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Nuclear Structure of Some Even - Even Nuclei in the framework of Three Parameters Rotational Model

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Abstract: *We investigate the nuclear structure of $^{96-98}$Ru using the modified three parameters rotational (TPR) model. In this model we used the origin equation of the nuclei energy without the additional part that used in the old model. This modified model can be applied on the all range of the nuclei mass numbers. To mention few, and as a limitation, we applied this model on the medium mass nuclei. Energy levels of low lying states have been calculated for ground state bands. The results were compared with experimental data and acceptable agreement are obtained. The nuclear moments of inertia were calculated and compared with available experimental data. From the results of calculation it is found that model is suitable for description of nuclear structure of these nuclei, specially the collectivity properties.*

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I. Introduction

A great deal of attention has been paid lately to the theoretical description of the collective states of the ground state bands of doubly even nuclei, due in part to the great success the variable moment of inertia (VMI) model has met with when reproducing the level excitation energies [1] the VMI model of Mariscotti et al. [2] is a phenomenological scheme which was built following the lines of the stretching model of Diamond et al. [3] assuming a different dependence of the moment of inertia on the variational parameter indeed, both models give the excitation energy of the state with angular momentum. There are arguments to justify this; where all nucleons move together as a whole preserving the nuclear volume, we think that this motion are more suitable to generate a collective behavior These considerations gave rise to the so-called collective model in which the individual nucleons do not play any role. It is the nucleus as a whole that determines its dynamics. The two parameters of VMI [4] and its phenomenological equivalent have achieved a permanent status for the representation of the energies of ground-state bands of deformed nuclei below the region of band crossing. It is remarkable that the formalism continues to give reasonable though not nearly as strikingly accurate, fits outside its original domain of definition. Recently, Bonatsos et al. [4] pointed out that a necessary condition on any extended formalism, namely that it agree with the known phenomenology at low angular momentum, required that at least one additional parameter be introduced into the analysis. Motivated by these investigations, we will study the energy levels, rotational frequencies, kinematic moments of inertia, and dynamic moments of inertia by solving the equation of the excitation energy of nuclei under consideration condition, which leads to the minimum of root-mean-square deviation (χ).

II. Mathematical Model

The excitation energy $E(I)$ and the angular momentum (Spin) $\hat{I}$ is expressed as [5]:

$$\hat{I}^2 = I(I+1) = \sum_{n=0}^{\infty} b_n E^n(I), \quad \dots (1)$$

if we restrict to three terms only, then

$$I(I+1) = b_0 + b_1 E(I) + b_2 E^2(I), \quad \dots (2)$$

where the following condition must be fulfilled

$${b_1}^2 > 4 b_0 b_2. \quad \dots (3)$$

The solution of Eq. (2) is given by

$$E(I) = E_0 + a\{\sqrt{[1 + bI(I+1)]} - 1\}. \quad ... (4)$$

Two possible types of nuclear moments of inertia have been suggested which reflect two different aspects of nuclear dynamics, the kinematic moment of inertia θ_1 and the dynamic moment of inertia θ_2, respectively, are given by

$$\theta_1 = \hbar^2 \hat{I}\left(\frac{d}{d\hat{I}}E(I)\right)^{-1}, \quad ...(5)$$

$$\theta_2 = \hbar^2 \left(\frac{d^2}{d\hat{I}^2}E(I)\right)^{-1}. \quad ...(6)$$

In the framework of nuclear collective rotational model, the rotational frequency $\hbar\omega$ of nuclear rotation is defined as the derivative of the energy $E(I)$ with respect to the angular momentum $\hat{I}$

$$\hbar\omega = \frac{d}{d\hat{I}}E(I). \quad ...(7)$$

We can extract the rotational frequency $\hbar\omega$, kinematic (dynamic)-moment of inertia θ_1 (θ_2) by using the experimental intra band $E2$ transition energies [6, 7].

III. Results and Discussion

The even-even Ruthenium isotopes $Ru(Z = 44)$ are one of the most important in the nuclear science. The structural features of atomic nuclei near the presumed doubly magic nucleus have attracted the attention of numerous theoretical and experimental studies in recent years [8, 9]. These studies were emphasized on the interpreting experimental data via different collective models [9]. Study of properties of nuclear excited states gives information about the forces between these nucleons and the behavior of this many particle system, the nucleus can be excited to higher rotational states by pumping energy to the nucleons and excite them to higher orbits. These excitations are due to collective motion of nucleons or single particle excitations. To understand these nuclear excitations and associated aspects of nuclear structural effects many nuclear models were developed over the past few decades. Fortunately, the power of computer permits calculations that use sophisticated approximations to minimize these problems and provide increasingly accurate models for describing nuclear and atomic properties; of which three parameters rotational (TPR) model. The parameters of the model a, b, E_0 and the calculated spin assignment of the ground state band for our selected in Ru are given in Table(1).

Table(1). Parameters of the Three Parameters Rotational (TPR) model for even- even $^{96–98}$Ru isotopes.

Nucleus	Parameters of the Three Parameters Rotational (TPR) model		
	a	b	E_0
^{96}Ru	29.34×10^{-1}	2.882×10^{-2}	6.908×10^{-1}
^{98}Ru	8.888×10^{-1}	2.578×10^{-1}	9.480×10^{-2}

In our calculations, all gamma ray transition energies were assumed from spin $I + 2$ to spin I. For each ground state band the three parameters of the model in question are determined by using a computer simulated search program to reproduce the observed energies, the observed energy levels E produce by parameters of the model $a, b,$ and E_0 the value of parameters of the model which leads to the minimum of root-mean-square deviation (χ), where N is the number of data points involved in the fitting procedure

$$\chi = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[E_i^{exper} - E_i^{theo}\right]^2} \quad ... (8)$$

Table(2). Excitation energies, kinematic moment of inertia, and square of rotational frequency for even- even $^{96–98}$Ru isotopes.

Nucleus	I^π	Energy E_{Levels} (MeV)		Kinematic Moment of Inertia $\frac{2\theta_1}{\hbar^2}$ (MeV^{-1})		Rotational Frequency $(\hbar\omega)^2$ (MeV^2)
		$Exp.$	Cal.	$Exp.$	Cal.	Cal.
^{96}Ru	**2^+**	**0.8326**	**0.9343**	**7.200**	**25.62**	**0.037**
	4^+	**1.5181**	**1.4405**	**20.44**	**29.70**	**0.091**
	6^+	**2.1498**	**2.1188**	**34.81**	**35.17**	**0.136**
	8^+	**2.9504**	**2.9016**	**37.45**	**41.48**	**0.167**
	10^+	**3.8167**	**3.7481**	**43.88**	**48.30**	**0.189**
	12^+	**4.4176**	**4.6348**	**76.54**	**55.45**	**0.203**
	14^+	**5.6798**	**5.5480**	**42.79**	**62.82**	**0.213**
	16^+	**6.4405**	**6.4793**	**81.47**	**70.32**	**0.220**

^{98}Ru	**2^+**	**0.6524**	**0.6245**	**9.200**	**13.93**	**0.124**
	4^+	**1.3978**	**1.4114**	**18.79**	**21.65**	**0.171**
	6^+	**2.2225**	**2.2630**	**26.67**	**30.02**	**0.186**
	8^+	**3.1902**	**3.1374**	**30.99**	**38.60**	**0.193**
	10^+	**4.0008**	**4.0223**	**46.86**	**47.29**	**0.197**
	12^+	**4.9140**	**4.9127**	**50.38**	**56.03**	**0.199**

Moment of inertia. The positive parity yrast levels are connected by a sequence of stretched $E2$ transition energies, which increases smoothly except around the back-bending region. The transition energy ΔE_γ should increase linearly with I for the constant rotor as $\Delta E_\gamma = ((I(I+1))/\theta)$. However it does not increase, but decreases for certain I values. This means that the interaction between two bands is small then one obtains a sudden transition which produce the beak-bending [10], for this purpose we have created a program to calculate rotational frequencies, kinematic and dynamic moments of inertia are examined for nuclei. In Fig. (1) the behavior of the calculated kinematic $\theta_1(I)$ and dynamic $\theta_2(I)$ moment of inertia corresponding to the calculated spin as a function of the rotational frequency $\hbar\omega$ are illustrated. From this figure, one observes that in the vast majority of these ground state bands the dynamic moment of inertia are seen to increase steady with rotational frequency, and the values of $\theta_1(I)$ for all bands are smaller than that the corresponding values of $\theta_2(I)$ for all ranges of frequencies. At the nearly same values of $\theta_1(I)$ and $\theta_2(I)$ can be found spin I_0, rotational frequency $\hbar\omega_0$ and the band-head moment of inertia θ_0 of the band-head for ground state as shown in Table (3).

Table(3). Moment of inertia, and rotational frequency for even- even $^{96-98}$Ru isotopes of the band-head for ground state.

Nucleus	I_0	Band-Head Moment of Inertia $\frac{\theta_0}{\hbar^2}$ (MeV^{-1}) Cal.	Rotational Frequency $\hbar\omega_0$ (MeV) Cal.
^{96}Ru	**2^+**	**11.83**	**0.073**
^{98}Ru	**2^+**	**4.363**	**0.124**

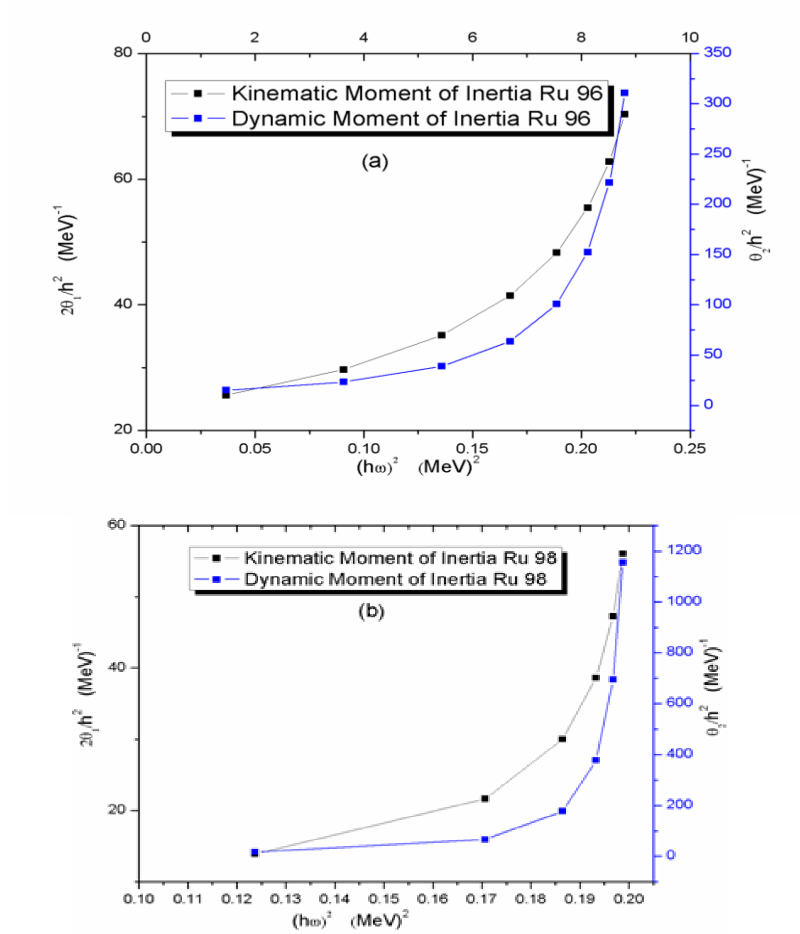

FIG. 1. Comparison between the kinematic moments of inertia and dynamic moments of inertia for (a) the nucleus ^{96}Ru and (b) the nucleus ^{98}Ru.

IV. Summary

To summarize, the three parameters rotational model is used to study the energies of$^{96-98}$Ru without the modified part. This modified model is used to analyze the even-even nuclei of mass region $A = 96 - 98$. The model connects directly the unknown spin and the energy of the level. For each nucleus the model parameters are determined by using a computer simulated search program to reproduce the observed energies. The calculated level spins, transition energies, rotational frequencies, kinematic and dynamic moments of inertia are examined for nuclei depending on my modified model. It is noticed that the comparison between our theoretical calculations and experimental data shows very good agreement and basically justifies our approach.

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