IOSR Journal of Business and Management (IOSR-JBM)
e-ISSN: 2278-487X, p-ISSN: 2319-7668. Volume 20, Issue 3. Ver. I (March. 2018), PP 23-30
www.iosrjournals.org

Travelling Salesman Problem (TSP) with Mixed Constraints and Multiple Job Facilities at each station

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ABSTRACT: *In this paper, mixed constraints (i.e., precedence and fixed position constraints) are considered in the usual travelling salesman problem with multiple job facilities at each station. The two constraints, i.e., precedence constraint(s) and fixed position constraint(s) should maintain feasibility among themselves. By precedence constraints, one means that the station(s) (cities or links or nodes) are visited (or completed) in such a way that a particular station is to be preceded (completed or visited) by another station and on the other hand, with fixed position(s) constraints, one means that the station(s) are visited in such a way that a particular station(s) is to be visited in a certain specific step(s).*
The aim of the paper is to find a tour of the salesman by using above two constraints in such a way that the total distance traveled is minimum while completing all the 'M' jobs, on the basis of first come first serve. The proposed algorithm is formulated and solved by the lexicographic search approach. It is seen that the time required for the search of the optimal solution is fairly less. Also the algorithm is applied to different order matrices with N = 5, 10, 25, 100, 150 and M = 6, 10, 20, 25, 40 to exhibit its effectiveness.

KEY WORDS: *Travelling Salesman Problem, Precedence Constraints, Fixed Position Constraints, Lexicographic Search Approach and Lower Bound.*

Date of Submission: 15-02-2018 Date of acceptance: 03-02-2018

I. Introduction

The importance of restricted contexts like precedence constraints, fixed position constraints, etc. in a routing problem has been very well discussed by Scrogg and Tharp [17]. It occurs frequently in many realistic situations. For example,

- A salesman has a quota of products to sell on a tour and he may want to visit the better prospects early. An early supply to a station may be made to the flood prone areas than the other stations to avoid the difficult situation, it might fall in; a CEO of a company may plan a tour and need to be in a certain station on a particular date, etc.
- During the evacuation of a population in anticipation of a disaster, medical personnel might need to visit some patients before they can be transported to shelters.
- With appropriate databases of the special needs population, an optimal evacuation plan can be created ahead of time, and this optimal plan can be adjusted as needed in the event of an actual disaster.

In this paper, an attempt has made to consider mixed constraints (i.e., precedence and fixed position constraints) in the usual travelling salesman problem with multiple job facilities at each station. By precedence constraints one means that the stations are visited in such a way that a particular station is to be preceded by another specific station. Precedence relation need not be immediate. Let us assume restricted relations are of the type $(a<b)$ or $(a<b<c)$ or $(a,b,c<d)$ etc. and these lead to $(b \rightarrow a=\infty)$, $(c \rightarrow b \rightarrow a=\infty)$, $(d \rightarrow a,b,c=\infty)$, etc. On the other hand by fixed position(s) constraints one means that the station(s) (cities or nodes) are visited in such a way that a particular station(s) is to be visited in a certain specific step(s).

The problem can be described as follows:

There are *'N'* stations to be visited and *'M'* distinct jobs to be performed by a salesman. The distance between each pair of stations and facilities for jobs at each station, are known. The salesman starts from a station (home station denoted as 'A_0') and returns back to it after completing all the jobs on the basis of performing the jobs as early as possible. The objective is to find a tour of the salesman by using the mixed constraints such that the total distance traveled is minimum while completing all the *'M'* jobs, on the basis of first come first serve.

For illustration, a matrix of order (10×10) with 20 jobs and mixed constraints, like precedence and fixed position constraints are considered in the usual travelling salesman problem. The problem is solved by lexicographic search approach, developed by Pandit [3, 4, 5, 13, 14]. The computational results with N = 5, 10, 25, 100, 150 and M = 6, 10, 20, 25, 40 are also calculated.

II. A Brief Review Of Literature

The procedure for solving the travelling salesman problem usually can be divided into three basic parts- a starting point, a solution generation scheme and a termination rule. The termination rule is such that, the iteration is stopped iff, a tour is optimal, and the method is exact. In approximate methods the tour reached at termination generally depends on the starting point, so it is possible to produce many final tours by using different starting points. The best of these final tours is then selected [1, 2].

The pioneers in solving the problem were Dantzig, Fulkerson and Johnson [7], Flood, M.M. [15], Crores, G.A. [10], Little, Murthy, Sweeney and Karel [17] and later on several algorithms have been developed for the solution of the usual travelling salesman problem. These include dynamic programming, integer programming, branch and bound, tour-to-tour approximations and the Gilmore-Gomery method [16] and lexicographic search.

III. Notation And Statement Of The Problem

Let $N =$ Set of stations (nodes) defined by $N = \{A_0, A_1, A_2, \cdots, A_{N-1}\}$; ' A_0 ' being the home station.

$K = \{J_1, J_2, \cdots, J_M\}$, a set of *'M'* jobs to be performed.

$d(A_i, A_j) =$ Distance or cost associated with the node pairs (A_i, A_j) and $(i, j = 0, 1, 2, \cdots, (N-1))$.

A salesman starts from station (A_0, say) and returns to it after completing all the jobs. He completes all the *'M'* either by visiting all the *'N'* stations and at each station only subset of *'M'* jobs can be completed; also he should not visit a station which is already visited. The number *'M'* and *'N'* are positive integers which are not necessarily equal. At home station no job is available and the travel distance from any station to itself is ∞, i.e., $d(A_i, A_i) = \infty$; $\forall\ A_i \in N$.

Moreover, the salesman has to obey mixed constraints involved in the tour. With mixed constraints the objective is to find a tour which completes all the *'M'* jobs and minimizes the total distance travelled.

The distance variables $x(A_i, A_j)$ may be 0 or 1 according as

$$x(A_i, A_j) = \begin{cases} 1, \text{ if the salesman visits station from the station.} \\ 0, \text{ otherwise.} \end{cases}$$

The two constraints, i.e., precedence constraint(s) and fixed position constraint(s) should not be contradictory. In other words, they should maintain feasibility among themselves. The precedence constraints introduced are: $(A_a < A_b)$, $(A_p < A_q)$ and $(A_u < A_v)$; where $(A_a, A_b, A_p, A_q, A_u, A_v \in S)$; $S = \{A_1, A_2, A_3, \cdots, A_n\}$ and the fixed position constraints introduced are: Stations *'x'* and *'y'* are to be visited at k^{th} step and l^{th} step respectively; *k* and *l* are any of $1, 2, \cdots, n$ and $k < l$.

Modify the cost (distance) matrix utilizing the mixed constraints. We reduce the modified cost matrix into an equivalent canonical matrix with elements $d'_{ij} = d_{ij} - \alpha_i - \beta_j$; where, $\alpha_i = \min_j\ d_{ij}$ and $\beta_j = \min_i\ (d_{ij} - \alpha_i)$. The new matrix is non-negative, with at least one zero in each row and each column.

IV. Mathematical Formulation

Mathematically, the problem may be stated as

$$\text{Minimize } Z = \sum_i \sum_j d(A_i, A_j)\ x(A_i, A_j)\ ; \quad (i, j = 0,1,2,\cdots,(N-1))$$

$$= \sum_i \sum_j d_{ij} x_{ij} \qquad \ldots \quad \ldots \quad \ldots \qquad (1)$$

(For simplicity we write $d(A_i, A_j) = d_{ij}$ and $x(A_i, A_j) = x_{ij}$)

subject to

$$\sum_{j=1}^{N-1} x_{0j} = 1\ , \qquad \sum_{j=1}^{N-1} x_{j0} = 1 \qquad \ldots \quad \ldots \quad \ldots \qquad (2)$$

Since the salesman starts from a depot ' A_0 ' and goes back to it.

$$\sum_{i \neq k} x_{ik} = \sum_{k \neq j} x_{kj} = 0, 1 \quad \forall\ k, j \qquad \ldots \quad \ldots \quad \ldots \qquad (3)$$

The unwanted sub-tours are eliminated by lexicographic search procedure.

In addition to the mathematical formulation the precedence constraints introduced are:

$$(A_a < A_b), (A_p < A_q) \text{ and } (A_u < A_v)\text{; where } (A_a, A_b, A_p, A_q, A_u, A_v \in S)\text{; } S = \{A_1, A_2, A_3, \cdots, A_n\}$$

and the fixed position constraints introduced are:

Station(s) '*x*' and '*y*' are to be visited at k^{th} step and l^{th} step, respectively; *k* and *l* are any of $1, 2, \cdots, n$ and $k < l$.

V. Numerical Illustration

For illustration, suppose there are 10 stations $(A_0, A_1, A_2, \cdots, A_9)$ and 20 jobs $(J_1, J_2, \cdots, J_{20})$; each station contains some job facilities, where an arbitrarily chosen distance matrix with job facilities at each station, is given in Table 1. Here the distance and job matrix is symmetric; but it is equally applicable to an asymmetric matrix.

Table 1: Distance and job matrix

Jobs ↓	Station	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
	A_0	∞	6	1	2	3	7	2	14	9	9
J_2, J_9, J_{10}, J_{11}	A_1		∞	17	4	2	12	8	20	13	11
J_5, J_{14}	A_2			∞	9	1	18	16	12	16	18
J_3, J_{13}, J_{17}	A_3				∞	7	16	6	11	13	2
$J_5, J_9, J_{11}, J_{18}, J_{20}$	A_4					∞	7	8	17	19	13
J_3, J_4, J_{15}	A_5						∞	5	12	4	2
J_1, J_9, J_{19}	A_6							∞	10	16	17
J_3, J_7, J_9	A_7								∞	10	11
$J_3, J_6, J_9, J_{11}, J_{16}$	A_8									∞	8
J_8, J_{12}, J_{15}	A_9										∞

Here, we consider the precedence relations and the fixed position relations as:

(i) Precedence relations: $(A_5 < A_7)$, $(A_3 < A_9)$, $(A_1 < A_3)$ and $(A_7 < A_9)$. All these constraints can be simply written as $(A_1 < A_3 < A_9)$ and $(A_5 < A_7 < A_9)$.

(ii) Fixed position relations: The stations A_5, A_7, A_2 and A_9 must be visited at the Step-2, Step-4, Step-5 and Step-7 respectively.

VI. Computational Procedure

6.1. *Part-I: Formation of modified, reduced cost matrices and alphabet table(s)*

Step 1: Modify the cost matrix by assigning $d_{ij} = \infty$ for which the tour between station '*i*' and station '*j*' is not possible, following the mixed constraints.

Step 2: Reduce the modified cost matrix into an equivalent canonical matrix (reduced modified cost matrix) (cf. Table 2) with the elements

$$d'_{ij} = d_{ij} - \alpha_i - \beta_j; \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

where $\alpha_i = \min_j d_{ij}$ and $\beta_j = \min_i (d_{ij} - \alpha_i)$

This new matrix is non-negative with at least one zero in each row and each column (cf. Table 2),

$$\text{i.e.} \quad \sum_{i \neq j} d_{ij} x_{ij} = \sum_{i \neq j} d'_{ij} x_{ij} + \left(\sum \alpha_i + \sum \beta_j = \gamma, \quad \text{a constant} \right) \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The second part of the equation (5) can be used as a bias of the matrix or in other words, this part is a fixed part of any tour.

Step 3: Using the reduced cost matrix, we list under column *i*, $(i = 0, 1, 2, \cdots, (N-1))$ the nodes $\{0, 1, 2, \cdots, (N-1)\}$ in order $J_0^{(i)}, J_1^{(i)}, \cdots, J_r^{(i)}, \cdots, J_s^{(i)}, \cdots, J_{N-1}^{(i)}$ such that $r < s$ if $d_{ijr} \leq d_{ijs}$. The ordering $(J_0, J_1, \cdots, J_{N-1})$ so obtained for a given node *i*, is defined as the alphabetic order in column *i*, the table, thus formed containing all the columns $0, 1, 2, \cdots, (N-1)$, is called the alphabet table (cf. Table 3).

Step 4: Construct another table using the 1st row of the reduced cost matrix, arranging the nodes $J_i^{(0)}$, $(i = 0, 1, 2, \cdots, (N-1))$ such that $r < s$ if $d_{r0} \leq d_{s0}$. The ordering thus obtained gives another alphabet table (cf. least column of Table 3). This arrangement of column matrix is used for lower bound setting. The alphabet table, thus enables us to list the tours in a systematic way such that the values of 'incomplete words' (leaders) at different stages also present a useful hierarchical structure. We can set 'lower bounds' to this incomplete word for quick convergence to the optimal solution. Finally, we obtain an initial trial solution S_n^* with value V_t.

6.2. *Part-II: Lexicographic search*

We start with a trial solution V_t with a sequence *T*.

Step 0 : $k=1$; L_k is a block just entered. Take a_1 as the first available entry.

Step 0a : $k=N$? If yes, go to 6. If no, go to 1.

Step 1 : Compute $V(L_k)$; $V(L_k)\le V_t$? If yes, go to 3 and else go to 1a

Step 1a : $k=1$? If yes, go to 9. If no, go to 2.

Step 2 : Move out of the current L_k and go to the next block of order $k-1$; go to 0.

Step 3 : Is $V(L_k)+d'(S_k)>V_t$? If yes, go to 4. If no, go to 5.

Step 4 : Move to next sub-block of L_{k-1}, with $a_k=\beta$ (say); with this new L_k. Go to 1.

Step 4a : Check for fulfillment of precedence constraints
Is $P(a_k)\subset S_k$ If yes go to 4b If no go to 1.

Step 4b : Check for fulfillment of fixed position constraints
Is $k < l$? If yes go to 5 If no go to 1.

Step 5 : Form $L_{k+1}=(L_k;\ a_{k+1}=r)$ where a_{k+1} is the first sub-block of L_k; put $k=k+1$. Go to 0a.

Step 6 : Check for the completion of all the *M* jobs. All *M* jobs are over ?
If yes, go to 7. If no, go to 1.

Step 7 : $[V(L_k)+d(a_k;A_0)]=V_T\le V_t$? If yes, go to 8. If no, go to 2 with $k=k+1$.

Step 8 : $V_t=V_T$, $L_n^*=T$ set $k=k-1$ and go to 2.

Step 9 : Is the first column of the alphabet table *A*, exhausted? If yes, go to 11. If no, go to 10.

Step 10 : Take the next available entry in L_k; go to 1.

Step 11 : Search is over; current *T* is the optimal sequence with value V_t.

Step 12 : Jobs are assigned in the optimal tour.

Flow chart of the solution procedure (algorithm) has been given in Fig.1 and Fig.2.

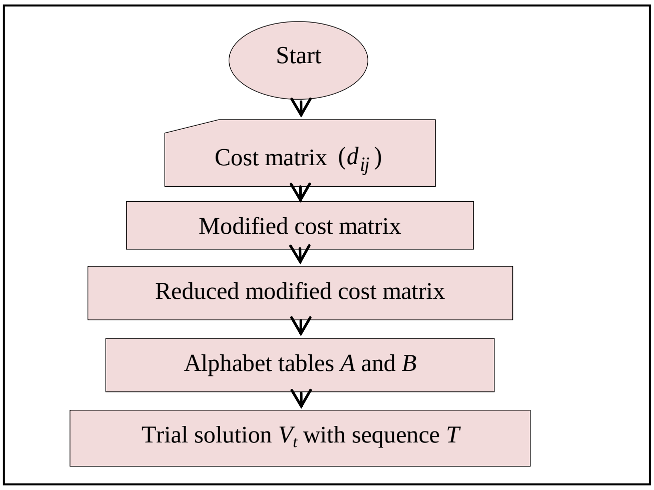

Fig. 1 : Formation of alphabet table and a trial Solution.

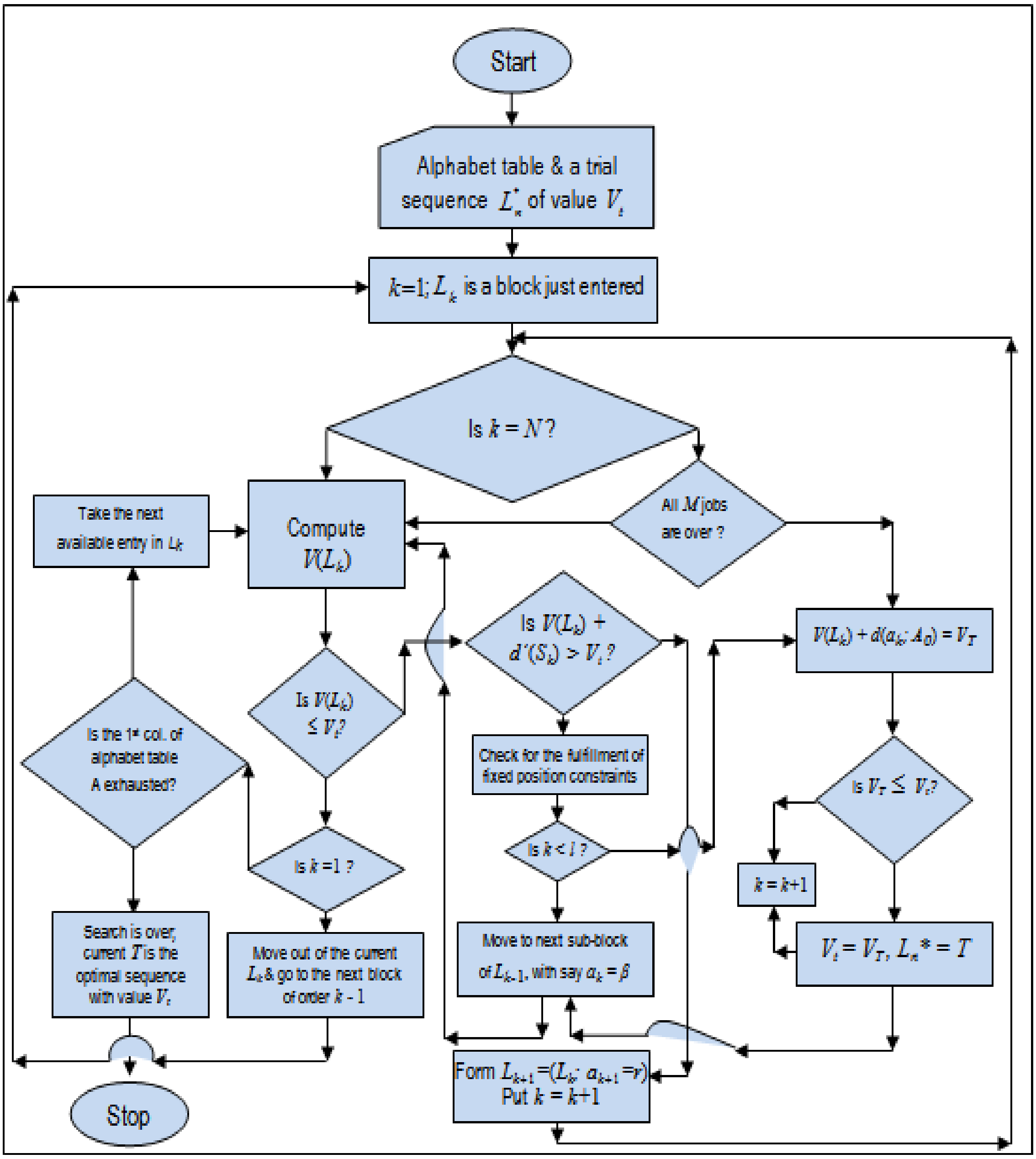

Fig. 2: Lexicographic search

VII. Solution

Due to precedence constraints the restricted and unrestricted stations are:

Restricted Stations: A_1, A_3, A_5, A_7, A_9 and Un-restricted Stations: A_2, A_4, A_6, A_8 .

Due to fixed position constraints the restricted, unrestricted stations with corresponding steps are shown in the following table:

Constraints	Stations (excluding home station)	Step Numbers (excluding last step)
Restricted	A_5 , A_7 , A_2 and A_9	Step-2, Step-4, Step-5 and Step-7 respectively
Un-restricted	A_1, A_3, A_4, A_6, A_8	Any of Step-2, Step-4, Step-7, Step-8 and Step-9

First we have to check whether the precedence relations $(A_1 < A_3 < A_9)$, $(A_5 < A_7 < A_9)$ and fixed position relations (A_5 at the step-2), (A_7 at the step-4), (A_2 at the step-5) and (A_9 at the step-7); are permissible or not. Since the stations A_5, A_7 and A_9 are contained in both the constraints (i.e., precedence and fixed position), so we have to check carefully whether, the constraints are permissible or not. More elaborately, the stations in precedence constraints the relation is $(A_5 < A_7 < A_9)$ and in fixed position constraints the relation is (step $2 \to A_5$) < (step $4 \to A_7$) < (step $9 \to A_9$); therefore, the constraints are consistent and it is solvable to find the minimum distance.

The implications due to precedence and fixed position constraints are:

Due to precedence constraints:					
$A_0 \to A_3 = \infty$ $A_0 \to A_7 = \infty$ $A_0 \to A_9 = \infty$	$A_1 \to A_0 = \infty$ $A_1 \to A_9 = \infty$	$A_3 \to A_0 = \infty$ $A_3 \to A_1 = \infty$	$A_5 \to A_0 = \infty$ $A_5 \to A_9 = \infty$	$A_7 \to A_0 = \infty$ $A_7 \to A_5 = \infty$	$A_9 \to A_1 = \infty$ $A_9 \to A_3 = \infty$ $A_9 \to A_5 = \infty$ $A_9 \to A_7 = \infty$
Due to fixed position constraints:					
$A_0 \to A_2 = \infty$ $A_0 \to A_5 = \infty$ $A_0 \to A_7 = \infty$ $A_0 \to A_9 = \infty$	$A_2 \to A_0 = \infty$ $A_2 \to A_5 = \infty$ $A_2 \to A_7 = \infty$ $A_2 \to A_9 = \infty$	$A_5 \to A_0 = \infty$ $A_5 \to A_2 = \infty$ $A_5 \to A_7 = \infty$ $A_5 \to A_9 = \infty$	$A_9 \to A_0 = \infty$ $A_9 \to A_2 = \infty$ $A_9 \to A_5 = \infty$ $A_9 \to A_7 = \infty$	$A_7 \to A_0 = \infty$ $A_7 \to A_5 = \infty$ $A_7 \to A_9 = \infty$	As from A_7 salesman has to visit immediately to A_2, therefore $A_7 \to A_1 = \infty$ $A_7 \to A_3 = \infty$ $A_7 \to A_4 = \infty$ $A_7 \to A_6 = \infty$ $A_7 \to A_8 = \infty$

Table 2: Modified reduced distance matrix

Station	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	α_i
A_0	∞	3	∞	∞	1	∞	0	∞	7	∞	2
A_1	∞	∞	15	0	0	10	6	12	11	∞	2
A_2	∞	15	∞	6	0	∞	15	∞	15	∞	1
A_3	∞	∞	7	∞	5	14	4	3	11	0	2
A_4	2	0	0	4	∞	6	7	10	18	12	1
A_5	∞	7	∞	10	3	∞	1	∞	0	∞	4
A_6	0	5	14	2	6	3	∞	2	14	15	2
A_7	∞	∞	0	∞	∞	∞	∞	∞	∞	∞	12
A_8	5	8	12	7	15	0	12	0	∞	4	4
A_9	∞	∞	∞	∞	5	∞	9	∞	0	∞	8
β_j	0	1	0	2	0	0	0	6	0	0	9 + 38 =47

Table 3: Alphabet table

$A_0 \downarrow$	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	LB ($\to A_0$)
Predecessor $\to$			A_1				A_5		$A_1, A_3,$ A_5, A_7	
Fixed Position $\to$		Step-5			Step-2		Step-4		Step-7	
$A_6 - 0$	$A_3 - 0$	$A_4 - 0$	$A_9 - 0$	$A_1 - 0$	$A_8 - 0$	$A_0 - 0$	$A_2 - 0$	$A_5 - 0$	$A_8 - 0$	$A_6 - 0$
$A_4 - 1$	$A_4 - 0$	$A_3 - 6$	$A_7 - 3$	$A_2 - 0$	$A_6 - 1$	$A_3 - 2$	$A_0 - \infty$	$A_7 - 0$	$A_4 - 5$	$A_4 - 2$
$A_1 - 3$	$A_6 - 6$	$A_1 - 15$	$A_6 - 4$	$A_0 - 2$	$A_4 - 3$	$A_7 - 2$	$A_1 - \infty$	$A_9 - 4$	$A_6 - 9$	$A_8 - 5$
$A_8 - 7$	$A_5 - 10$	$A_6 - 16$	$A_4 - 5$	$A_3 - 4$	$A_1 - 7$	$A_5 - 3$	$A_3 - \infty$	$A_0 - 5$	$A_0 - \infty$	$A_1 - \infty$
$A_2 - \infty$	$A_8 - 11$	$A_8 - 15$	$A_2 - 7$	$A_5 - 6$	$A_3 - 10$	$A_1 - 5$	$A_4 - \infty$	$A_3 - 7$	$A_1 - \infty$	$A_2 - \infty$
$A_3 - \infty$	$A_7 - 12$	$A_0 - \infty$	$A_8 - 11$	$A_6 - 7$	$A_0 - \infty$	$A_4 - 6$	$A_5 - \infty$	$A_1 - 8$	$A_2 - \infty$	$A_3 - \infty$
$A_5 - \infty$	$A_2 - 15$	$A_5 - \infty$	$A_5 - 14$	$A_7 - 10$	$A_2 - \infty$	$A_2 - 14$	$A_6 - \infty$	$A_2 - 12$	$A_3 - \infty$	$A_5 - \infty$
$A_7 - \infty$	$A_0 - \infty$	$A_7 - \infty$	$A_0 - \infty$	$A_9 - 12$	$A_7 - \infty$	$A_8 - 14$	$A_8 - \infty$	$A_6 - 12$	$A_5 - \infty$	$A_7 - \infty$
$A_9 - \infty$	$A_9 - \infty$	$A_9 - \infty$	$A_1 - \infty$	$A_8 - 18$	$A_9 - \infty$	$A_9 - 15$	$A_9 - \infty$	$A_4 - 15$	$A_7 - \infty$	$A_9 - \infty$

Table 4: Lexicographic search table (partial)

Route No.↓	A_0↓	Precedence Relations $(A_1 < A_3 < A_9)$ and $(A_5 < A_7 < A_9)$									Remark
Fixed Position→	Step-1	Step-2 A_5	Step-3	Step-4 A_7	Step-5 A_2	Step-6	Step-7 A_9	Step-8	Step-9	Step-10	
1	$A_6\,\overline{(0)}\,0$	$A_5\,\overline{(3)}\,3$	$A_8\,\overline{(3)}\,0$	$A_7\,\overline{(3)}\,0$	$A_2\,\overline{(3)}\,0$	$A_4\,\overline{(3)}\,0$	$A_9\,\overline{(15)}\,12$				NF
							×				
2						$A_1\,\overline{(18)}\,15$					NF
						×					
					×						
				×							
3			$A_4\,\overline{(6)}\,3$	$A_7\,\overline{(16)}\,10$	$A_2\,\overline{(16)}\,0$	$A_1\,\overline{(31)}\,15$					NF
						×					
					×						
				×							
4			$A_1\,\overline{(10)}\,7$	$A_7\,\overline{(22)}\,12$	$A_2\,\overline{(22)}\,0$	$A_4\,\overline{(22)}\,0$					NF
5						$A_3\,\overline{(28)}\,6$	$A_9\,\overline{(28)}\,0$	$A_8\,\overline{(28)}\,0$	$A_4\,\overline{(43)}\,15$	$A_0\,\overline{(45)}\,2$	$45 = V_t$
Contd.											
10	$A_4\,\overline{(1)+0}\,1$	$A_5\,\overline{(7)+0}\,6$	$A_1\,\overline{(14)+0}\,7$	$A_7\,\overline{(26)+0}\,12$	$A_2\,\overline{(26)+0}\,0$	$A_3\,\overline{(32)+0}\,6$	$A_9\,\overline{(32)+0}\,0$	$A_8\,\overline{(32)+0}\,0$	$A_6\,\overline{(44)+0}\,12$	$A_0\,\overline{(44)}\,0$	$44 = V_t$
Contd.											
12	$A_1\,\overline{(3)+0}\,3$	$A_5\,\overline{(13)+0}\,10$	$A_8\,\overline{(13)+0}\,0$	$A_7\,\overline{(13)+0}\,0$	$A_2\,\overline{(13)+0}\,0$	$A_4\,\overline{(13)+0}\,0$					NF
13						$A_3\,\overline{(19)+0}\,6$	$A_9\,\overline{(19)+0}\,0$	$A_4\,\overline{(24)+0}\,5$	$A_6\,\overline{(31)+0}\,7$	$A_0\,\overline{(31)}\,0$	$31 = V_t$
Contd.											
25	$A_8\,\overline{(7)+0}\,7$	$A_5\,\overline{(7)+0}\,0$	$A_1\,\overline{(14)+0}\,7$	$A_7\,\overline{(26)+0}\,12$	$A_2\,\overline{(26)+0}\,0$	$A_4\,\overline{(26)+0}\,0$					NF
						×					
					×						
				×							
			×								
		×									
	×										

The Search is complete.

Note: 'NF' means 'not feasible' i.e., the leader is not permissible, i.e., due to the precedence constraints, one or more station(s), $(A_1$ or A_3 or $A_7)$ will remain unvisited.

The optimal path occurs at route number 12 (cf. Table 4) and is:

$$A_0 \rightarrow A_1 \rightarrow A_5 \rightarrow A_8 \rightarrow A_7 \rightarrow A_2 \rightarrow A_3 \rightarrow A_9 \rightarrow A_4 \rightarrow A_6 \rightarrow A_0$$

Therefore, the value of the path =31 + γ =31 +47 =78 units.

When the salesman starts from home station to perform his jobs, during his tour, we assume that the salesman has to perform his jobs as early as possible (i.e., on the first come first served basis), from the visit of first station onwards. The job performance table for the path is:

Table 5: Optimum solution

Station	Jobs available	Jobs to be performed	No. Jobs to be performed	Distance
A_0	-	-	-	-
A_1	J_2, J_9, J_{10}, J_{11}	J_2, J_9, J_{10}, J_{11}	4	6
A_5	J_3, J_4, J_{15}	J_3, J_4, J_{15}	3	12
A_8	$J_3, J_6, J_9, J_{11}, J_{16}$	J_6, J_{16}	2	4
A_7	J_3, J_7, J_9	J_7	1	10
A_2	J_5, J_{14}	J_5, J_{14}	2	12

A_3	J_3, J_{13}, J_{17}	J_{13}, J_{17}	2	9
A_9	J_8, J_{12}, J_{15}	J_8, J_{12}	2	2
A_4	$J_5, J_9, J_{11}, J_{18}, J_{20}$	J_{18}, J_{20}	2	13
A_6	J_1, J_9, J_{19}	J_1, J_{19}	2	8
A_0	-	-	-	2
Total	-	-	20	78

VIII. Computational Experience

A computer program of the algorithm has been developed in C languages and is tested on the system HP COMPAQ dx2280 and Intel Pentium D Processors. Random numbers are used to construct the cost matrix. The following table (Table: 6) gives the list of the problems tried along with the average CPU run time (in seconds) for solving them.

Table 6: CPU run time (in seconds)

Serial number	Number of stations	Number of jobs	No. of problems tried in the respective dimensions	Average CPU run time (in sec.)	
				Alphabet Table	Search Table
1	5	8	6	0.00000	0.0000
2	10	15	6	0.05494	0.0437
3	15	15	6	0.08932	0.3126
4	20	20	6	0.10989	1.2349
5	25	25	6	0.10989	1.9438
6	50	40	6	1.54920	3.4250

IX. Conclusion

For efficiency of the proposed algorithm, a large number of problems are tested and it is found that the algorithm is workable in all the cases. Also, it is observed that the time required for the search of the optimal solution is fairly less.

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Dr. Nazimuddin Ahmed "Travelling Salesman Problem (TSP) with Mixed Constraints and Multiple Job Facilities at each station." IOSR Journal of Business and Management (IOSR-JBM) 20.3 (2018): 23-30.