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Do GARCH Models help in Pricing Option Contracts more efficiently?

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Abstract
Volatility forecasting is a grandioseventure. It is one such problem which has to be faced by almost every investor while taking decisions regarding their investments.GARCHfamily models have been one the most venerated group of volatility forecasting models amongst practitioners as well as academicians. The present study aims to use the GARCH models to price option contracts by taking data from the Indian options market for the first decade since their introduction. The Black and Scholes option pricing model is used to value the Nifty Index option by extracting the volatility forecasts for the underlying asset, that is Nifty, through GARCH models.
Keywords: *GARCH, volatility, option contracts, stock market*

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I. Introduction

Option contracts were launched in India in 2001. Since then there has been no looking back. It has made its residuum in the world market. Today India has virtually become a monopoly in the world of derivatives trading. One quandary every option trader has to face is to predict the underlying asset's volatility.To elucidate this there are many econometric models providing a measure of the underlying asset's volatility. From simple Random walk model to more complex ones like GARCH, HAR, Stochastic volatility models, Implied volatility (IV), etc are available for becoming a trader's choice. Out of these available models, GARCH family models have been quite acknowledged ones. For example, Akigray (1989), Doidge and Wei (1998), Ederington and Guan (2000), Jorion (1995), Madhusudan Karmakar (2004), Naimy and Hayek (2018), etc have found GARCH models to be better performers in different markets and across various assets. Consonantly, an option trader has many model choices available in order to value an option contract. For example, Black and Scholes(BS) model, Binomial model, Stochastic volatility, Affine jump-diffusion, Levy and time-changed Levy process, etc. Amongst these BS model is the most basic and simple model which has become time-honored because of its simplicity. Indian option markets, though globally quite incandescent, have not been tested much by the academicians. Present study targets to fill this gap by testing the BS model on Indian call options on Nifty by picking data from the first decade of their introduction. The study aims to empirically test the performance of the BS model in Indian options market by measuring volatility of the underlying asset Nifty through three models, that is, the Random walk model, GARCH (1,1) model and the GARCH (p,q) model.

Specifically, the paper is branched into following sections: first section is about literature review, second section is about data and methodology, third section presents the results and the last section is about summary and conclusions.

II. Literature Review

Since its concoction, BS model has been a benchmark model for both academician as well as traders. Black and Scholes (1973) tested the model's performance by taking data on six month call options with underlying asset's volatility measured through historic standard deviations. The study could identify overvalued and undervalued options creating a profit before transaction costs.

Latane and Rendleman (1976) laid the BS model next to the newly invented Implied volatility measure in order to check the model's performance. The estimators of volatility of the underlying asset was taken to be the Weighted implied standard deviations and through these the over/undervalued options and proper hedge positions were determined. It was found that BS model generally overpricethe call option contracts. Geske and Roll (1984) applied the American variant of the Black and Scholes model. They found both original and reverse biases related to the strike prices. The original BS model which can be applied on European option contracts identified underpricing of near-the-maturity American call options. Galai (1977) tested the performance of the

BS model and concluded that the model can effectively locate deviations for identifying expost hedge strategies leading to profits. Statistically significant strike price biases through the BS model were identified by Rubinstein (1985), who compared six option pricing models. Similarly, Whaley (1982) compared three option pricing models and found that the BS model performed well and within tolerance that gets imposed by trading imperfections on the market.

III. Data and Methodology

The present study tries to analyze the S&P CNX Nifty index call option contracts from 1 July 2002 to 30 June 2011. Previous studies have shown biases in the BS model while valuing potions. These biases were mostly identified in the first decade since BS model was introduced. To match this, we have restricted the data on Nifty option contracts to 2011 in the present study, to know whether the same biases existed in Indian markets in the initial decade. To apply the BS model a trader needs six pieces of information out of which only the volatility of the underlying asset Nifty is not directly observable. Therefore present study uses three volatility forecasts models, that is the random walk, GARCH(1,1) and the GARCH (p,q) model to have three different volatility forecast of Nifty and then take these as input into the BS model to forecast option contract prices. The BS model can be applied through the following equation:

$C = S\,N(d_1) - Xe^{-rt}\,N(d_2)$ -------------- [1]

Where

$$d_1 = \frac{\ln[S/X] + [r + \sigma^2/2]t}{\sigma\sqrt{t}}$$

$d_2 = d_1 - \sigma\sqrt{t}$, C is value of the call option, S is price of underlying security, X is exercise price, t is time to expiration, σ^2 is variance rate of return for the underlying security, r is short term interest rate which is continuous and constant through time and N (d_i) is cumulative normal density function evaluated at d_i . The whole sample is divided into two parts. The first part is used to estimate the model parameters and is called the in-sample period starting from 1July 2002 to 30 June 2008. The second part is called the out-of-sample period which starts from 1 July 2008 to 30 June 2011 for which volatilities from the three models would be forecasted and used in the BS model. For estimating actual volatility, we can use the squared daily returns or the absolute daily returns, that is,

$\sigma_t^2 = r_t^2$,or ………….(2)

$\sigma_t = |r_t|$ …………(3)

According to the random walk model the best forecast of time t volatility is the observed volatility in time t-1;

$$\sigma^2{}_t = \sigma^2{}_{t-1} \quad \text{…………………..(4)}$$

So the best forecast for tomorrow's volatility is today's volatility;

$$\hat{\sigma}_{t+1} = \sigma_t \quad \text{--------------------(5)}$$

Where σ_t alone is used as a forecast for σ_{t+1}.

The Nifty time series exhibit some statistical properties, which allow for the application of GARCH model for forecasting. The skewness statistic is -0.85 indicating nifty return distribution to be asymmetric. Kurtosis being equal to 9.5 designated the time series as leptokurtic. The Jarque-Bera test of normality rejects the normality assumption. The result indicates that the Nifty return series is not normally distributed but is leptokurtic and skewed suggesting that the use of an ARCH/GARCH model may be appropriate.

The sample values of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the Nifty log return series for 30 lags were and a significant autocorrelation is found at lag 1, lag 10and lag 14. Thus the correct model may include lag 1, lag 10 or lag 14 or a combination of them for defining the AR structure. After comparing the different combinations, AR(1,14) model was found to be having minimum BIC value. Thus, the mean equation was first structured by including the first and the fourteenth lag, but since the results indicated the intercept term, c, to be insignificant at 5% level of significance, therefore, finally the mean equation was defined by excluding the intercept term as follows:

$r_t = \beta_1 r_{t-1} + \beta_2 r_{t-14} + \varepsilon_t,$ ………………….(6)

In order to estimate the variance equation, first the ARCH (q) models are estimated for "p" up to 9 lags which are described through the following variance equation:

Variance equation: $\sigma_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2$ ……..……..(7)

The optimal lag length (p) for daily return data, according to both the AIC and the BIC criteria is found to be 1, as both are minimized at lag one. Thus, the optimal order for the ARCH (q) model is taken to be one and the results of applying the model to the Nifty log return series showed that all the coefficient estimates of the parameters are statistically significant as none of the p-values are greater than the 5% significance level. So, next we tried to fit the GARCH (p,q) model employing the same equation for the log returns, but the equation for the variance includes p new terms.

The log likelihood values and the AIC, and BIC criterion were calculated for 81 models in the GARCH (p,q) class for $p \in [1,2 \dots 9] and\ q \in [1,2, \dots 9]$. The minimum AIC and BIC values were present for the GARCH (4,2) model and the log likelihood values vary only to a very small extent amongst the various models. Therefore, we apply the GARCH (4,2) model and the estimation output for the same is summarized in table [1]. It can be observed from the table that none of the estimated coefficient values of the variance equation are insignificant at 5% level of significance. The sum of all the coefficient values for various p and q lagged terms is less than 1, thereby indicating that the variance process is stationary.

Table 1: Estimation output for the GARCH (4,2) model.

Particulars	**Value**	**Particulars**	**Value**
	Mean Equation		
Variable	Coefficient	t-statistics	p-value
β_1	0.1023	3.7345	0.0000
β_2	0.0459	1.8765	0.0673
	Variance Equation		
α_0	1.88E-05	4.1239	0.0000
α_1	0.1243	8.1154	0.0000
α_2	0.1123	5.4396	0.0000
γ_1	-0.4523	-3.766	0.0000
γ_2	0.8234	7.2912	0.0000
γ_3	0.4987	4.4356	0.0000
γ_4	-0.2125	-2.1908	0.0222
	Other Measures		
Akaike info criterion	-5.7625	Log likelihood	4416.82
Schwarz criterion	-5.7465	Durbin-Watson stat	2.033
		TR^2 statistic	4.552
		Probability	0.4234

The model is then tested for misspecification. The histogram is not of bell-shaped form. the histogram and skewness indicates a long left tail.

We next implemented GARCH(1,1) model and the estimation output for the same is summarized in table [2]. The sum of all the coefficient values of the variance equation is less than one. This can be taken as an indicationfor stationarity of the variance process.

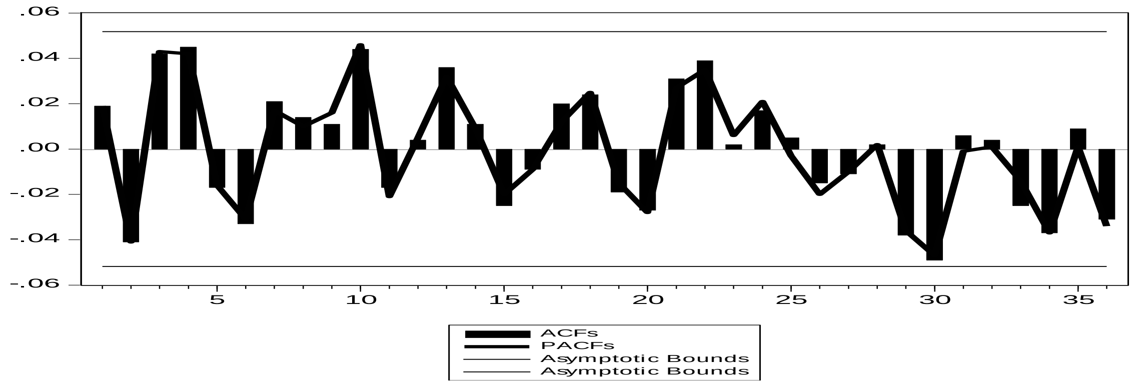
Table 2: Estimation output for the GARCH (1,1) model.

Variable	Coefficient	Std. Error	z-Statistic	Prob.
β_1	0.1125	0.0245	3.9235	0.0000
β_2	0.0566	0.0234	1.8916	0.0695
	Variance Equation			
α_0	8.341E-06	1.68E-06	5.229	0.0000
α_1	0.1152	0.0178	10.674	0.0000
γ_1	0.8112	0.0234	46.060	0.0000
Akaike info criterion	-5.7665	Schwarz criterion		-5.7437
Durbin-Watson stat	2.1131	TR^2 Stat		7.111
Probability		0.2341		

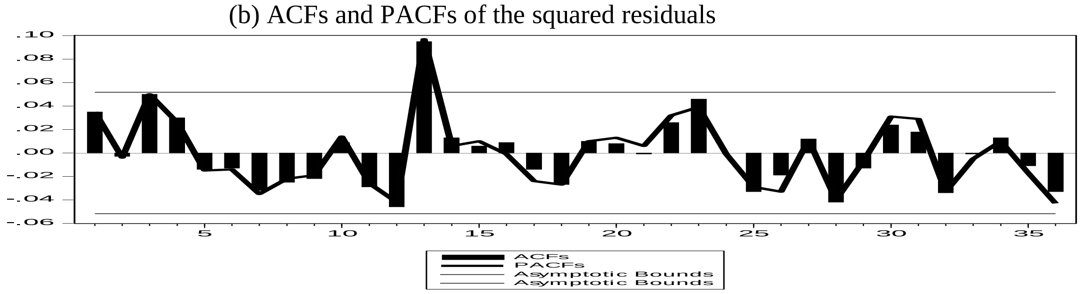
The GARCH (1,1) model reduces the excess kurtosis to 1.52. The null hypothesis of a normal distribution was rejected by the JB statistic. The ACFs and the PACFs shown in Figure 2, of the residual and the squared residual series remains within the asymptotic bounds, thereby, indicating linear independency. The Ljung-Box Q-Statistics also indicated the same results. In other words, the GARCH (1,1) model also is able to remove the ARCH effects from the data.

Figure 2: The autocorrelation functions (ACF) and the partial autocorrelation functions (PACF) for the residuals and the squared residuals of the GARCH (1,1) model.

(a) ACFs and PACFs of the residuals



(b) ACFs and PACFs of the squared residuals



Now we can apply the BS model. We include only one-month option contracts and therefore the left observations are segregated into various categories according to their moneyness. A call option is categorized as at-the-money (ATM) if it's $S_A/X_A \in$ (0.97-1.03), out-of-the-money (OTM) if $S_A/X_A \leq$ 0.97 and in-the-money (ITM) if $S_A/X_A \geq$ 1.03. Animproved partition resulted in six money ness categories for which the results will be reported.

IV. Empirical Results

The present study revisits the performance of BS model by measuring volatility of the underlying asset through three models, namely the Random walk model, the GARCH(p,q) model and the GARCH(1,1) model. Literature has shown that BS model reflects systematic biases in respect to moneyness, maturity time and volatility of the underlying asset. The volatility of the Nifty index was forecasted through the three models and the forecasted values were taken as inputs into the BS model for pricing Nifty index option contracts. A comparison is made between the model prices and the actual market prices of these options and the absolute and percentage pricing errors were calculated as:

Absolute error = $|market\ price - model\ price|$ ………….[8]

Percentage error = $\frac{market\ \ price\ - model\ \ price}{market\ \ price}$ ……………….[9]

The average absolute and percentage errors for the out-of-sample period are summarized in table 3 according to various moneyness categories. Seven categories of moneyness were identified: deep out-of-the-money options (M<0.94), not so deep out-of-the-money options ($0.94 \leq M < 0.97$), near-the-money options ($0.97 \leq M < 1$ and $1 \leq M < 1.03$), not so deep in-the-money options ($1.03 \leq M < 1.06$), deep in-the-money options ($M \geq 1.06$) and lastly "all-option based" category which is found by averaging pricing errors across all the moneyness categories.

Table 3: (a) Absolute pricing errors

Moneyness / Model	All-options based	M<0.94	$0.94 \leq M < 0.97$	$0.97 \leq M < 1$	$1 \leq M < 1.03$	$1.03 \leq M < 1.06$	$M \geq 1.06$
RW	46.32	59.72	56.34	58.45	51.57	40.71	31.86
GARCH(11)	21.01	23.20	21.30	19.60	18.16	21.98	23.37
GARCH(42)	21.69	23.77	22.32	20.51	19.30	22.50	23.47

(b) Percentage pricing errors

Moneyness / Model	All-options based	M<0.94	$0.94 \le M < 0.97$	$0.97 \le M < 1$	$1 \le M < 1.03$	$1.03 \le M < 1.06$	$M \ge 1.06$
RW	0.1148	-0.225	0.1525	0.2724	0.1649	0.0724	0.0439
GARCH(11)	0.0349	-0.0372	0.0136	0.0245	0.0417	0.0505	0.0427
GARCH(42)	0.0390	-0.0160	0.0280	0.0315	0.0425	0.0520	0.0435

Literature has shown that the BS model reflects various biases. Present study is an effort to identify the biases in BS model when volatility of the underlying asset (that is Nifty index) is measured by RW, GARCH(1,1) and GARCG(p,q) models which belong to one of the most accepted category of models in the literature.

Analysis of table 3 above shows that BS model with GARCH (1,1) model inputs demonstrates a sweeping performance across all categories of option contracts except percentage errors of deep OTMs. In absolute terms, the pricing errors are much lower than the BS values with RW model across-the-board.BS model with GARCH (1,1) is able to reduce the errors almost to half (in case of ATMs more than half) if compared with BS model with the Random walk volatility inputs. Amongst all the moneyness categories, BS model with GARCH (1,1) performs the best with minimizing the absolute differences for the two NTM category optionsconfirming the previous finding that BS model performs best for near-the-money options. Between the two GARCH models, GARCH (1,1) is a better performer which also confirms the previous finding about GARCH models that generally equating p and q equal to one provides a satisfactory performance and increasing them beyond one over fits the model thereby reducing the model's performance.

In the percentage errors category, a positive figure indicates overpricing while a negative one indicates underpricing. According to percentage errors BS model with RW volatilities is the worst performer. Here again BS model with the GARCH (1,1) model is the better one though the BS model with GARCH (p,q) volatilities is a close performer in the NTM and ITM Categories. This again reiterate the previous finding that it is enough to include the first lag in both the mean and variance equations of the GARCH models. The percentage errors are negative only for deep OTMs no matter which model out of the 3 models is used for forecasting the underlying asset's volatility. Thus the BS model, irrespective of the volatility forecast underprices the deep OTMs and overprices all other categories of options. This is in sharp contrast to previous findings (like Macbeth and Merville(1979), Chiras and Manaster (1978)) that BS model overprices OTMs and underprices ITMs, which might be because the market in the first decade of its inception is still not mature. It would be interesting to see how the BS model's performance change with the three volatility models by comparing it with the performance in the second decade.

Percentage as well as absolute errors for "all-options based" category shows it is not always better to go averaging across categories irrespective of the fact whether one is dealing with ITMs, OTMs or NTMs. Contradictory to the previous finding that BS model price NTMs most efficiently, the percentage errors indicate that it prices OTMs (whether deep or not) most efficiently(given that volatility is measured through GARCH family)). Moreover, BS model tends to overprice more as compared to underpricing the option contracts. This can be seen by comparing the figures (-0.225, that is the highest underpricing) for underpricing with that of overpricing (0.2724, that is the highest overpricing). So the BS model's performance is exceedingly sensitive to the category of options priced as well as the volatility model used.This is a strong insinuation for the market playerswhile taking decisions to trade an option contract.

V. Conclusion

Indian options market has grown prodigiously at the world dais since its kickoff. There are many option pricing models available to a trader amongst which the BS model is one of the most popular and simple option pricing model. The BS model can be applied with 6 input variables out of which the volatility of the underlying asset is the most difficult to measure. BS model has shown hitherto systematic biases when referred to in relation to moneyness, time left to maturity and volatility of the underlying asset. Present study is an attempt to revisit these biases by taking three volatility measuring models namely the RW model, the GARCH (1,1) model and the GARCH (p,q) model.

By taking data on Nifty index call option contracts present study reconfirms some previously identified biases whereas reject others. By using the GARCH models for measuring volatility, a contrasting finding was that BS model underprice deep OTMs and overprice both NTMs and ITMs. It was confirmed that the BS model is at its best in providing the value of near-the-money options as mentioned in prior literature. GARCH (1,1) volatility forecasting model is sufficient enough and going for GARCH (p,q) model doesn't enhance the performance as far as the BS model is concerned. The present study assert that the BS model does show some moneyness and volatility related biases. The BS model's performance can vary depending upon the model used for predicting volatility of the underlying security. Various stakeholders should be cautious of the fact that which category of options they are valuing and how they are measuring the volatility of the underlying security. Overall, the GARCH (1,1) model is a better choice out of the three models tested in the present study, if the objective is to value the call option contracts through the BS model.

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