching In Science. V. 2, No. 2, Pp. 107-116, 2022.

[2]. Quigley, Mt. Concrete Materials In Primary Classrooms: Teachers' Beliefs And Practices About How And Why They Are Used. Mathematics Teacher Education And Development. V. 23, No. 2, Pp. 59-78, 2021.

[3]. Karaismailoglu, F, Yildirim, M. The Effect Of 3d Modeling Performed Using Tinkercad Or Concrete Materials In The Context Of The Flipped Classroom On Pre-Service Teachers’ Spatial Abilities. Research In Science & Technological Education. V. 42, No. 1, Pp. 1-20, 2023.

[4]. Komikesari, H, Mutoharoh, M, Dewi, Ps, Et Al. Development Of E-Module Using Flip Pdf Professional On Temperature And Heat Material. Journal Of Physics: Conference Series. V. 1572, No. 1, Pp. 12-17, 2020.

[5]. Rozal, E, Ananda, R, Zb, A, Et Al. The Effect Of Project-Based Learning Through Youtube Presentations On English Learning Outcomes In Physics. Al-Ishlah: Jurnal Pendidikan. V. 13, No. 3, Pp. 1924-1933, 2021.

[6]. Delgado, T, Bhark, Sj, Donahue, J. Pandemic Teaching: Creating And Teaching Cell Biology Labs Online During Covid-19. Biochemistry And Molecular Biology Education. V. 49, No. 1, Pp. 32-37, 2021.

[7]. Marougkas, A, Troussas, C, Krouska, A, Et Al. Virtual Reality In Education: A Review Of Learning Theories, Approaches And Methodologies For The Last Decade. Electronics. V. 12, No. 13, Pp. 1-21, 2023.

[8]. Kumar, Vv, Carberry, D, Beenfeldt, C, Et Al. Virtual Reality In Chemical And Biochemical Engineering Education And Training. Education For Chemical Engineers. V. 36, Pp. 143-153, 2021.

[9]. De Geus, K, Beê, R, Corrêa, V, Et Al. Immersive Serious Game-Style Virtual Environment For Training In Electrical Live Line Maintenance Activities. In: Proceedings Of The 12th International Conference On Computer Supported Education (Csedu 2020), V. 2, Pp. 42-53, 2020.

[10]. Pirker, J., Dengel, A., Holly, Et Al. Virtual Reality In Computer Science Education: A Systematic Review. In Proceedings Of The 26th Acm Symposium On Virtual Reality Software And Technology, Canada, Pp. 1-8, 2020.

[11]. Pottle, J. Virtual Reality And The Transformation Of Medical Education. Future Healthcare Journal. V. 6, No. 3, Pp. 181-185, 2019.

[12]. Chen, Fq, Leng, Yf, Ge, Et Al. Effectiveness Of Virtual Reality In Nursing Education: Meta-Analysis. Journal Of Medical Internet Research. V. 22, No. 9, Pp. 1-13, 2020.

[13]. Soliman, M, Pesyridis, A, Dalaymani-Zad, D, Et Al. The Application Of Virtual Reality In Engineering Education. Applied Sciences, V. 11, No. 6, P. 1-14, 2021.

[14]. Fabris, Cp, Rathner, Ja, Fong, Ay, Et Al. Virtual Reality In Higher Education. International Journal Of Innovation In Science And Mathematics Education. V. 27, No. 8, Pp. 69-80, 2019.

[15]. Melinda, V, Widjaja, Ae. Virtual Reality Applications In Education. International Transactions On Education Technology. V. 1, No. 1, Pp. 68-72, 2022.

[16]. Akhunova, Nkk. Possibilities Of Using Virtual Reality Technologies In Education. Asian Journal Of Multidimensional Research (Ajmr). V. 10, No. 3, Pp. 549-555, 2021.

[17]. Andrade, Cb, Santos, Jcb, Moura, Vcs. Comparação Entre Os Métodos De Ensino Aprendizagem, Com E Sem A Utilização De Materiais Manipuláveis. Revista Científica Doctum: Educação. V. 1, No. 2, Pp. 1-15, 2017.

[18]. Coxeter, Hsm. Regular Polytopes, New York: Dover, 1973.

[19]. Weisstein, Ew. Miscellaneous Polyhedra. Mathworld--A Wolfram Web Resource. [Online] Http://Mathworld.Wolfram.Com/Topics/Miscellaneouspolyhedra.Html (Accessed 31 July 2024).
[20]. Siqueira, Ph. Visualization Of Archimedian And Platonic Polyhedra Using A Web Environment In Augmented Reality And Virtual Reality. International Journal For Innovation Education And Research. V. 9, No. 11, Pp. 1-13, 2021.
[21]. Fileformat. Wavefront Obj File Format Summary. [Online] Https://Www.Fileformat.Info/Format/Wavefrontobj/Egff.Htm (Accessed 31 July 2024).
[22]. A-Frame. A-Frame Documentation Of Virtual Reality. [Online] Https://Aframe.Io/Docs/1.3.0/Introduction/ (Accessed 31 July 2024).
[23]. Wenninger, Mj. Dual Models. Cambridge University Press: Cambridge, 2003.
[24]. Hemul. Equirectangular Image Produced By Hemul75. [Online] Https://Depositphotos.Com/Photo/360-Panorana-Of-Empty-Modern-Interior-Room-3d-Rendering-445536720.Html (Accessed 31 July 2024).
[25]. Hemul. (2024) Equirectangular Image Produced By Hemul75. [Online] Https://Depositphotos.Com/Photo/360-Panorana-Of-Empty-Modern-Interior-Room-3d-Rendering-445535812.Html (Accessed 31 July 2024).