

Estimating Weights in Analytic Hierarchy Process Using LP and Optimizing Of Human Resource Allocation

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Abstract: The Analytic Hierarchy Process estimates the weights of alternatives by deriving priorities of a comparison matrix. In this paper, we used two phases Linear Programming (LP) models to estimate the weights of a pairwise comparison matrix derived within the framework of the Analytic Hierarchy Process when eigenvector method fails to satisfy element dominance properties. The first phase brings a consistency bound and it is used in the second phase to derive priority vector. The priorities obtained for the alternatives served as the coefficients of the objective function of linear programming to optimize a human resource problem as an example. Here we show which positions to fill, how to allocate resources.

Date of Submission: 16-06-2018

Date of acceptance: 02-07-2018

I. Introduction

The success of any organization lies upon its ability to make critical decisions on growth and sustainability. However, decision-making is a complex process as it involves multiple stakeholders with different opinions and interests. To avoid making ad-hoc decisions, decision-makers are required to evaluate every alternative to the problem. With the Analytic Hierarchy Process¹¹, the problem is modelled by the decision maker and is structurally decomposed into a hierarchy consisting of levels of criteria, sub-criteria and alternatives with homogeneous clusters of factors¹⁵. Subsequently, an assessment of the usefulness of elements at each hierarchical level is made.

AHP is a very suitable multi-criteria decision-making tool proving to be effective in different application areas such as planning, optimization, selecting a best alternative and in the allocation of resources. A list of applications of AHP can be found in William H et al²¹. It is also a reliable tool in resolving conflicts¹⁷. AHP is critical in defining decision making processes taking into consideration decision maker's input, judgments, views and feelings²⁰. AHP finds applications in new product development¹, business performance³, project selection⁵, human resource management⁷.

Specifying the hierarchy is of crucial importance; the hierarchical structure gives a clear overview of the complex relationship existing in the problem. This is important because it enables the decision maker to take into consideration every aspect in each level of the hierarchy. It also allows a decision maker to take into consideration a set of evaluation criteria and alternative options from among which the best selection is to be made. The focus of the problem, usually the goal, is the highest level of the hierarchy. There are subsequent levels of criteria down further that include sub-criteria, and finally the level of alternatives from which decisions are generated. Elements with a global composition may be included along the top levels of the hierarchy.

In each hierarchical level division, a pairwise comparison matrix is developed with $n(n-1)/2$ number of comparisons, where 'n' is the number of criteria or alternatives in each level^{8,18}. Using a fundamental scale developed by Saaty¹⁶, decision makers are able to assign the corresponding importance of one criterion relative to the other^{4,12,13}. Finally, the weights of the elements being compared are estimated. In the end all the pairwise comparison results are synthesized and the decision is made in accordance with the final overall ranking of the alternatives.¹¹

In summary, AHP applications in decision making involve four main procedures, namely: decomposition of the problem, making judgments in the pairwise comparison matrices and checking their inconsistency, improving it to derive the priority weights, and the determination of the final global weights for all the elements in the model, including the alternatives, the synthesis step.

With each comparison matrix, the decision maker commonly uses eigenvector method (EM) or additive normalization (AN) or logarithmic least square method (LLS) to generate a priority vector. These methods give the estimated relative weights of the elements as a result of the judgments. To produce the final weight for the

alternatives, weights generated at different levels of the hierarchy are synthesized according to the principle of hierarchic structure.

In the next section, AN, EM, LLS and LP methods are explained. The relative weights in a problem have been estimated using all these methods. In the subsequent section, the necessity of using AHP and LP models in human resource selection is covered. Finally, a case has been discussed in detail developing the hierarchy, used interval LP approach for determination of weights.

II. Estimating Weights

The following notations we have used for a pairwise comparison matrix:

Let $A = (a_{ij})$ for all $i, j = 1, 2, \dots, n$ denote an $n \times n$ pairwise comparison matrix, where a_{ij} is the importance of element i over the j th element. All the entries in matrix A are positive ($a_{ij} > 0$) and reciprocal $a_{ij} = 1/a_{ji}$ for all $i, j = 1, \dots, n$. In pairwise comparison matrix A , a_{ij} can be a single number that estimates $\frac{w_i}{w_j}$ or an interval specified with a lower bound l_{ij} or an upper bound u_{ij} or a mixed of both. In case of interval, a_{ij} is the geometric mean the of the interval bounds. The decision maker wants to compute a vector $w = (w_1, w_2, \dots, w_n)$ of weights associated to pairwise comparison matrix A .

The matrix A is considered to be consistent when $a_{ij} = a_{ik} a_{kj}$ for all $i, j, k = 1, 2, \dots, n$, which implies that the decision maker is coherent (no error) in his judgments to develop the comparison matrix.¹⁷ Assuming A contains no error and w_i is the weight of the i^{th} element, then we have

$$a_{ij} = \frac{w_i}{w_j}, \quad i, j = 1, 2, \dots, n \quad (1)$$

Summing over all j , we obtain

$$\sum_{j=1}^n a_{ij} w_j = n w_i, \quad i = 1, 2, \dots, n \quad (2)$$

Which, in matrix notation, is equivalent to

$$Aw = nw. \quad (3)$$

The vector w is the principal eigenvector of the matrix A corresponding to the eigenvalue n , alternatively, we can say that the matrix A is consistent when $Aw = nw$.⁸

Additive normalization (AN)

In obtaining the priority vector w using AN method, columns are first normalized such that elements of each column of the matrix A is divided by the sum of that column; then in each resulting row, normalized elements are summed up and divided by the number of elements in each row which is arithmetic average of the row¹⁹. The following equations (4) to (6) describe the above process;

$$a'_{ij} = a_{ij} / \sum_{i=1}^n a_{ij} \quad i, j = 1, 2, \dots, n \quad (4)$$

$$w_i = \left(\frac{1}{n}\right) \sum_{j=1}^n a'_{ij}, \quad i = 1, 2, \dots, n \quad (5)$$

It can be observed that

$$\sum_{i=1}^n w_i = 1 \quad (6)$$

If A is consistent, then the columns of the normalized matrix $N = (a'_{ij})$ of A are identical. If A is not consistent, then we can write $Aw = \lambda_{max} w$, where λ_{max} is the principal eigenvalue and given by

$$\lambda_{max} = \sum_{i=1}^n \sum_{j=1}^n a_{ij} w_j \quad (7)$$

The consistency index (CI) is given by

$$CI = (\lambda_{max} - n)/(n - 1) \quad (8)$$

while the consistency ratio(CR) is given by

$$CR = CI/RI \quad (9)$$

The random index (RI), which depends on the order of the matrix, is the average CI of a large number of randomly generated matrices. A CR of 0.10 or less is considered acceptable.

Eigenvector method (EM)

The principal eigenvector λ_{max} of A is determined by solving the determinant,

$$\det(A - \lambda_{max} I) = 0 \quad (10)$$

Then using the value of λ_{max} , the eigenvector $w = (w_1, w_2, \dots, w_n)$ is find out from

$$(A - \lambda_{max} I)w = 0 \quad (11)$$

The consistency of the matrix is checked using equations (8) and (9), in Section 2.1.

Logarithmic least square method (LLS)

This method has also been developed to estimate the vector of weights¹⁹. With LLS, the weights w_i , for $i = 1, \dots, n$, are chosen to minimize the objective

$$\sum_{i=1}^n \sum_{j=1}^n [\ln a_{ij} - (\ln w_i + \ln w_j)]^2 \quad (12)$$

Given that $a_{ij} = 1/a_{ji}$ for all $i, j = 1, 2, \dots, n$, (13)

the LLS is quite simple w_i for $i = 1, \dots, n$ is given by the geometric mean of the row i .

Linear programming approach (LP)

There are two desirable properties of a pairwise comparison matrix – element dominance (ED) and row dominance (RD).

ED is said to be preserved if $a_{ij} > 1$ implies $w_i \geq w_j$. If a_{ij} is exactly equal to 1, then an argument can be made for either $w_i \geq w_j$ or $w_j \geq w_i$. RD is said to be preserved if $a_{ik} \geq a_{jk}$ for all k and $a_{ik} > a_{jk}$ for some k Implies $w_i \geq w_j$.

EM and LLS both satisfy RD (but not the ED). In the LP approach, we can incorporate ED and RD as constraints so that there is no violation of ED.

The two-stage LP approach² is described in following two sub-sections.

First stage: LP to establish the consistency bound

In general, any estimate of relative preference a_{ij} can be written as

$$\frac{w_i}{w_j} = a_{ij} \varepsilon_{ij}, \quad i, j = 1, 2, \dots, n \quad (14)$$

If the decision maker is consistent then ε_{ij} is equal to 1. Defining three transformed decision variables for the model: $x_i = \ln(w_i)$, $y_{ij} = \ln(\varepsilon_{ij})$, and $z_{ij} = |y_{ij}|$.

The first stage LP can be written as:

$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^n z_{ij} \tag{15}$$

S.t.

$$x_i - x_j - y_{ij} = \ln a_{ij}, \quad i, j = 1, 2, \dots, n; \quad i \neq j, \tag{16}$$

$$z_{ij} \geq y_{ij}, \quad i, j = 1, 2, \dots, n; \quad i < j, \tag{17}$$

$$z_{ij} \geq y_{ji}, \quad i, j = 1, 2, \dots, n; \quad i < j, \tag{18}$$

$$x_1 = 0 \tag{19}$$

$$x_i - x_j \geq 0 \quad i, j = 1, 2, \dots, n; \quad a_{ij} > 1 \tag{20}$$

$$x_i - x_j \geq 0 \quad i, j = 1, 2, \dots, n; \quad a_{ij} \geq a_{jk}, \quad \text{for all } k; \tag{21}$$

$$a_{ik} > a_{jk} \text{ for some } k$$

$$z_{ij} \geq 0 \quad i, j = 1, 2, \dots, n \tag{22}$$

$$x_i, y_{ij}, \quad \text{unrestricted for } i, j = 1, 2, \dots, n \tag{23}$$

The objective function (15) which is $\sum_{i=1}^{n-1} \sum_{j=i+1}^n z_{ij}$, minimizes the sum of logarithms of positive errors in natural log space, whereas the constraint (16) is defining the errors. Equations (17) and (18) are the degree of over estimation, (19) sets one of the weight w_1 to zero, (20) preserves element dominance and (21) for row dominance. For a perfectly consistent comparison matrix, z^* is equal to zero.

The objective function provides consistency index

$$CI(LP) = \frac{2z^*}{n(n-1)} \tag{24}$$

When a_{ij} is given in interval,

we replace equation (20) with the following constraint:

$$x_i - x_j \geq \ln l_{ij} \quad i, j = 1, 2, \dots, n; \quad i < j$$

and equation (21) is replaced with

$$x_i - x_j \leq \ln u_{ij} \quad i, j = 1, 2, \dots, n; \quad i < j$$

Second stage: LP to generate a priority vector

The first stage LP minimizes the product of all errors ε_{ij} , but multiple optimal solutions may exist. In the second stage LP, the solution that minimizes the maximum errors ε_{ij} is selected. The second stage LP can be presented as:

$$\text{Min } z_{max} \tag{25}$$

S.t.

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^n z_{ij} = z^* \tag{26}$$

$$z_{max} \geq z_{ij}, \quad i, j = 1, 2, \dots, n; \quad i < j, \tag{27}$$

and all first stage LP constraints.

z^* is the optimal first stage solution value, z_{max} is the maximum value of error the z_{ij} . Constraint (26) ensures that only those solutions that are optimal in the first stage LP are feasible in the second stage model.

III. Empirical Illustration: Human Resource Allocation

Let us consider an organization, which faces different decision-making, challenges in its day-to-day operations and it is in the company’s best interest to manage risky situations that may come from wrong decision-making choices. Proper decision making choices will ensure the company’s sustainability and prosperity. Since a company’s activities help to ensure its longevity, managers and decision-makers, organization needs to evaluate weights and priorities of each activity in relation to its consequent outcome. Their aim is to equip the companies with strategic decision-making techniques for sustainable growth, and consequently to achieve a competitive upper hand in the market. Decision makers must, therefore, adopt and apply different business practices, methods, and various tools that prove effective in decision-making.

In any organization, selection of employees to fill different departmental posts is a crucial and sensitive matter⁹. Decision makers are well aware of the costs involved in the improper selection of employees on one hand, and the associated benefits of equipping the company with the right employees on the other. It is therefore vital for companies to not only select and hire employees whose skills and capabilities align well with the goals and objectives of the company, but also to achieve optimality in the return of their investment in human resources¹⁴. Having limited resources, the need for selection of potential employees must also incorporate the need to provide the company with employees whose values, mission and vision match those of the companies.

An effective tool is required, one that will be able to take into consideration both tangible and intangible aspects of criteria to take into consideration when selecting employees. The key is to find the proper measurement of the weights and priorities of different criteria such that error and bias in decision-making is minimized. With this tool, the right kind of employees will be chosen to man the right kind of tasks in the organization and ultimately help the organization to achieve the optimum value of its investment in human resources¹⁴. AHP proves to be such a useful tool for selection of employees, and the application of Linear Programming will ensure the company achieves the optimality it requires^{5,14}. Our purpose here is to illustrate AHP by deriving priorities and applying LP models and to formulate LP models to optimize returns when the elements of comparison matrix are in the interval. The company has identified the Human Resource Management Department and Support System Departments to fill different positions for a firm is listed in Table 1.

The organization wants to select and employ qualified, goal-oriented efficient and personnel. They seek to ensure the integration of business objectives with the right personnel who possess the required skills and ability to achieve the goal and object of the company. They however has a limited budget of 60,000 USD to spend in terms of salaries and they also have few positions to be filled (Table 3). They seek personnel that will seek to incorporate the mission and mission of the company.

The company seeks to employ a Human Resources Manager, a post that is currently vacant in the Human Resources Department and IT Personnel and a Legal Officer position under the Support Systems Department. These positions, once filled will help the organization to fulfil the demand. It will also help the organization to have quality products and it will help the organization to make effective management of employees. In Patel et al¹⁰ have discussed the use of LP in determining weights when there was no interval response to construct a pairwise comparison matrix.

Table 1: Personnel requirements

Positions	Variable	Salary in USD	Personnel required	Department
T_1	Human Resources Manager	2,000	1	HR
T_2	IT Technician	1,200	2	SS
T_3	Legal Officer	1,500	1-2	SS

The problem has been elaborated with hierarchical disintegration, development of the pair-wise comparison matrix, priority weights derivation and the global weight synthesis. A partial hierarchy is depicted in Fig.1 with local weights.

A numerical example to derive weights

Let us consider a pairwise comparison matrix of the interval (Matrix-1). From this pairwise comparison matrix, we find the geometric mean on the intervals and natural logarithms of the geometric means to write linear programming phase-1.

Table 2:pair-wisecomparaison matrix.

	Demand Fulfillment (DF)	Quality Product (QP)	Employee Management (EM)
DF	1	[2,4]	[4,6]
QP	[1/4,1/2]	1	[3,5]
EM	[1/6,1/4]	[1/5,1/3]	1

Then we solve the two-phased linear equations and the weights given are given below.

First phase model is given as:

$$\text{Min } z_{12} + z_{13} + z_{23}$$

S.t.

$$x_1 - x_2 - y_{12} = 1.040$$

$$x_2 - x_1 - y_{21} = -1.040$$

$$x_1 - x_3 - y_{13} = 1.59$$

$$x_3 - x_1 - y_{31} = -1.59$$

$$x_2 - x_3 - y_{23} = 1.354$$

$$x_3 - x_2 - y_{32} = -1.354$$

$$z_{12} - y_{12} \geq 0$$

$$z_{21} - y_{21} \geq 0$$

$$z_{13} - y_{13} \geq 0$$

$$z_{31} - y_{31} \geq 0$$

$$z_{23} - y_{23} \geq 0$$

$$z_{32} - y_{32} \geq 0$$

$$x_1 - x_2 \geq \ln(2) = 0.693$$

$$x_1 - x_2 \leq \ln(4) = 1.386$$

$$x_1 - x_3 \geq \ln(4) = 1.386$$

$$x_1 - x_3 \leq \ln(6) = 1.792$$

$$x_1 - x_2 \geq \ln(3) = 1.099$$

$$x_1 - x_2 \leq \ln(5) = 1.609$$

$$x_1 = 0$$

$$z_{ij} \geq 0$$

$$x_i, y_{ij} = \text{unrestricted } i, j = 1, 2, 3.$$

We get $z^* = 0.804$. The second Stage LP model for this interval judgment is given by:

$$\text{Min } z_{max}$$

S.t.

$$z_{12} + z_{13} + z_{23} = 0.804$$

$$x_1 - x_2 - y_{12} = 0.040$$

$$x_2 - x_1 - y_{21} = -0.040$$

$$x_1 - x_3 - y_{13} = 1.59$$

$$x_3 - x_1 - y_{31} = -1.59$$

$$x_2 - x_3 - y_{23} = 1.354$$

$$x_3 - x_2 - y_{32} = -1.354$$

$$z_{12} - y_{12} \geq 0$$

$$z_{21} - y_{21} \geq 0$$

$$z_{13} - y_{13} \geq 0$$

$$z_{31} - y_{31} \geq 0$$

$$z_{23} - y_{23} \geq 0$$

$$z_{32} - y_{32} \geq 0$$

$$z_{max} - z_{12} \geq 0$$

$$z_{max} - z_{13} \geq 0$$

$$z_{max} - z_{23} \geq 0$$

$$x_1 - x_2 \geq \ln(2) = 0.693$$

$$x_1 - x_2 \leq \ln(4) = 1.386$$

$$x_1 - x_3 \geq \ln(4) = 1.386$$

$$x_1 - x_3 \leq \ln(6) = 1.792$$

$$x_1 - x_2 \geq \ln(3) = 1.099$$

$$x_1 - x_2 \leq \ln(5) = 1.609$$

$$x_1 = 0$$

$$z_{max} \geq 0$$

$$z_{ij} \geq 0$$

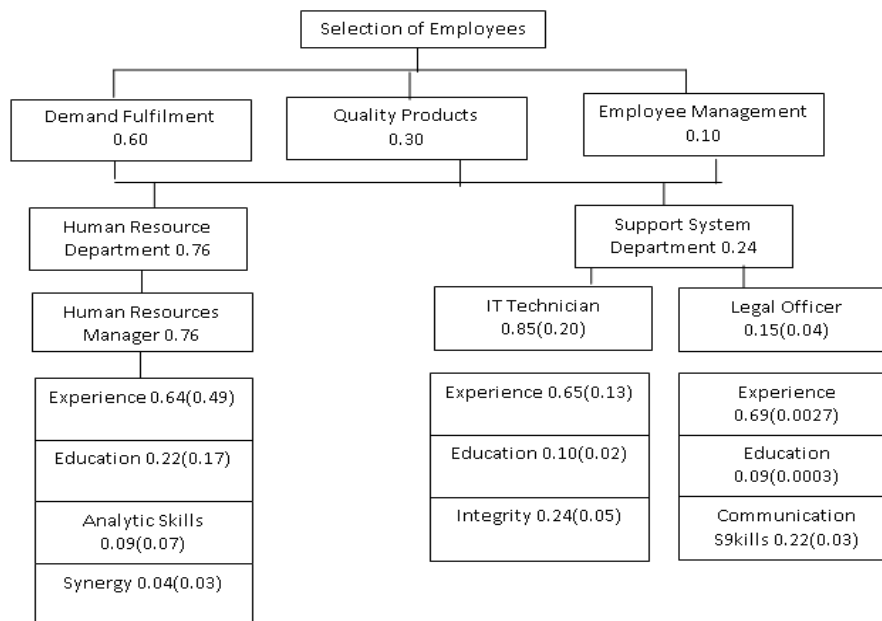
$$z_{ij} \geq 0$$

$$x_i, y_{ij} = \text{unrestricted } i, j = 1, 2, 3.$$

In the second phase $z^* = 0.3469, x_1 = 0, x_2 = -0.693$ and $x_3 = -1.792$.

The weights obtained for this 3×3 matrix are $w_1 = 0.599988, x_2 = 0.300038, x_3 = 0.099974$. These values are for the Purpose of selection in Fig 6 level 2 of the hierarchy.

Fig.1: Tree structure for Human Resources Allocation with Weights.



Intensities and scores of criteria

To implement absolute measurement mode in AHP, in each selection criterion for every post can be found in level 5 of Figure 1. Criteria are further sub-divided into different levels of intensity. These intensities are located at level 6. For example, for evaluation of Human Resources Manager have the following intensities: (i) experience is divided in to three intensities of high (corresponds to 3+ years of experience), medium (1-3 years), and low (less than one year); (ii) education is divided in to master, degree and diploma; (iii) analytical skills into excellent, good and fair; (iv) synergy in to high, medium and low.

The rating for each position is given in Table 5. The priorities of the intensities are derived from pairwise comparisons using LP approach and idealized by dividing each by the largest so that the largest becomes 1 and rest follows proportionally.

Table 3: Intensities for Scores of Criteria

Criteria by job (global priorities)	Intensities and idealized priorities
Human Resources Managers	
Experience(0.49)	High (1) Medium (0.363) Low (0.132)
Education(0.16)	Masters (1) Degree (0.55) Diploma (0.302)
Analytic Skills(0.07)	Excellent (1) Good (0.25) Fair (0.125)
Synergy(0.03)	High (1) Medium (0.573) Low (0.219)
IT Technician	
Experience (0.13)	High (1) Medium (0.363) Low (0.132)
Education (0.02)	Masters (1) Degree (0.55) Diploma (0.302)
Integrity (0.05)	High (1) Medium (0.210) Low (0.088)
Legal Officer	
Experience (0.0027)	High (1) Medium (0.363) Low (0.132)
Education (0.0003)	Masters (1) Degree (0.55) Diploma (0.302)
Communication Skills (0.0009)	High (1) Medium (0.573) Low (0.219)

After conducting a series of the interview on job performance skills, personality traits, communication skills, each candidate was evaluated by a group of experts (minimum 3) according to the posts they applied for and the selection criteria under each post.

Table 4: Rating for potential applicant for the Human Resources Manager position

Applicants	Human Resources Manager			
	Experience	Education	Analytical skill	Synergy
	0.49	0.16	0.07	0.03
x_1	High	Degree	Good	High
x_2	High	Masters	Excellent	High
x_3	Medium	Masters	Good	Medium
x_4	Medium	Degree	Good	Medium

Table 5: Total Scores for Applicants in Human Resources Manager Position

Applicants	Human Resources Manager				Total Score
	Experience	Education	Analytical skill	Synergy	
	0.49	0.16	0.07	0.03	
x_1	1	0.55	0.25	1	0.6255
x_2	1	1	1	1	0.75
x_3	0.363	1	0.25	0.573	0.3725
x_4	0.363	0.55	0.25	0.573	0.3005

Similarly, we can find out total score for each applicant for the post of IT technician and legal officer.

Manpower allocation

Two linear models are presented in this section for the best Human Resource Allocation.

Model 1: Optimization for individual applicants

The objective function coefficients are the total scores given in Table 3, 4 and 5. The decision variables x_1, x_2, \dots, x_{12} are binary, subject to salary constraint, upper and lower bound constraints on positions are given by this model:

$$\begin{aligned} \text{Max } & 0.625x_1 + 0.75x_2 + 0.372x_3 + 0.30x_4 + 0.2x_5 + 0.16x_6 + 0.071x_7 + 0.068x_8 + 0.0035x_9 \\ & + 0.0039x_{10} + 0.002x_{11} + 0.0037x_{12} \\ \text{S.t. } & 2x_1 + 2x_2 + 2x_3 + 2x_4 + 1.2x_5 + 1.2x_6 + 1.2x_7 + 1.2x_8 + 1.5x_9 + 1.5x_{10} + 1.5x_{11} + 1.5x_{12} \\ & \leq 6 \quad \text{(Salary constraint)} \\ & x_1 + x_2 + x_3 + x_4 = 1 \quad \text{(Human Resources Manager)} \\ & 0 \leq x_5 + x_6 + x_7 + x_8 = 1 \quad \text{(IT Technician)} \\ & 1 \leq x_9 + x_{10} + x_{11} + x_{12} \leq 2 \quad \text{(Legal Officer)} \\ & x_j, j = 1, 2, \dots, 12 \quad \text{are binary.} \end{aligned}$$

Model 1 was solved using Excel Solver to select x_2 from human resource manager, x_5 from IT technician and x_{10} from the legal officer.

Model 2: Optimizing different positions

In this approach, coefficients of the objective function are the priorities of the three positions given in the fourth level of hierarchy of Fig 1. The positions are denoted as y_1 to y_3 . The previous model did the selection of applicants based on their rating, considering the relative importance of the post, whereas this model determines the optimal number of jobs, and then selects the best applicants for those positions. Coefficients of objective function for model 2 are from fourth level of Fig 1 adjusted by multiplying 1/3 with Human Resources department and 2/3 with Support Systems department with the corresponding weights of the criteria. The model is given below and the results are tabulated in Table 6.

$$\begin{aligned} \text{Max } & 0.253y_1 + 0.16y_2 + 0.026y_3 \\ \text{S.t. } & 2y_1 + 1.2y_2 + 1.5y_3 + \leq 6 \quad \text{salary constraint} \\ & y_1 = 1 \quad \text{Human Resources Manager} \\ & y_2 = 2 \quad \text{IT Technician} \\ & y_3 \geq 1 \quad \text{Legal Officer, Lower bound} \\ & y_3 \leq 2 \quad \text{Legal Officer, Upper bound} \\ & y_j \quad \text{are integers.} \end{aligned}$$

Table 6: The optimal solution of LP Model 2

Variable	Position	Optimal Solution for Model 2	Salary in 000
x_7	Human Resources Manager	1	2.0
x_5	IT Technician	1	1.2
x_{10}	Legal Officer	1	1.5
	Total	3	4.7

IV. Summary and Conclusions

AHP can measure intangibles and LP proves to be effective in optimizing the resource allocation problem by also considering tangible measurements. This paper has used both tangible and intangible measures. After converting intangibles by using the AHP technique, priority has also been calculated using LP interval model. Element dominance and row dominance have been incorporated as constraints in LP. LP has several advantages over the additive normalization or eigenvectors or LSS methods for determining priorities. In Section 3, the case has been presented to fill the vacant post. The relative weight of each factor, subfactor and the ratings of each alternative (applicants) with respect to each subfactor, to give overall ratings calculated. Combined AHP and LP models seem to provide an effective tool.

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Maryam Olfati "Estimating Weights in Analytic Hierarchy Process Using LP and Optimizing Of Human Resource Allocation." IOSR Journal of Business and Management (IOSR-JBM) 20.6 (2018): 50-58.