IOSR Journal of Business and Management (IOSR-JBM)
e-ISSN: 2278-487X, p-ISSN: 2319-7668. Volume 22, Issue 9. Ser. IV (September 2020), PP 59-64
www.iosrjournals.org

AnInventory Model withTime Dependent Demand, Weibull-Distributed Deteriorationunder Salvage Value and Shortages.

Dr.BiswaranjanMandal
Associate Professor of Mathematics
Acharya Jagadish Chandra Bose College, Kolkata

Abstract: *This paper deals with the development of an inventory model for deteriorating items following Weibull distributed. A time varying price dependent demand rate is considered here. The model is solved with salvages value associated to the units deteriorating during the cycle. Shortages are allowed and fully backlogged.Finally the model is illustrated with the help of a numerical example, some particular cases are derived and the sensitivity of the optimal solution towards the changes in the values of different parameters is also studied.*

Key Words:*Inventory, deterioration, Weibull distributed, time-varying demand, salvages value and shortages.*
Subject classification: *AMS Classification No. 90B05*

Date of Submission: 04-09-2020 Date of Acceptance: 19-09-2020

Introduction:

The effect of deterioration for items cannot be disregarded in many inventory systems and it is a general phenomenon in real life. Deterioration is defined as decay or damage such that the items cannot be used for its original purpose. Food items, drugs, pharmaceuticals and radioactive substances are few examples of items in which appreciable deterioration can occur during the normal storage period of the units and consequently this loss must be taken into account while analyzing the system. Efforts in analysing mathematical models of inventory in which a constant or variable proportion of the on-hand inventory deteriorates with time have been undertaken by Ghare and Schrader [1], Covert and Philip [2], Shah [3], Giri and Goyal [4], Mandal et al [5] etc. to name only a few. But till the recent past authors have considered that deteriorated units are the complete loss to the organization which in reality is not a common practice. In this connection, Poonam Mishra and Shah [6] studied an EOQ model for inventory management of time dependent deteriorating items with salvage value. Ajanta Roy [7] developed an inventory model for deteriorating items with price dependent demand and time-varying holding cost. Jaggi et al [8] and Mohan R et al [9] studied an EOQ model for deteriorating items with salvage value assuming deterioration and demand rate in constant behaviour.

For these sort of situations, efforts have been made to develop a realistic inventory model with time varying price dependent demand under Weibull distributed deterioration rate. Also salvage value associated to the units deteriorating during the cycle is considered in the proposed model. This work is generalised by allowing shortages which are fully backlogged. Some particular cases are derived in the model

Finally anumerical example has beenpresentedto illustrate the theory and lastly a sensitivity study of the optimal solution to changes in the parameters has been examined and discussed.

Assumptions and Notations:
The mathematical models are developed under the following assumptions and notations:

(i) Replenishment size is constant and replenishment rate is infinite.
(ii) Lead time is zero.
(iii) T is the fixed length of each production cycle.
(iv) C_h is the inventory holding cost per unit per unit time.
(v) C_o is the ordering cost/order.
(vi) C_d is the deterioration cost per unit per unit time.
(vii) p is the purchasing cost per unit item.
(viii) The salvage value $kp, 0 \le k < 1$is associated with deteriorated units during a cycle time.
(ix) C_s is the shortage cost unit per unit time.
(x) TC is the average total cost per unit time.

(xi) The instantaneous rate of deterioration of the on-hand inventory is

$$\theta(t) = \alpha\beta t^{\beta-1}, 0 \le \alpha \le 1, \beta \ge 1$$

(xii). The demand rate R(t) is assumed as

R(t) = $R_o t^{-p}, R_0 > 0, 0 < p < 1$

(xiii). Shortages are allowed and fully backlogged

Formulation and Solution of the Model:

Let Q be the total amount of inventory produced or purchased at thebeginning of each period and after fulfilling backorders let us assume we get anamount S(>0) as initial inventory. Due to reasons of market demand anddeterioration of the items, the inventory level gradually depletes during theperiod (0, t_1) and ultimately falls to zero at t = t_1. Shortages occur during timeperiod (t_1, T) which are fully backlogged. Let I(t) be the on-hand inventory at any time t. The differential equations which the on-hand inventory I(t) governed by the following :

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -R(t), 0 \le t \le t_1 \quad (1)$$

And $$\frac{dI(t)}{dt} = -R(t), t_1 \le t \le T \quad (2)$$

The initial condition is $I(0) = S$ and $I(t_1) = 0$ (3)

Putting the values of $\theta(t) = \alpha\beta t^{\beta-1}$ and R(t) = $R_o t^{-p}$, we get

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -R_o t^{-p}, 0 \le t \le t_1 \quad (4)$$

And $$\frac{dI(t)}{dt} = -R_o t^{-p}, t_1 \le t \le T \quad (5)$$

Now solving the equations (4) and (5) using the initial condition (3) and neglecting the second and higher powers of α [since $O(\alpha^2)$ is very small as $0 \le \alpha < 1$], we get

$$I(t) = R_o(1-\alpha t^{\beta})[\frac{1}{1-p}(t_1^{1-p} - t^{1-p}) + \frac{\alpha}{1+\beta-p}(t_1^{1+\beta-p} - t^{1+\beta-p})],\ 0 \le t \le t_1 \quad (6)$$

And $$I(t) = \frac{R_o}{1-p}(t_1^{1-p} - t^{1-p}), t_1 \le t \le T \quad (7)$$

Since I(t_1) = 0, we get from equation (6) the following expression

$$S = R_o[\frac{1}{1-p} t_1^{1-p} + \frac{\alpha}{1+\beta-p} t_1^{1+\beta-p}] \quad (8)$$

The number of items backlogged at the beginning of the period is

$$Q - S = \int_{t_1}^{T} R_o t^{-p} dt$$

Or, $$Q = R_o[\frac{1}{1-p}T^{1-p} + \frac{\alpha}{1+\beta-p}t_1^{1+\beta-p}] \quad \text{(using (8))} \tag{9}$$

Therefore the total cost per unit time TC consists of the following:

Ordering cost (OC) = C_o

Purchasing cost (PC) = pI(0) = pS = p $R_o[\frac{1}{1-p}t_1^{1-p} + \frac{\alpha}{1+\beta-p}t_1^{1+\beta-p}]$

Holding cost (HC) = $C_h \int_0^{t_1} I(t)dt$

$$= C_h R_o[\frac{1}{2-p}t_1^{2-p} + \frac{\alpha\beta}{(1+\beta)(2+\beta-p)}t_1^{2+\beta-p}] \text{ (neglecting } O(\alpha^2)\text{)}$$

Cost due to deterioration (CD) = $C_d \{I(0) - \int_0^{t_1} R(t)dt\} = C_d \frac{R_o\alpha}{1+\beta-p}t_1^{1+\beta-p}$

Salvage cost (SV) =kp $\frac{R_o\alpha}{1+\beta-p}t_1^{1+\beta-p}$

Cost due to shortage (CS) = $C_s \int_{t_1}^{T}(T-t)R(t)dt = \int_{t_1}^{T}(T-t)R_o t^{-p}dt$

$$= C_s R_o[\frac{1}{(1-p)(2-p)}T^{2-p} - \frac{1}{1-p}Tt_1^{1-p} + \frac{1}{2-p}t_1^{2-p}]$$

The average total cost per unit time of the system will be

TC(t_1) = $\frac{1}{T}$ **[OC +PC +HC+CD-SV+CS]**

$$= \frac{1}{T}[C_o + pR_o\{\frac{1}{1-p}t_1^{1-p} + \frac{\alpha}{1+\beta-p}t_1^{1+\beta-p}\} + C_h R_o\{\frac{1}{2-p}t_1^{2-p} + \frac{\alpha\beta}{(1+\beta)(2+\beta-p)}t_1^{2+\beta-p}\} +$$

$$C_d \frac{R_o\alpha}{1+\beta-p}t_1^{1+\beta-p} - kp\frac{R_o\alpha}{1+\beta-p}t_1^{1+\beta-p} + C_s R_o\{\frac{1}{(1-p)(2-p)}T^{2-p} - \frac{1}{1-p}Tt_1^{1-p} + \frac{1}{2-p}t_1^{2-p}\}]$$

(10)

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1} = 0$

This gives

$$p(1+\alpha t_1^{\beta}) + C_h(t_1 + \frac{\alpha\beta}{1+\beta} t_1^{1+\beta}) + C_d \alpha t_1^{\beta} - kp\alpha t_1^{\beta} + C_s(t_1 - T) = 0 \quad (11)$$

For minimum the sufficient condition $\frac{d^2TC(t_1)}{dt_1^{\,2}}$ >0 would be satisfied.

Let $t_1 = t_1^*$ be the optimum value of t_1.

The optimal values Q^* of Q, S^* of S and TC^* of TC are obtained by putting the value $t_1 = t_1^*$ from the expressions (9), (8) and (10).

Particular Cases:

(a). If the deterioration of the items is switched off then $\alpha = 0$

In this case the total amount of inventoryQ and initial inventoryS become

Q = $\frac{R_o}{1-p} T^{1-p}$ and S = $\frac{R_o}{1-p} t_1^{1-p}$

The average total cost per unit time of the system will be

$$TC(t_1) = \frac{1}{T}[\ C_o + \frac{pR_o}{1-p} t_1^{1-p} + \frac{C_h R_o}{2-p} t_1^{2-p} + C_s\ R_o\{\frac{1}{(1-p)(2-p)} T^{2-p} - \frac{1}{1-p} T t_1^{1-p} + \frac{1}{2-p} t_1^{2-p}\}]$$

(b). When k = 0 i.e. no salvage value associated to the deteriorate units.

Here Q, S and TC are given by the following

Q = $R_o[\frac{1}{1-p} T^{1-p} + \frac{\alpha}{1+\beta-p} t_1^{1+\beta-p}]$, S = $R_o[\frac{1}{1-p} t_1^{1-p} + \frac{\alpha}{1+\beta-p} t_1^{1+\beta-p}]$

$$\text{And TC} = \frac{1}{T}[\ C_o + p\,R_o\{\frac{1}{1-p} t_1^{1-p} + \frac{\alpha}{1+\beta-p} t_1^{1+\beta-p}\} + C_h R_o\{\frac{1}{2-p} t_1^{2-p} + \frac{\alpha\beta}{(1+\beta)(2+\beta-p)} t_1^{2+\beta-p}\} + C_d \frac{R_o\alpha}{1+\beta-p} t_1^{1+\beta-p} + C_s\ R_o\{\frac{1}{(1-p)(2-p)} T^{2-p} - \frac{1}{1-p} T t_1^{1-p} + \frac{1}{2-p} t_1^{2-p}\}]$$

Numerical Example:

To illustrate the developed inventory model, let the values of parameters be as follows:

C_o = \$100 per order; p = \$0.1 per unit; C_h = \$0.2 per unit; C_d = \$0.1 per unit; C_s = \$20 per unit; k = 0.1; α = 0.4; β = 2; R_o = 1000; T = 1 year

Solving the equation (11) with the help of computer using the above parameter values, we find the following optimum outputs

t_1^* = 0.98year; Q^* = 1240.81 units; S^* = 1219.79 units and TC^* = Rs 351.76

It is checked that this solution satisfies the sufficient condition for optimality.
The numerical results are also furnished to illustrate the particular cases of the model

Particular case	t_1^* (year)	Q^* (units)	S^* (units)	TC^* (Rs)
Absence of deterioration	0.99	1666.67	1096.10	323.53
No salvage value	0.98	1860.64	1218.40	363.10

It is observed that the total cost decreases when there is no deteriorated items in the model, whereas the cost increases in case of no salvage value of the items.

Sensitivity Analysisand Discussion.

We now study the effects of changes in the system parameters C_o, p, C_h, C_d, C_s, k, α β and R_o on the optimal total cost (TC^*), optimalordering quantity (Q^*) and optimal on-hand inventory (S^*) in the present inventory model. The sensitivity analysis is performed by changing each of theparameters by – 50%, – 20%, +20% and +50%, taking one parameter at a timeand keeping remaining parameters unchanged. The results are furnished in table A.

Table A:Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	% change in		
		Q^*	S^*	TC^*
C_o	-50 -20 +20 +50	No effect	No effect	-0.14 -0.06 0.06 0.14
p	-50 -20 +20 +50	-0.05 -0.02 0.02 0.05	0.003 0.001 -0.002 -0.005	-0.19 -0.08 0.08 0.21
C_h	-50 -20 +20 +50	0.002 0.0006 -0.0009 -0.002	0.001 0.002 -0.003 -0.007	-0.16 -0.06 0.06 0.16
C_d	-50 -20 +20 +50	0.0003 0.000008 -0.0003 -0.0003	0.001 0.00008 -0.001 -0.001	-0.02 -0.007 0.007 0.02
C_s	-50 -20 +20 +50	-0.006 -0.002 0.0009 0.002	-0.02 -0.007 0.003 0.007	-0.01 -0.003 0.002 0.004
k	-50 -20 +20 +50	-0.0003 -0.000008 0.000008 0.00002	-0.001 -0.00008 0.00008 0.0002	0.002 0.0007 -0.0007 -0.002
α	-50 -20 +20 +50	-0.05 -0.02 0.02 0.05	-0.05 -0.02 0.02 0.05	-0.05 -0.02 0.02 0.05
β	-50 -20 +20 +50	0.06 0.02 -0.01 -0.03	0.06 0.02 -0.01 -0.03	0.02 0.01 -0.01 -0.02
R_o	-50 -20 +20 +50	-0.50 -0.20 0.20 0.50	-0.47 -0.16 0.26 0.58	-0.36 -0.14 0.014 0.36

Analyzing the results of table A, the following observations may be made:

(i) TC^* increases or decreases with the increase or decrease in the values ofthe system parameters C_o, p, C_h, C_d, C_s, α and R_o. On the other hand it increases or decreaseswith the decrease or increase in the values of the system parameters k and β. The results obtained show that TC^* are very lowsensitive to changes inthe value of the system parameters.

(ii) Q^* increases or decreases with the increase or decrease in the values of the system parameters p, C_s, k, α and R_o. On the other hand it increases or decreases with the decrease or increase in the values of the system parameters C_h, C_d ,and β. However Q^* is very low sensitive to changes in the values of the parameters p, C_h, C_d, C_s, k, α and β, but moderate sensitive towards the changes of R_o.

(iii) S^* increases or decreases with the increase or decrease in the values of the system parameters C_s, k, α and R_o. Whereas it increases or decreases with the decrease or increase in the values of the system parameters p, C_h, C_d, and β. However S^* is very low sensitive to changes in the values of the parameters p, C_h, C_d, C_s, k, α and β, but moderate sensitive towards the changes of R_o.

From the above analysis, it is seen that all the parameters are almost low sensitive except demand parameter R_o. Hence the estimationof the demand parameter R_o needs some adequate attention.

References:

[1]. Ghare P.M. and SchradeG.F(1963). A model for exponential decayinginventory. Journal of Industrial Engineering, 14, 238-243.

[2]. Covert R.P. and Philip G.C.(1973). An EOQ model for items with Weibull distribution deterioration, AIIE Trans., 5, 323-326.

[3]. Shah Y. K. (1977). An order level lot-size inventory model for deteriorating items, AIIE Trans., 9, 108-112.

[4]. Giri B.C. and Goyal S.K.(2001). Recent trends in modelling of deteriorating inventory, European Journal of Operations Research, 134, 1-16.

[5]. Mandal B. Bhunia A.K. and Maiti M.(2003). An inventory system of ameliorating items for price dependent demand rate, Computers and Industrial Engineering, 45(3), 443-456.

[6]. Poonam Mishra and Shah N.H.(2008). Inventory management of time dependent deteriorating items with salvage value, Applied Mathematical Sciences, 16(2), 793-796.

[7]. Ajanta Roy(2008). An inventory model for deteriorating items with price dependent demand and time-varying holding cost. AMO, 10(1).

[8]. Jaggi C.K. and Aggarwal S.P.(1996). EOQ for deteriorating items with salvage value, Bulletin of Pure and Applied Sciences, 15(1), 67-71.

[9]. Mohan R. and Venkateswarlu R.(2013). Inventory management models with variable holding cost and salvage value, IOSR J. of Business and Management (IOSR-JBM), 12(3), 37-42.

Dr.BiswaranjanMandal"AnInventory Model withTime Dependent Demand, Weibull-Distributed Deteriorationunder Salvage Value and Shortages." *IOSR Journal of Business and Management (IOSR-JBM)*, (9)22, 2020, pp. 59-64.