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Tracking Signal Performance in Monitoring Manufacturing Processes

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Abstract: *Manufacturing processes that consist of time series data are frequently monitored by forecast-based quality control schemes. These control schemes are based on the application of a time series forecast to the process and monitoring the resultant forecast errors with a tracking signal. This study compares the performance of tracking signals in their ability to detect the presence of changes in the process mean (step shift) and additive outliers in an autocorrelated manufacturing process. The criteria used are the Average Run Length (ARL) and Cumulative Distribution Function (CDF) of the run lengths. The CDF is offered an alternative performance evaluation criterion, for forecast-based schemes. Based on the CDF criterion, the Smoothed Error Tracking Signal offers the greatest probability of early detection of a step shift and an additive outlier in an autocorrelated process.*

Keywords: *tracking signals, autocorrelation, process monitoring*

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I. Introduction

The occurrence of large unusual observations is not uncommon in time series data. These outliers may be due to recording errors or to one-time unique situations such as an unexpected change in demand for a product or a change in a production system. Fox (1972) defines two types of outliers may occur in practice. An additive outlier corresponds to an external disturbance that affects the value of a single observation. An innovational outlier refers to an internal disturbance that changes the value of an observation and all other successive observations. Typically, in process control environments, monitoring schemes are compared based on their ability to detect step shifts or innovational outliers in the level of a process. However, which monitoring scheme detects the presence of an additive outlier most quickly is also of interest.

Autocorrelation implies the existence of a relationship between consecutive observations and can be of two types. A process that tends to drift over time is characteristic of positive autocorrelation and results when successive observations are similar in value. Negative autocorrelation is depicted by a sawtooth pattern and results when consecutive observations are dissimilar. High volume manufacturing processes along with an increased frequency of sampling by automated gages, gives rise to autocorrelated data.

The presence of autocorrelation creates unique problems for production monitoring schemes. Positive autocorrelation tends to increase the frequency of out-of-control signals that are detected by monitoring schemes. Positive autocorrelation occurs most often in production environments and chemical operations (Woodall and Faltin (1993)).

In the forecasting and time series fields, tracking signals are used to monitor forecasting systems. Alwan and Roberts (1988) have proposed a method for monitoring autocorrelated data that involves the application of a time-series forecast to the process and monitoring the forecast errors. Unusual behavior in the process should result in a large error that is reflected as a signal on a tracking signal.

Traditionally, monitoring tools have been compared on the basis of Average Run Lengths (ARLs). The ARL is the expected number of observations required to detect an out-of-control situation. However, simple exponential smoothing forecasts recover quickly from step increases in the time series process that it monitors. This would suggest that the performance of forecast-based schemes should be based on the probability of "early detection". As an average measure that is inflated by long run lengths, the ARL is an inadequate measure of quick recovery, that is characterized by short run lengths. Hence the cumulative distribution function (CDF) of the run lengths is offered as an alternative criterion to the average run length (ARL) for the selection of an appropriate monitoring scheme. The CDF provides the cumulative probability of a signal occuring by the ith time period after a disturbance.

This paper compares the performance of a Smoothed Error (ETS) tracking signal and a Cumulative Sum (CUSUM) tracking signal in monitoring forecast errors from exponential smoothing forecasts applied to autoregressive process data of order one, denoted by AR (1), in the presence of a changes in the process mean and additive outliers. Superville and Yorke (2012) compared the performance on control charts in detecting additive outliers. The study shows that the ETS tracking signal offers the highest probability of early detection of a shift in the mean and an additive outlier in AR (1) manufacturing processes.

II. A Model For Autocorrelated Data

A time series model that is widely used in inventory and quality control applications is the autoregressive integrated moving average (ARIMA) model. The ARIMA(p,d,q) model is denoted by

$$\Phi_p (B) \nabla^d X_t = \Theta_q (B)\varepsilon_t \qquad (1)$$

where $\Phi_p (B) = (1-\phi_1 B-\phi_2 B^2-...-\phi_p B^p)$ is an autoregressive polynomial of order p, $\Theta_q (B) = (1-\theta_1 B-\theta_2 B^2-...-\theta_q B^q)$ is a moving average polynomial of order q, B is the backshift operator, ∇ is the backward difference operator and ε_t, the white noise, is a sequence of independent normal random errors with mean zero and variance σ^2, denoted by $\varepsilon_t \sim N(0, \sigma^2)$.

A special case of the ARIMA(p,d,q) model that has been found to be useful in production and quality control environments is the ARIMA(1,0,0), referred to as the first-order autoregressive model and denoted by AR(1). It is represented by

$$X_t = \xi + \phi X_{t-1} + \varepsilon_t . \qquad (2)$$

Without loss of generality it is assumed that $\varepsilon_t \sim N(0,1)$. It is also assumed that an AR(1) model is applicable in this article. Montgomery and Mastrangelo (1991) show that a number of chemical and manufacturing processes conform to this model.

The simple exponential smoothing forecast, also known as an exponentially weighted moving average (EWMA) forecast is given by

$$F_{t+1} = \alpha_F X_t + (1-\alpha_F)F_t , \qquad 0\le \alpha_F \le 1 . \qquad (3)$$

where X_t represents the process observation at time period t, and F_{t+1} represents the one-step-ahead forecast for observation X_{t+1} at time period t. The forecast error at time period t, denoted by e_t, is defined as

$$e_t = X_t - F_t.$$

Alwan and Roberts (1988) have observed that processes that do not drift too rapidly are well modeled by simple exponential smoothing. For the AR(1) model, Cox (1961) has shown that optimal simple exponential smoothing in terms of minimum mean square forecast error is given by

$$\alpha_F = 1- ½[(1-\phi)/\phi], \qquad 1/3 < \phi \le 1 \qquad (4)$$

where ϕ is the parameter of the AR(1) process. This result is used in the simulations discussed in the following sections.

III. Tracking Signals

In this study, the Smoothed Error (ETS) and Cumulative Sum (CTS) tracking signals are applied to exponential smoothing forecast errors and their performances evaluated.

The Smoothed Error Tracking Signal

Trigg's (1964) Smoothed Error (ETS) tracking signal is given by

$$ETS_t = \left| E_t / MAD_t \right| \qquad (5)$$

where

$$E_t = \alpha_1 e_t + (1-\alpha_1)E_{t-1} , \qquad 0\le \alpha_1 \le 1 \qquad (6)$$

and

$$MAD_t = \alpha_2 \left| e_t \right| + (1-\alpha_2)MAD_{t-1} , \qquad 0\le \alpha_2 \le 1 . \qquad (7)$$

Typically, $E_0 = 0$ and MAD_0 is set equal to its expected value which is approximately equal to $0.8\sigma_e$ (where σ_e is the standard deviation of the forecast errors). A signal occurs if ETS_t exceeds a critical value K_1. Gardner (1983) suggests that the value of K_1 should be set to achieve a desired in-control ARL.

The Cumulative Sum Tracking Signal

Brown's (1959) Cumulative Sum (CTS) tracking signal is given by

$CTS_t = |SUM_t / MAD_t|$ (8)

where

$SUM_t = e_t + SUM_{t-1}$. (9)

The value of MAD_0 is set equal to its expected value as with ETS_0. The value of SUM_0 is set equal to zero. A signal occurs if the value of CTS_t exceeds a critical value K_2. Gardner (1983) suggests that the value of K_2 should be set to achieve a desired in-control ARL.

Concerning the choice of parameters for the forecast model (α_F), and tracking signals (α_1 and α_2), McKenzie (1978) and Gardner (1985) recommend that $\alpha_F \geq \alpha_1$, with $\alpha_1 = 0.1$ commonly used in practice. Small values of α_1 allow the ETS to respond more quickly to small disturbances in the demand process. Traditionally, the smoothing parameters in the numerator and denominator of the ETS have been set equal to each other, that is, $\alpha_1 = \alpha_2$. More recently, McClain (1988) has suggested that the smoothing parameter in the MAD,α_2, be smaller than the parameter in the numerator, α_1, so that the variance of the forecast errors may be stabilized.

IV. Evaluation Criteria: Arl Vs. CDF

The ARL is a criterion on which the relative performance of both tracking signals has been based. However, exponentially smoothed forecasts tend to recover quickly from disturbances in the time series that it monitors. In general, the rate of forecast recovery depends on the type of shift, the underlying model and the forecasting tool in use. In most cases, forecast recovery is shown to significantly impact ARLs.

The necessity of quick detection of process shifts leads one to the cumulative probability of a signal following a process shift as a meaningful criterion for the comparison of forecast-based monitoring schemes. The use of the cumulative distribution functions (CDF) as an evaluation criterion is not new. Barnard (1959), Bissell (1968) and Gan (1991) recommend its use on independent observations. Referred to as a 'response to a change in demand', McClain (1988) advocates its use for forecast-based schemes which incorporate tracking signals. The CDF measures the cumulative percentage of disturbances in a time series that are detected early.

V. Design Of The Simulation Study

In this simulation study, two monitoring schemes were compared. They are the ETS and CTS. ARLs and CDFs are provided for each monitoring scheme for outliers of size $3.0\sigma_p$, where $\sigma^2_p = \sigma^2 / (1-\phi^2)$, is the variance of an AR(1) process.

The initial values of the smoothed-error for the ETS (equation 5) and the sum of errors for the CTS (equation 8) are set to zero as suggested by Gardner (1985) and McClain (1988). The smoothing constants $\alpha_1$and α_2 were set to 0.10 as suggested by McKenzie (1978).

The simulation study was conducted as follows:

i) AR(1) series with autoregressive parameter values of ϕ =0.0, 0.5, 0.7, and 0.9 and N(0,1) are generated by the IMSL (1991, p.1350-1351) subroutine RNARM / DRNARM. The parameter ξ was set equal to zero, without loss of generality,

ii) the first fifty observations are used to allow for a burn-in period,

iii) the forecast is started at time period 2 with its initial value set equal to the first observed data point,

iv) fifty (50) preliminary sequences of forecast errors are used to estimate the variance of the forecast errors for a step increase of zero (the in-control state),

v) tracking signals and control charts are constructed based on the estimates obtained in step (iv). The initial MAD values are set to $0.8\sigma_e$ (σ_e is the standard deviation of the forecast errors) as suggested by Montgomery, Johnson and Gardiner (1990),

vi) the monitoring schemes are applied to the forecast errors,

vii) steps (i)-(iii), (v) and (vi) are repeated 1000 times. For each monitoring scheme, the run length for each simulation iteration is recorded. These run lengths are used to obtain the ARLs and CDFs after a shift of size 0.0, $1\sigma_p$ and $3\sigma_p$ and then an additive outlier of size $3.0\sigma_p$.

VI. Simulation Results

Table I displays simulated ARLs and CDFs for the ETS and CTS tracking signals applied to the optimal exponential smoothing forecast errors from an AR(1) process with ϕ ranging from 0.0 to 0.9, after a shift of size 0.0, $1\sigma_p$ and $3\sigma_p$. Table 2 displays simulated ARLs and CDFs for the ETS and CTS tracking signals applied to the optimal exponential smoothing forecast errors from an AR(1) process with ϕ ranging from 0.0 to 0.9, with outliers simulated as $3\sigma_p$.

For step shifts (Table 1), the results may be summarizes as follows:

1. For step shifts of 1σ and 3σ, the ARLs for the autocorrelated cases (ϕ>0) are substantially larger for the independent case (ϕ=0). This is a result of the quick recovery of the forecast of the forecast illustrated by short run length and the inability of ARLs to adequately reflect these short run lengths. In the calculation of the ARL, longer run lengths mask shorter run lengths.
2. Based on CDFs, the ETS provides a higher probability of early detection of a step shift than the CTS for the autocorrelated cases (ϕ>0). This occurs although the CTS has shorter ARLs than for these cases. The detection of a step shift early is critical, since the forecast recovers quickly. This suggests the use of the ETS for autocorrelated cases.
3. Neither tracking signal provides a very high probability of early detection. By the 4th time period after a step shift of 3σ, the CTS detects 47.2% of shifts and the ETS detects 49.8% for ϕ=0.5. These detection probabilities decrease dramatically for the cases where ϕ>0.5.

For outliers (Table II), the results can be summarized as follows:

1. For the independent case (ϕ=0), the ETS has a substantially smaller ARL (4.3) compared to the CTS (21.9). The ETS also has a larger probability of early detection of an outlier (92.2% by the sixth observation compared to 0% for the ETS). Recall that the ARL, as an average measure, is inflated by long run lengths. It is unable to adequately reflect short run lengths that are indicative of quick forecast recovery. For forecast-based schemes, ARLs are not informative.
2. Based on CDFs, the ETS provides a higher probability of early detection of an outlier for the autocorrelated cases where ϕ=0.5, 0.7 and 0.9. This occurs although the ETS control chart may have a longer ARL than the CTS. As an example, consider the case where ϕ=0.9. The ETS provides a higher probability of early detection on the first observation after the outlier (30.2% compared to 0.9% for the CTS) despite having a longer ARL (61.9) than the CTS (52.6). The detection of an outlier early, that is, within the first few observations after the occurrence of an outlier is critical since the forecast recovers quickly. This suggests the use of the ETS chart for the autocorrelated cases.

VII. Conclusions

This paper has compared tracking signals for monitoring autocorrelated observations in the presence shifts in a process mean and additive outliers. The quick recovery property of forecasting tools suggests that the performance of tracking signals applied to forecast errors be based on the CDF on the run lengths and not on the ARL. The Smoothed Error Tracking Signal is recommended over the CUSUM tracking signals as it offers the highest probability of early detection of change in a process mean and an additive outlier in an AR(1) process.

TABLE I. Average Run Lengths and Percentage of Signals detected by the *i*th observation after a shift of size Δ. The ETS and CTS tracking signals are applied to forecast errors from AR(1) processes with autoregressive parameters ϕ and an in-control ARL of 250.

ϕ	Δ	Monitoring Scheme	ARL	Number of time periods after an outlier 1	2	3	4	5	6
0	0.0	CTS	252	0.2	0.3	0.3	0.3	0.4	0.4
		ETS	248	1.1	1.7	2.4	2.6	3.2	3.7
	1.0	CTS	26.8	0.7	0.7	0.8	0.9	1.0	1.2
		ETS	11.2	0.9	0.2	4.6	7.1	11.7	17.4
	3.0	CTS	21.9	0.4	0.4	0.4	0.4	0.4	0.4
		ETS	4.4	3.0	11.1	28.8	53.6	54.2	91.1
0.5	0.0	CTS	250	0.2	0.9	1.2	1.6	2.0	2.4
		ETS	250	0.3	0.9	1.2	1.8	2.1	2.2
	1.0	CTS	37.2	0.6	2.4	3.5	5.7	7.2	9.3
		ETS	213.2	1.3	2.7	4.7	6.2	8.3	10.3
	3.0	CTS	5.7	3.9	14.9	30.5	47.2	59.9	70.5
		ETS	70.6	8.6	24.6	38.3	49.8	55.9	61.2

	0.0	CTS	251	0.5	1.5	1.8	2.1	2.5	2.6
		ETS	251	0.6	0.9	1.7	2.1	2.3	2.7
	1.0	CTS	90.2	1.0	2.3	3.2	4.0	5.2	6.7
0.7		ETS	238.9	1.0	2.6	3.7	4.3	4.9	5.4
	3.0	CTS	11.8	5.0	10.8	16.3	22.3	27.9	33.5
		ETS	158.6	9.6	16.3	20.1	24.1	27.0	29.4
	0.0	CTS	251	0.1	0.1	0.1	0.1	0.1	0.4
		ETS	251	0.5	0.8	1.2	1.5	2.0	2.2
	1.0	CTS	181.6	0.3	0.3	0.3	0.6	0.6	0.8
0.9		ETS	233.5	1.5	2.7	2.7	3.5	3.9	4.3
	3.0	CTS	108.6	0.2	0.7	1.1	1.7	2.4	3.1
		ETS	179.1	6.2	7.4	9.0	9.8	10.2	11.0

VIII. Conclusions

This paper has compared tracking signals for monitoring autocorrelated observations in the presence shifts in a process mean and additive outliers. The quick recovery property of forecasting tools suggests that the performance of tracking signals applied to forecast errors be based on the CDF on the run lengths and not on the ARL. The Smoothed Error Tracking Signal is recommended over the CUSUM tracking signals as it offers the highest probability of early detection of change in a process mean and an additive outlier in an AR(1) process.

TABLE II. Average Run Lengths and Percentage of Signals detected by the *i*th observation after an outlier of size $3\sigma_p$. The ETS and CTS tracking signals are applied to forecast errors from AR(1) processes with autoregressive parameters ϕ and an in-control ARL of 250.

ϕ	Monitoring Scheme	ARL	Number of time periods after an outlier 1	2	3	4	5	6
0.0	ETS	4.3	3.9	12.7	30.5	53.9	75.7	92.2
	CTS	21.9	0.0	0.0	0.0	0.0	0.0	0.0
0.5	ETS	43.1	12.6	32.8	48.3	60.0	67.5	72.6
	CTS	4.5	5.4	21.6	40.3	58.7	73.0	83.2
0.7	ETS	90.0	20.2	34.1	40.5	46.6	49.4	52.4
	CTS	6.4	9.9	22.6	34.5	45.9	54.2	61.7
0.9	ETS	61.9	30.2	35.6	38.0	39.5	39.8	40.3
	CTS	52.6	0.9	2.0	3.1	5.0	7.6	10.3

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