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Analytical Solution To The Yukawa's Potential Using Confluence Hypergeometry Functions

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Abstract: *Analytical solution to the Schrodinger equation is of paramount importance in the field of science. In this research we have been able to resolve the Schrodinger equation with the Yukawa's potential into a hypergeometric function and obtain a useful analytical approximate solution.*
Keywords: *Yukawa's potential, Schrodinger equation, Whittaker's confluent hypergeometric function.*

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I. Introduction

Solutions to Schrodinger equation are possible only for some potentials [1-5]. In a more recent researches different authors have develop different analytical solutions for several potentials [6-8]. Many of which are important in the field of science today. Obtaining dynamical solutions to Schrodinger equation are of great interest in physics and chemistry such is the exact solution to the hydrogen atom and harmonic oscillator [10-12]. The Yukawa potential or Static Screened Coulomb Potential (SSCP) [13] is given by:

$$V(x) = V_0 \frac{e^{-x/a}}{x} \quad (1)$$

Where, $V_0 = \text{aZ}$is the fine-structure constant, $Z$is the atomic number, and$a = (137.037)^{-1}$is the screening parameter. This potential is often used to compute bound-state normalizations and energy levels of neutral atoms [14-16] which have been studied over the past years.
In addition, although the Yukawa interaction(equation 1) is an idealized one, it nonetheless forms a reasonable basis for the description of physical phenomenasuch as the photoelectric disintegration of the deuteron and the so-called deuteron-stripping process [17].

II. Materials And Methods

In this research work we set out to solve the Schrodinger equation using an approximate method by resolving the equation into a hypergeometric function.The applications of this novel treatment to solving Schrodinger equation, have been proven a success. With the successful application of this method and bearing in mind the significance of a reliable algebraic solution for Yukawa-type potentials, that is clearly reported, we demonstrate here how such interaction potentials can be simply treated within the framework of the present formalism.Given the Schrodinger equation in the x-direction as shown in equation (2).

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x) \qquad (2)$$

Then it could be resolve that

$$\frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}V(x)\psi(x) = -k^2\psi(x) \qquad (3)$$

Where,

$$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad (4)$$

The potential in (equation 3) can be replaced with the Yukawa's potential (equation 1) which is given by:

$$V(x) = V_0 \frac{e^{-x/a}}{x}$$

At large 'a' this reduces to

$$V(x) = \frac{V_0}{x} \qquad (5)$$

Putting the resolve Yukawa's potential into equation (3) we get:

$$\frac{d^2\psi}{dx^2} - \frac{2mV_0}{\hbar^2 x}\psi(x) = -k^2\psi(x)$$

$$\frac{d^2\psi}{dx^2} - \frac{\phi^2}{x}\psi(x) = -k^2\psi(x) \qquad (6)$$

Where, $\phi^2 = \frac{2mV_0}{\hbar^2}$ (7)

Equation (7) can be written as:

$$\frac{d^2\psi}{dx^2} + \left(k^2 - \frac{\phi^2}{x}\right)\psi(x) = 0 \qquad (8)$$

We the subjected equation (8) to the Wolfram Mathematica 9.0 program. The solution to the equation is give in hypergeometric function as:

$$\psi(x) = e^{x\left[-\sqrt{-k^2}+\sqrt{k^2}\right]-x\sqrt{k^2}} xC_1F_{(a,b,c)} + e^{x\left[-\sqrt{-k^2}+\sqrt{k^2}\right]-x\sqrt{k^2}} xC_2F_{(a,b,c)} \quad (9)$$

Where, C_1 and C_2 are constant, while $F_{(a,b,c)}$ is the Whittaker's confluent hypergeometric function. Assuming that $C_2 = 0$, equation (9) reduces to:

$$\psi(x) = e^{x\left[-i\sqrt{k^2}\right]} xC_1F_{(a,b,c)} \qquad (10)$$

Equation (10) is the Eigen function to the assumed solution in equation (8)

Where, $a = 1 + \frac{-4\sqrt{k^2} - 2\left(-2\sqrt{k^2} + \phi\right)}{4\sqrt{-k^2}} = 1 - \frac{\phi^2}{2ik}$ (11)

$$b = 2$$

$$c = 2xk \qquad (12)$$

The complex plane real value is gotten given a real equation (10) for the actual plane wave

$$\psi(x) = e^{-ikx} xC_1F_{(a,b,c)} = xC_1F_{(a,b,c)}\cos kx \qquad (13)$$

Using the following transformation of hypergeometric function

$$F_{(n,2,c)} = x^{1-d} F_{(a-d+1,2-d,c)} \quad (14)$$

Setting d = 0

$$F_{(n,2,c)} = xF_{(a-1,2,c)} \qquad (15)$$

Hence we have that

$$\psi(x) = F_{(n,2,c)} C_1 \cos kx \qquad (16)$$

Where, $n = a - 1$

III. Results And Discussion

The Yukawa potential is very deep and the wave function becomes very sharply peaked near the origin. This causes a great deal of difficulty in the numerical solution of the Schrödinger equation, which is reflected in the instability of the wave function thus obtained.TheWhittaker's confluent hypergeometric function for special cases of the confluent hypergeometric function can be express as:

$F_{(a,a+1,c)} = c^{-1}$ (17)

For the special case generated by equation (17) we let n = 1 in equation (15), hence we have:

$$F_{(1,2,c)} = (2xk)^{-1} \qquad (18)$$

The wave function then becomes:

$$\psi(x) = \frac{1}{2xk} xC_1 Coskx = \frac{1}{2k} C_1 Coskx \qquad (19)$$

The normalization constant C_1, in the above equation can be gotten by normalizing the wave function:

$$\frac{C_1}{4k^2}\int_0^{\pi} \cos^2 kx dx = 1$$

$$C_1 = \frac{8k^2}{\pi} \qquad (20)$$

On the condition that k must be an integer.

$$\psi(x) = \frac{4k}{\pi} Cos\alpha x \qquad (21)$$

The Eigen value was then obtained from equation (11).

$$a = 1 - \frac{\phi^2}{2ik}$$

$$k = -\frac{\phi^2}{2in}, \text{ Here n = a-1= 1, 2, 3…} \qquad (22)$$

Recall that, $\phi^2 = \frac{2mV_0}{\hbar^2}$ and $k = \sqrt{\frac{2mE}{\hbar^2}}$

Hence,

$$E = \frac{mV_0^2}{2n^2\hbar^2} \qquad (23)$$

This show that the Eigenvalue depend onV_0.

IV. Conclusion

In this paper, we have been able to infuse the Yukawa's potential into the Schrodinger equation and obtained an approximate value for the Eigen function and the Eigenvalue in accordance with reviewed literature. This was obtained for a large value of 'a'.

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