

## Plasmapause and Increase in Whistler Mode Wave's Growth

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**Abstract:** Singh et al have studied in detail the Whistler mode VLF waves intensity enhancement, by assuming change in  $V_{II}$ , volume of flux tube,  $E_{II}$  change due to a change of position of plasmapause. Enhancing this work, we in place of energy increase, consider the wave growth increase of whistler mode waves. The results are in agreement with reported observations and suggest that plasmapause plays an important role in wave propagation. It is shown that this guidance is possible at both the inner and outer edges of the Plasmapause and that more efficient guide occurs as the Plasmapause gradients become stronger. In the case of strong gradients, waves coming from a wide latitudes range ( $\sim 8^\circ$ ) are focused tightly about the plasmapause field lines, resulting in a wave intensity increase of about 3dB near the magnetic equator plane<sup>12</sup>. Since plasmapause has inner & outer edges, we can say that it has a width ( $\Delta_{pp}$ ) which depends upon  $Kp_{max}$  values, MLT as well as  $L$  the McIlwain parameters.  $\Delta_{pp}$  variation is discussed in next section.

**Keywords:** Wave Propagation, Plasmapause, Plasma instability, Whistler mode waves.  
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### I. Introduction

Singh et al<sup>1</sup> by using flux tube volume, studied intensity enhancement of VLF waves due to Plasmapause. Guidance of whistler mode waves by discrete columns of enhanced ionization in the magnetosphere has long been known<sup>2</sup>. These ionization columns are called ducts and those ducts guide whistlers from one hemisphere to the other along the geomagnetic field lines<sup>3, 4</sup>. The existence of such ducts has been established by a large number of indirect measurement of natural as well as manmade signals<sup>5, 6</sup>. Satellite observations have also shown the presence of such ducts<sup>3, 4</sup>. The ray theory for guidance of whistler mode signals an along field aligned columns of enhanced or depressed ionization is well developed<sup>3-6</sup>. In this case propagation wave normals are approximately field aligned and fall generally in the transmission cone of the lower ionosphere boundary. This is why those signals have high probability for ground observation at high/middle latitudes. One more useful and important feature of these signals is echoing. VLF waves propagating along the field line are reflected from the ionosphere or the ground and bounce back and forth between the two hemispheres in the same duct<sup>2</sup>.

Plasmapause was first indicated by Whistler measurements<sup>7-8</sup> with details of relative plasmashere show an abrupt decrease in plasma density over the equator at a distance of  $4R_o$  ( $L=4$ ). The decrease was approximately 100 times in a distance of only about  $0.15R_o$ . The position of this decrease varies with magnetic activity such that ( $R_o$  being earth radius of 6372 Km).

Plasmapause position  $L_{pp}=5.6-0.46 kp_{max}$ . (1)

Recently after analyzing the ISEE data Carpenter and Park, Carpenter and Anderson<sup>10</sup> and Park et al<sup>11</sup> have shown that plasmapause can have position anywhere between  $L=2.0-7.8$  depending upon  $Kp_{max}$ . Inan and Bell<sup>12</sup> have shown that the plasmapause can act as a one sided duct to guide whistler mode signals along the earth's field lines in a manner similar to normal ducted propagation. In some matters this guidance is analogues to the Whispering gallery mode as discussed by many workers earlier<sup>13, 14</sup>. Plasmapause guidance of VLF waves has also been addressed previously in the literature<sup>15</sup>. Walter and Scarabuccl<sup>15</sup> discussed that VLF energy could be guided along plasmapause boundary and give rise to the type of whistlers known as knee whistler<sup>5</sup>. Plasmapause has also been shown to be able to guide waves in VLF range<sup>16</sup>. Aikyo and Ondoh<sup>17</sup> have shown that plasmapause gradients could act to guide VLF hiss along the plasmapause from geomagnetic equatorial plane to the ionosphere. Inan and Bell<sup>1</sup> showed that the plasmapause guidance is a form of gradient trapping of VLF wave energy and guiding is possible at both the inner and outer edge of the plasmapause. It was observed that efficient guiding occurs as the gradient becomes stronger.

Their study indicate that the vicinity of plasmapause represents a natural and readily accessible region of VLF wave guidance and focusing where both passive and active VLF experiments can be studied through ground as well as in –situ measurements.

During wave particle interactions taking place in the plasmasphere/magnetosphere wave growth/energy/diffusion of energetic electrons takes place. In this case either the waves are amplified or absorbed. In this case either the waves are amplified or absorbed. In this paper we study behavior of plasmapause as a source of VLF wave growth of interaction between waves of 3, 4, 5KHz at different L- shells.

## II. Method Of Calculation And Ionospheric Model

Though plasmapause can have location anywhere between L=2 and L=7.8 (L is McIlwain parameter) we study plasmapause amplification at L=3 to L=6 (corresponding to geomagnetic latitude  $\Delta$  between 55°-62° as intense VLF events with good occurrence rates are observed mostly within this latitude range. The general resonant condition between electrons and whistler mode is given by [13-19]

$$r^2 - 1 = (f_{He}/f_{pe})^2 (f_{he}/f)(1 + f_{hp}/f) \quad (2)$$

where  $f_{He}$ =electrons gyrofrequency

$$873.6/L^3 \text{ (KHz)} \quad (3)$$

$f_{pe}$ = Electron Plasma frequency(kHz)

$$= \sqrt{80.63N} \text{ (electron/cc)} \quad (4)$$

F=interacting frequency in KHz

$f_{Hp}$ = proton's gyrofrequency

$$= 0.478/L^3 \text{ (KHz)} \quad (5)$$

The cyclotron mode resonance condition is

$$= 2\pi(f\mu - f) \quad (6)$$

$$KC = 2\pi f \cdot \mu \quad (7)$$

$$\text{So, } V_{\parallel}/C = f_H - f/f \cdot \mu \quad (8)$$

Where  $\mu$  is refractive index expressed as

$$\mu^2 = f_p^2 / f (f_{He} - f) \quad (9)$$

where  $\omega$  is wave frequency,  $K_{\parallel}$  is the parallel component of propagation vector  $V_{\parallel}$ , the parallel resonant velocity,  $n$  is the order of cyclotron mode,  $\Omega_{He}$ , electron gyrofrequency and  $\gamma$  is the relativistic factor  $\gamma$  is calculated after using the expression given by Tsurutani et al<sup>19</sup>[see eqn. 2] Under resonant conditions, energy exchange can occur between the wave and electrons and energetic electrons are constraint to diffuse along resonant diffusion surfaces which are described by

$$V = V \sin \alpha \quad (10)$$

$$V^2 + (V_{\parallel} - V_{ph})^2 = 2E^*/m \quad (11)$$

Where  $V$  is the electron velocity perpendicular to the field line,  $V_{ph}$  is the phase velocity ( $=c/\mu$  where  $c$  is the velocity of light and  $\mu$  is the refractive index of the medium expressed in the equation 9 and  $E^*$  is the scaling energy which is conserved during the interaction  $m$  is the mass of electron. The parallel resonant energy of the electron is computed from the following expression.

$$E_{\parallel} = 511(\gamma - 1) \text{ keV} \quad (12)$$

Or

$$E_{\parallel} \text{ (Kev)} = 250(V_{\parallel}/c)^2 \quad (13)$$

Wave growth  $\gamma$  is computed using following expression of kennel & Petschek<sup>21, 22</sup>  
Energy reduction of an electron from high pitch angle can be express as<sup>20</sup>

$$(\alpha = 90^\circ)/E(\alpha = 0^\circ) = (1 + E_m/E^*)^2 \quad (14)$$

The expression clearly shows that for good wave amplification it is necessary that  $E_m > E^*$ .  $E^*$  is obtained when  $V_{\parallel} \rightarrow 0$  in equation (11). Thus  $E^*$  and  $E_m$  (the magnetic energy per particle) are computed from following formula<sup>20, 21</sup>.

$$E^* = \frac{m}{2} [V_{\parallel}^2 \tan^2 \alpha_0 + (c/\mu)^2] \quad (15)$$

$$E_m = B^2/2 \mu_0 N \quad (16)$$

Here B is geomagnetic field strength (in Tesla),  $\mu_0$  permeability of free space ( $=4\pi \cdot 10^{-7}$  unit) and N is electron number density in per cubic meter.

The electron density values used in this study are taken from diffusive equilibrium ionospheric model given recently by Carpenter and Anderson<sup>10</sup>, after analyzing ISEE data. Table 1 depicts variation of electron density with L parameter. Interacting wave frequency values are 3, 4, 5 kHz in our case.

Wave growth  $\gamma$  is computed using following expressions of Kennels & Petschek<sup>21</sup>.

$$\gamma = 2\pi^2 \eta \cdot A \cdot f_{He} \tag{17}$$

Where  $\eta$  = no. of hot electron/cold electrons

$$= N(\text{hot})/N(\text{Cold}) \tag{18}$$

A = pitch angle anisotropy

$$= 1/2 \ln(\text{cosec } \alpha_0) \tag{19}$$

Where  $\alpha_0$  = Half loss cone pitch angle

The gain in wave intensity can be computed<sup>22</sup> using the expression

$$\text{Gain (dB)} = 10 \log e^G \tag{20}$$

$$\text{Where } G = 2 \cdot \gamma \cdot L \cdot R_e / V_g \tag{21}$$

Here  $R_e$  is the radius of earth (6379km) and

$V_g$  is the group velocity given by

$$V_g = 2 \cdot c \cdot (f_{He}/f_{pe}) \cdot (f/f_{He})^{1/2} \cdot (1 - f/f_{He})^{3/2} \tag{22}$$

$\alpha_0$  values do not depend upon frequency. For a mirror height of 100-140 km its values do not change significantly and are  $11.63^\circ$ ,  $8.62^\circ$ ,  $5.5^\circ$ , and  $4.58^\circ$ ,  $3.87^\circ$  at  $L=2.5, 3.0, 4.0, 4.5$  respectively<sup>23</sup>.

### III. TABLES

Table 1: Variation of electron density in  $\text{elect./cm}^3$  with L as well as  $L_p$  (Plasmapause position) used in the study.

L	N(No plasmapause)	N(plasmapause)
2.5	1500	150
3	900	90
4	500	50
4.5	300	30
5	200	20

Table 2: Values of  $E_{||}$ ,  $E_m$ ,  $E^*$  (all in keV) for 3kHz. Bracketted values refer to plasmapause position.

L	$E_{  }$	$E_m$	$E^*$
2.5	4.23 (24.23)	6.69 (23.60)	0.56 (19.06)
3	3.91 (21.42)	3.74 (20.59)	0.37 (17.23)
4	2.56 (15.02)	1.19 (13.41)	0.22 (10.61)
4.5	2.42 (13.65)	0.98 (11.92)	0.13 (9.18)

5	2.39 (11.69)	0.78 (10.52)	0.07 (8.02)
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Table 3: Value of  $E_{\parallel}$ ,  $E_m$ ,  $E^*$ (Kev) at different frequencies for  $L = 4$  values corresponding to plasmapause position.

<b>f(KHz)</b>	<b><math>E_{\parallel}</math></b>	<b><math>E_m</math></b>	<b><math>E^*</math></b>
3	2.56 (49.21)	1.98 (14.97)	0.28 (2.94)
4	1.43 (35.42)	1.98 (14.97)	0.26 (3.12)
5	0.83 (24.94)	1.98 (14.97)	0.23 (3.64)

Table 4: Gain (dB) for 3 KHz at  $L=2.5, 3, 4, 4.5, 5$

<b>L</b>	<b>No pp</b>	<b>pp</b>
2.5	75.54	7.54
3.0	73.23	7.32
4	69.02	6.90
4.5	66.86	6.68
5	65.34	6.53

Table 5:- Gain (dB) for different frequencies at  $L=4$

<b>f in KHz</b>	<b>pp</b>	<b>No (pp)</b>
3	69.02	6.90
4	68.63	6.86
5	68.39	6.83

#### IV. CONCLUSION

We calculate  $V_{\parallel}$ ,  $E^*$ ,  $E_m$  etc. at a given  $L$  position. Then this position is considered to be plasmapause position. Since in case of a plasmapause position, electron densities suddenly decrease/drops, we further compute above discussed parameters for reduced number density. We drop the cold plasma density by a factor of 10. Carpenter and Anderson<sup>10</sup> have shown that plasmapause electrons density can vary between 10%-50% of quiet time values. In all cases we find that  $E_m > E^*$  a favorable condition for adequate wave amplification through cyclotron mode resonance. Table 2 shows  $E_{\parallel}$  and  $E_m$  values at different  $L$  for 3 KHz frequency. These parameters go on decreasing with increase in  $L$ . Values of these parameters are higher when a location is a plasmapause position than at non plasmapause position. Table 3 shows values at  $L=4$  for all wave frequencies. It is evident from the table that  $E_{\parallel}$ ,  $E^*$  decrease with increase in frequency ( $f$ ) and we get higher values for  $L=L_{pp}$  in comparison to normally found ones and satisfying the condition of wave amplitude.

Inan and Bell<sup>12</sup> have shown that due to plasmapause guiding, wave intensity is increased by 3dB. we have adopted the formulation of Throne and Horne<sup>10</sup> and their energy transfer ratio, eq.(4), is quite a high value<sup>20</sup> because they take high pitch angle to be  $\approx 0^\circ$  whereas Inan<sup>24</sup> has shown that electron releases maximum energy at  $\alpha=60-65^\circ$  and minimum pitch angle should have been  $\alpha(\text{min.})-\alpha_0$  where  $\alpha_0$  is angle of loss cone (Ref.24-26). Such considerations may bring down the value of 6dB to 3dB easily.

Denby et al<sup>27</sup> analyzed the Ariel 3/4 satellite observations of the GBR (16KHz) and NAA (17.8KHz) transmitters above the ionosphere in the conjugate hemisphere. They found that a plasmapause guide both GBR and NAA signals and guiding efficiency depends upon the location of plasmapause position. For 16 kHz signals, the efficiency of guiding falls for  $L_{pp} > 3.0$  and guiding effect ceased at  $L_{pp} > 3.5$ . It is clear from above that for plasmapause guiding  $L_{pp}$  is beneficial when  $f \leq f_H/2$  and as  $f_H$  increases, efficiency at given  $L_{pp}$  decreases i.e. focusing and power gain will go on decreasing if frequency is increased and gain will be more if  $L_{pp}$  is less. It is to be emphasized here that in our case energy gain of 3dB (average) for VLF waves propagating through plasmapause are due to energy transfer by energetic electrons to the waves, whereas in Inan and Bell<sup>12</sup>, energy gain is due to focusing. Under plasmapause condition (or actually disturbed conditions), the parallel resonant energy in Table 1 and 2 are larger at every value, than the values under normal conditions. Recroft<sup>28</sup> have shown

that under disturbed conditions, the energy of energetic electrons in magnetosphere is considerably enhanced. They have also shown the density of thermal plasma plays a crucial role in determining the energy of gyroresonant electrons as is evident in this paper. Radiation belt electrons (1keV-1MeV) can become precipitated out when they drift through the plasmapause<sup>28</sup>.

Whereas plasmapause density gradients in the magnetosphere act on the wave to force it to propagate along the field line, another kind of density gradients (called negative latitudinal gradients) play an important role in observing VLF waves at ground. Singh<sup>29</sup> has shown that negative latitudinal gradients are not only useful for non-conducted waves but ducted VLF signals as well. Since ducts terminate at an altitude of 1000km above earth's surface<sup>3,4</sup>.

Authors presented a theoretical model which explains the problem of how whistlers are observed on the ground at low latitude in the absence of suitable ducts in the ionosphere this model too exhibit the physics of time reversal but latitudinal gradients. Even then his work is found to be more relevant so far observation of VLF emissions/whistlers at low latitude ground stations is concerned.

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